



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

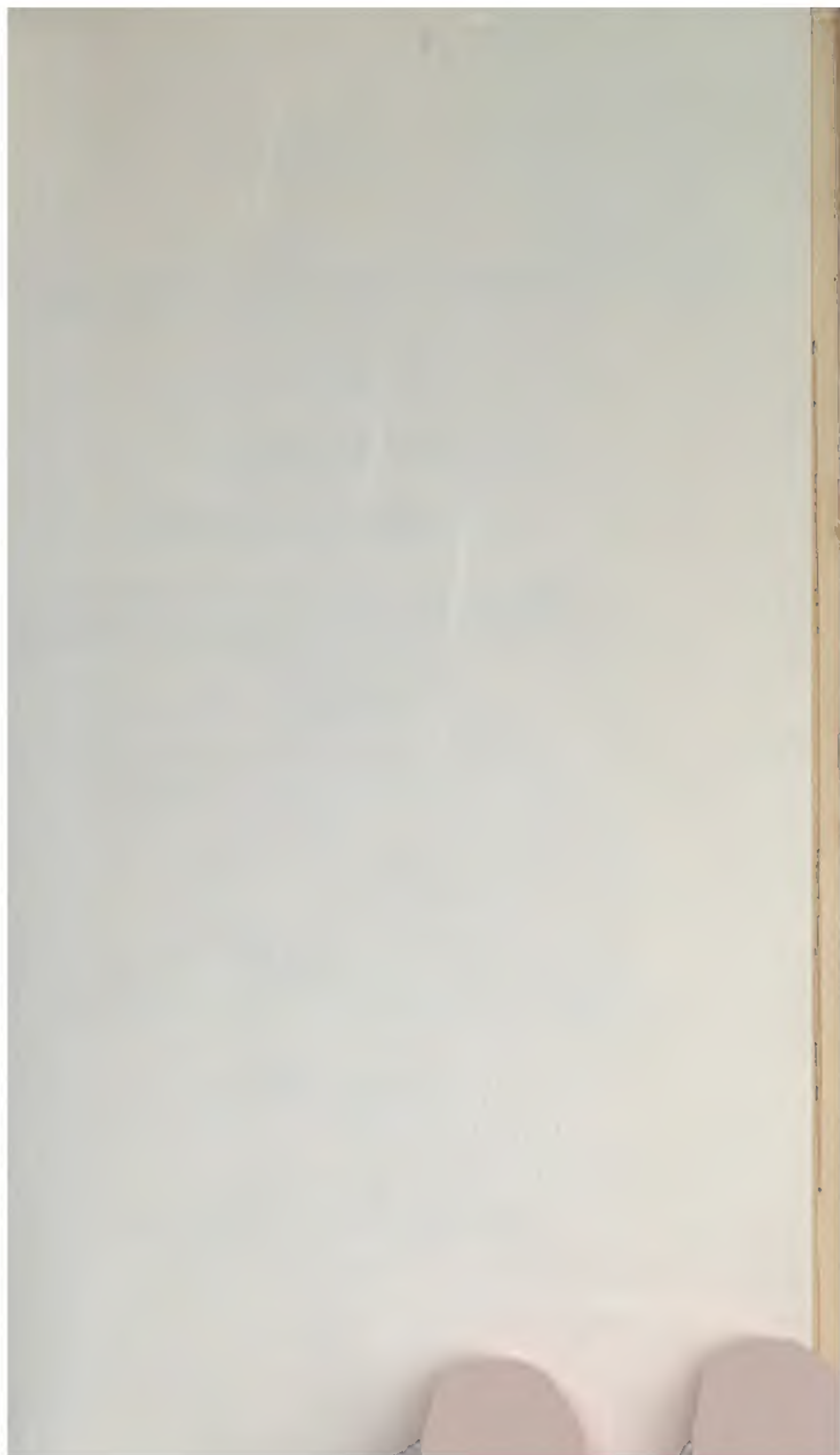
Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>













A TREATISE
ON
THE STRENGTH OF TIMBER,
CAST IRON,
MALLEABLE IRON,
AND OTHER MATERIALS:

WITH RULES FOR APPLICATION IN ARCHITECTURE,
THE CONSTRUCTION OF SUSPENSION BRIDGES, RAILWAYS, &c.;

AND
AN APPENDIX,
ON
THE POWER OF LOCOMOTIVE ENGINES,
AND THE EFFECT OF INCLINED PLANES AND GRADIENTS.

WITH SEVEN PLATES.

BY PETER BARLOW, F.R.S.

MEM. INST. OF FRANCE; OF THE IMP. AND ROYAL ACADEMIES OF PETERSBURGH, AND
BRUSSELS; OF THE AMER. SOC. ARTS; AND HON. MEM. INST. CIVIL ENGINEERS.

New Edition, Revised.

LONDON:
JOHN WEALE, 59, HIGH HOLBORN.

M.DCCC.XLV.

op

TA 410

B 25

1845

PRINTED BY W. HUGHES,
KING'S HEAD COURT, GOUGH SQUARE.

TO THE
CHAIRMAN AND DIRECTORS
OF THE
LONDON AND BIRMINGHAM RAILWAY COMPANY,

This Work

IS RESPECTFULLY DEDICATED

BY THEIR OBEDIENT SERVANT,

THE AUTHOR.

CONTENTS.

ON THE STRENGTH OF TIMBER.

	ART.	PAGE
Experiments on Direct Cohesion, by Musschenbroek	2 ..	2
———— on the effect of Seasoning	5 ..	7
———— on Direct Cohesion, at the Royal Military Academy	9 ..	15
———— on Lateral Adhesion	13 ..	22
Mechanism of the Transverse Strain	17 ..	25
Resulting Formulæ	28 ..	43
Mechanical Action of the Fibres to resist a transverse Strain	30 ..	48
Resulting Formulæ	45 ..	65
Theoretical Deflection of Beams as depending on their length	47 ..	67
Investigation relating to the Elastic Curve	50 ..	70
Another mode of investigating the amount of Deflection	51 ..	73
Deflection of Beams supported at each end	54 ..	79
The same, when uniformly loaded	55 ..	82
Deflection of Beams as depending on their breadth and depth	56 ..	84
Experiments on Deflection	57 ..	85
Dupin's Experiments and Deductions	63 ..	94
General Practical Deductions	66 ..	96

	ART.	PAGE
Experiments on the Transverse Strength of Timber .	67 ..	98
———— by Buffon	68 ..	102
———— by Beaufoy	70 ..	106
———— by Peake and Barrallier	72 ..	108
———— by Couch, on various formed Bars . .	75 ..	113
———— on Woods of different kinds, made at the Royal Military Academy	80 ..	120
Description of Apparatus	81 ..	<i>ib.</i>
Experiments on Fir Battens, supported at each end	84 ..	126
Miscellaneous Experiments on Fir Battens, cross-cut in the centre	91 ..	133
———— grooved along the middle	92 ..	134
———— on triangular Fir Battens	93 ..	135
Experiments on Battens, fixed at each end . . .	94 ..	136
———— fixed at one end at different angles of inclination	95 ..	137
———— on Oak Battens	96 ..	138
———— on Ash Battens	98 ..	140
———— on Beech Battens	99 ..	141
———— on solid and hollow Cylinders . . .	100 ..	142
Determination of Practical Data	102 ..	144
Table of Data	104 ..	148
Additional Experiments, by P. W. Barlow, on Woods of various kinds	105 ..	153
Experiments on the Strength of bent Timber . . .	106 ..	154
———— on bent Timber, in its natural state, compared with Timber bent artificially . . .	109 ..	161
———— on Trussed Girders	110 ..	162
On the Resistance to Pressure	111 ..	164
Crushing Force	112 ..	<i>ib.</i>
Resistance of Columns to Flexure	113 ..	165
Experiments on Vertical Pressure, by Girard . . .	114 ..	169
Solution of Practical Problems	115 ..	172
On the Strength of Cements and Building Materials .	116 ..	186
Experiments on the Resisting Power of various Build- ing Materials to a Crushing Force	119 ..	191
On the Force necessary to overturn Walls and Columns	120 ..	192
On the Pressure of Banks and Dimensions of Revet- ments	121 ..	193

STRENGTH OF CAST IRON.

	ART.	PAGE
Direct Cohesion of Cast Iron	123 ..	201
On the Strength of Hydrostatic Presses	124 ..	202
Direct Cohesion of various Metals	126 ..	210
Experiments on the Resistance of Iron Bars to a Wrenching Force	127 ..	213
————— on the Resistance of Cast Iron to a Crushing Force	129 ..	216
Transverse Strength of Cast Iron	131 ..	217
Deflection of Cast Iron	133 ..	221
Experiments on the Transverse Strength of Beams of various Sections, by Eaton Hodgkinson, Esq.	136 ..	226
Miscellaneous Experiments, by G. Rennie, Esq.	137 ..	245
Experiments on the Transverse Strength of Steel	139 ..	247

STRENGTH OF MALLEABLE IRON.

Description of the Testing Machine in His Majesty's Dockyard, Woolwich	141 ..	250
Experiments on the Strength of Chains	143 ..	261
————— on the Direct Cohesion of Malleable Iron	144 ..	262
————— on Bars and Cables	146 ..	268
————— on Iron Bolts and Bars in the Testing Machine, by the Author	147 ..	271
————— on Iron Wire, by Telford	150 ..	275
Comparison of these results, with Theoretical De- ductions	153 ..	286
On the Catenary Curve	155 ..	295
Table for computing the several cases of Strain, &c. of Suspension Bridges	157 ..	299
Transverse Strength of Malleable Iron	158 ..	306
Experiments on the Extension of Iron under dif- ferent degrees of Tension	159 ..	310
————— to determine the Comparative Resistance of Malleable Iron to Extension and Compression	162 ..	319
Comparative Strengths of Parallel Rails of equal sections but of different figures	166 ..	333

	ART.	PAGE
Rule for computing the Strength	168 ..	338
Mechanical Solution of Strength	171 ..	343
Rail of Maximum Strength	174 ..	348
On the Longitudinal Figure of Rails	175 ..	354
Report addressed to the Directors of the London and Birmingham Railway Company	180 ..	365
Experiments on Railway Bars	181 ..	375
Miscellaneous Experiments on the Motion and Pres- sure of Locomotive Engines and Trains on the Liverpool and Manchester Railway	188 ..	398
Description of the Apparatus employed	189 ..	401
Experiments on the Deflection of Railway Bars	193 ..	416
On the proportional increased Strength with in- creased Distance of Bearing	195 ..	426
Sections for different Distances	195 ..	429
Tables of the Weight of Iron, per mile	196 ..	432
Experiments on the Stability of Blocks	198 ..	440
Best Form of Rail	199 ..	445
Best Form of Chairs	200 ..	448
Formation of Joints	201 ..	452
Best Mode of Fixing	202 ..	456

 APPENDIX.

On the Deflection of Railway Bars	1 ..	460
On the Power of Locomotive Engines	6 ..	466
On the Effect of Gradients	10 ..	470
Table of Specific Gravities of Building Materials		484

' P R E F A C E .

THE first edition of my 'Essay on the Strength and Stress of Timber' was published in 1817, since which it has gone through three editions : another edition having been called for, I have thought it right to remodel the whole, and to introduce into it a great variety of matter not found in the original Work. At the time of the first publication, the construction of suspension bridges was in its infancy, and the application of malleable iron for the purposes of railways unknown. These, and various novel applications of timber, iron, and other materials, to different mechanical works, have rendered it necessary to investigate, experimentally and theoretically, many subjects which were not known when the first edition of this Work was published, and which it was difficult to introduce without remodeling the whole.

This has been accordingly done, and it is hoped that the utility of the Work has been thereby greatly increased. The arrangement which it has now been thought proper to adopt may be thus stated : the first subject treated of, is

the strength of direct cohesion of the fibres of timber, with an account of the experiments of Musschenbroeck, Du Hamel, Emerson, and others; and lastly, of those made by the Author, with a description of the apparatus by which the results were obtained.

The next division treats of the mechanism of the transverse strain to which timber and other materials are exposed when loaded in any part of their length, and the mechanical action of the fibres to resist this strain. We then investigate theoretically the laws of deflections under all the varieties of position and fixing, to which timber and iron are subjected in architectural and other constructions. Having thus examined theoretically the nature of the several strains and the consequent deflections, we proceed to a detail of various experiments by Buffon, Girard, Beaufoy, &c., on the transverse strength of timber; and lastly, the original experiments of the Author, which laid the foundation of the first edition, and on which is founded the Table of Data adopted in the subsequent part of this division of the Work. Another section is employed in the detail of experiments on bent timber, as used in ship-building—on the effect of boiling and steaming timber; experiments by Girard on vertical pressure, and a series of illustrative problems and examples. A short chapter follows on the strength of cement and building materials, as stone, brick, &c., and on the subject of revetment walls.

The next division treats on the direct strength of cast iron and its application in the construction of hydrostatic presses; also on the direct strength of copper, brass, yellow metal, &c., from experiments made by Mr. Kingston on the testing machine in Woolwich Dockyard; and others by Messrs. George Rennie, Tredgold, and Duleau.

The following chapter treats on the transverse strength and deflection of cast iron beams under a great diversity of forms, principally from a highly interesting and valuable paper by Eaton Hodgkinson, Esq., in volume v. of the 'Manchester Memoirs.' We come now to the subject of malleable iron; and as the experiments on this material were principally made on the testing machine in his Majesty's Dockyard, Woolwich, it was thought that an accurate drawing and description of this machine would be acceptable to the reader: two new plates have been therefore introduced, illustrating its entire construction and operation.

A detail of experiments is then given on the strength of direct cohesion of iron bars and bolts, the testing strengths of the different descriptions of iron cables used in the British navy; Mr. Telford's experiments on iron wires; and lastly, a Table by Davies Gilbert, Esq., for the calculation of the several particulars connected with the construction of suspension bridges.

The next subject of investigation is the application of malleable iron to the purposes of railway bars, being the

substance of two Reports by the Author, addressed to the Directors of the London and Birmingham Railway Company, with the addition of several subsequent experiments on railway bars of various forms and dimensions, and of miscellaneous experiments on the effect of locomotive engines and trains on the bars of the Liverpool and Manchester line.

These form the subject of the principal matters treated of in the body of the Work, but an Appendix is added, on the practical action of locomotive engines, and on the effect of inclined planes and gradients, with a view to the comparison of the mechanical advantages and disadvantages of rival lines of railway.

May 10th, 1837.

•

A TREATISE
ON
THE STRENGTH OF MATERIALS.

ON THE STRENGTH OF TIMBER.

1. **THERE** are four distinct strains to which a beam of timber, a bar of metal, or any other hard body, may be exposed, and in which the mechanical effort to produce the fracture, and the resistance opposed to it by the fibres or particles, are differently exerted ; while each of these again is subject to various modifications, according to the manner in which the bodies are supported or fixed, the positions in which they are placed, and the direction of the forces or strains to which they are exposed.

These four distinct cases or strains may be stated as follow :

1st. A body may be torn asunder by a stretching force applied in the direction of its fibres, as in the case of ropes, stretchers, king-posts, tie-beams, &c.

2ndly. It may be broken across by a transverse strain, or by a force acting either perpendicularly or

obliquely to its length, as in the case of levers, joists, &c.

3rdly. A beam or bar may also be destroyed by a pressure exerted in the direction of its length, as in the case of pillars, posts, and truss-beams.

4thly. It may be twisted or wrenched by a force acting in a perpendicular direction, at the extremity of a lever or otherwise, as in the case of the axle of a wheel, the lever of a press, &c.

These several cases will form the subject of inquiry in the following pages.

Experiments on the Strength of Direct Cohesion of the Fibres of different kinds of Wood.

2. It is usual to distinguish by the expression *force of direct cohesion of bodies*, or simply *direct cohesion*, that force by which the fibres or particles of a body resist a separation, and which must ultimately be traced to that unknown cause we are accustomed to speak of under the denomination of *corpuscular attraction*.

This is by far the simplest strain of the four above alluded to with regard to its mechanical action ; but the most difficult to submit to experiment, in consequence of the enormous forces that are requisite to produce the rupture even on pieces of small dimensions, and the great difficulty there is in applying those forces in the direct line of the fibres of the body ; and if this is not done, the first rupture may

be occasioned by some unequal action of the weight on a part of the fibres only, or by some force of torsion whereby a part of them may be wrenched asunder.

The consequence in either case is, that the force of direct cohesion will be estimated at less than its real value; and it is probably owing to this circumstance that so little agreement is found in the results of such experiments as have been made with a view to this determination. The strength of different woods of the same kind, and of different parts of the same timber, are also very different, as has been shown by the experiments of Musschenbroeck, Robison, Buffon, and others; but, as regards this difference, we still unfortunately meet with strange discrepancies. Musschenbroeck's experiments were made with great care, and he has given a very minute detail of them, particularly those on ash and walnut. In these he states the weights required to tear asunder slips taken from the four sides of the tree, and on each side in a regular succession from the centre to the circumference. His pieces were all formed into slips fitted to his apparatus, and cut down to the form of parallelopipedons of $\frac{1}{5}$ th an inch square, and therefore $\frac{1}{25}$ th of a square inch section; and the several weights required to produce the rupture when the rods are reduced to a square inch, are as stated in the following Table:

3. *Musschenbroeck's results on the Strength of Direct Cohesion.*

	lbs.		lbs.
Locust-tree . . .	20,100	Pomegranate . . .	9,750
Jugeb	18,500	Lemon	9,250
Beech, Oak . . .	17,300	Tamarind	8,750
Orange	15,500	Fir	8,330
Alder	13,900	Walnut	8,130
Elm	13,200	Pitch Pine	7,650
Mulberry	12,500	Quince	6,750
Willow	12,500	Cypress	6,000
Ash	12,000	Poplar	5,500
Plum	11,800	Cedar ¹	4,880
Elder	10,000		

In these experiments, it was found, that the wood immediately surrounding the pith or heart was the weakest. Dr. Robison also asserts, under the article **STRENGTH**, 'Encyclopædia Britannica,' from his own observation on *very large* oaks and firs, that the heart was weaker than the exterior parts. He observes also, that the wood next the bark, commonly called the *white*, or *sap*, is again weaker than the rest; and that, generally, the greatest strength is found between the centre and the sap.

With regard to our experiments, they were not particularly directed towards this inquiry; but, in most cases, the heaviest wood was found the strongest; and this was generally the case with those parts that grew nearest the centre of the trunk, and nearest to the root, provided it was so far removed from the latter as not to be very cross-grained. M. Girard²

¹ See Musschenbroeck's 'System of Natural Philosophy,' published after his death, by Lulofs, 3 vols. 4to; or the French translation of the same, by Sigaud de la Fond, Paris, 1760.

² *Traité Analytique de la Résistance des Solides.*

is also of the same opinion, stating it as a well-established fact, that the strongest part of a tree is nearest the centre.

4. From this contrariety of results, it is difficult to draw any satisfactory conclusion: the probability is, that much depends upon the age of the timber, and on the soil in which it was grown. While the tree is advancing in its growth, the last-formed wood, that is, the exterior parts, are probably weaker than the heart; but when a tree has attained complete maturity, and approaches, though imperceptibly, towards decay, the circumstances may be reversed; the exterior parts, or last-formed wood, becoming harder and stronger, while the central parts are beginning to experience that dissolution which ultimately pervades the whole. It may be observed, that Dr. Robison states his timbers to be *very large*; and Musschenbroeck's must have likewise been of considerable size, from the number of slips he was able to cut out between the centre and circumference: both which circumstances seem to give a degree of probability to the above suggestions.

Very nearly the same view is taken of this subject by Du Hamel, in his work, 'Sur l'Exploitation des Bois,' where the same ideas are given, not (as those above) merely as conjectures, but as facts, drawn from numerous experiments and observations. The author concludes his chapter on this subject as follows: "Si ce que nous venons d'avancer est vrai, il faut nécessairement que le bois qui est vers le centre

TABLE OF EXPERIMENTS.

Instituted in order to ascertain the Weight of a Cubic Foot of different kinds of Wood; the Foreign when first imported, those of the Growth of England when felled: also the Weight of each when fully seasoned; showing, at the same time, the Loss sustained in Dimensions during the Process of Seasoning.

BY MR. BENJAMIN COUCH, OF PLYMOUTH DOCKYARD.

STRENGTH OF TIMBER.

Species (in the language of Commerce).	Country where produced.	What part of the tree the pieces experi- mented on were cut from.	DIMENSIONS.				Weights in air of a cubic foot, as ascertained.	
			When first planned for experi- ment.		When seasoned.		When first planned for experiment.	When seasoned.
			Length.	Breadth and thick- ness, or diameter.	Length.	Breadth and thick- ness, or diameter.		
Riga Masts, superior	Russia	Butt ^s	ft. 7 6	inches. 18 diameter ⁴	ft. 7 6	inches. 17½ diameter	ounces. 672	ounces. 644
Riga Masts, inferior	Russia	Top	4 5	11 by 11	4 5	10½ by 10½	546	552 ⁵
Pitch Pine Mast .	Baltimore	Butt	12 0	12 diameter	12 0	11½ diameter	577	494
		Top	6 6	8½ diameter	6 6	8½ diameter	464	464
		Butt	2 6	10 by 10	2 6	9½ by 9½	755	741
		Top	6 0	7½ by 7½	6 0	7½ by 7½	518	524
Yellow Pine Mast {	North America	Butt	3 4	18½ diameter	3 4	18½ diameter	628	597
		Top	7 6	10 by 8	7 6	9½ by 7½	540	529
White Pine Mast {	Canada	Butt	2 4	18 by 18	2 4	17½ by 17½	683	461
		Top	5 0	16 by 16	5 0	15½ by 15½	495	420
Northern Pine Mast {	American States	Butt	3 6	12 by 12	3 6	11½ by 11½	555	405
		Top	9 6	12 by 12	9 6	11½ by 11½	638	448
New York	New York	Butt	3 6	17½ diameter	3 6	17½ diameter	656	549
		Top	4 0	7½ by 7½	4 0	7½ by 7½	432	416

Also, in consequence of the closeness of their pores, they are more dense and heavy, become extremely hard, and resist the attack of worms.

The specific gravity of a tree growing in such soil as that above described, is to that of a similar tree in a wet marshy situation, frequently as 7 to 5; and the weights which a similar beam will support without breaking, in the two cases, are in about the ratio of 5 to 4.

May not this account for the superior quality of the Sussex oak? which I am informed by Mr. Hookey, timber-master in Deptford Dockyard, he has always found to be the best for strength and durability: that the next in quality is that which grows in the south-west parts of Kent, and the north-east parts of Hampshire.

5. As to the density of the top and bottom of the same tree, and of the centre and external parts, much depends upon the age of the timber when felled; but generally, in a sound tree, the density is found to decrease from the butt upwards, and from the centre to the circumference. On the former point, the following experiments, the result of many years' observation, which have been made with great care by Mr. B. Couch, timber-master in Plymouth Dockyard, are highly valuable; and they are given in preference to those of Du Hamel; not only on account of their containing a greater variety of woods, but because the results are given in weights and measures which are more familiar to English engineers.

6. To the same gentleman I am indebted for the following Table relative to the loss of weight sustained by oak in seasoning. The eight pieces on which the experiments were made, were English oak, varying from 3 inches to $10\frac{3}{8}$ inches in thickness, and from 24 inches to 40 inches in length; the particulars of which are stated in the three upper lines in the following Table; the dimensions there given being those of the pieces when first taken from the saw-pits in their rough state, viz., without planing; and not being originally cut for the purpose of these experiments, most of the dimensions are found partly fractional.

These several pieces were laid on the beams of a smith's shop, and placed at such a distance from the forges that the fire might only operate sufficiently to keep the air dry. They were converted from trees just received from the forest, and were weighed every month, from February, 1810, to August, 1812; at which latter period, it was observed that the larger pieces lost but little of their weight, and the weighing of them monthly was therefore discontinued, and only performed annually, as shown in the annexed Table: from which it appears that the

Total weight, February, 1810, was $972\frac{1}{4}$ lbs.

Ditto, August, 1815 $630\frac{1}{4}$

Weight lost $341\frac{3}{4}$

That is, more than one-third of the weight is lost in seasoning.

The specific gravity of No. 1, before seasoning, was 1074, and after that process only 720 ; and it is probable that the specific gravity of oak is always within these limits ; or, at least, that it seldom much exceeds the greatest, or falls below the least of these numbers.

TABLE OF EXPERIMENTS

RELATIVE TO THE LOSS OF WEIGHT IN SEASONING ENGLISH OAK.
BY MR. COUCH.

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Length.....	24 $\frac{1}{2}$	25 $\frac{1}{2}$	30 $\frac{1}{2}$	31 $\frac{1}{2}$	39 $\frac{1}{2}$	30 $\frac{1}{2}$	37 $\frac{1}{2}$	38 $\frac{1}{2}$
Breadth.....	16 $\frac{1}{2}$	14 $\frac{1}{2}$	16 $\frac{1}{2}$	12 $\frac{1}{2}$	16 $\frac{1}{2}$	12 $\frac{1}{2}$	14 $\frac{1}{2}$	14 $\frac{1}{2}$
Depth.....	10 $\frac{1}{2}$	9 $\frac{1}{2}$	8 $\frac{1}{2}$	7 $\frac{1}{2}$	6	5 $\frac{1}{2}$	4	3
Periods of Weighing.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
February, 1810..	163 $\frac{1}{2}$	133	164	104 $\frac{1}{2}$	163 $\frac{1}{2}$	77 $\frac{1}{2}$	92	74 $\frac{1}{2}$
March.....	154 $\frac{1}{2}$	123 $\frac{1}{2}$	155 $\frac{1}{2}$	99	148 $\frac{1}{2}$	71 $\frac{1}{2}$	82 $\frac{1}{2}$	65 $\frac{1}{2}$
April.....	149 $\frac{1}{2}$	118	151 $\frac{1}{2}$	96	142 $\frac{1}{2}$	68 $\frac{1}{2}$	78	60 $\frac{1}{2}$
May.....	144 $\frac{1}{2}$	113 $\frac{1}{2}$	147	92 $\frac{1}{2}$	135 $\frac{1}{2}$	66 $\frac{1}{2}$	75	56 $\frac{1}{2}$
June.....	140 $\frac{1}{2}$	109 $\frac{1}{2}$	143 $\frac{1}{2}$	90 $\frac{1}{2}$	130 $\frac{1}{2}$	64	71 $\frac{1}{2}$	53 $\frac{1}{2}$
July.....	137 $\frac{1}{2}$	106 $\frac{1}{2}$	141	88 $\frac{1}{2}$	127	62	69 $\frac{1}{2}$	51 $\frac{1}{2}$
August.....	135 $\frac{1}{2}$	104 $\frac{1}{2}$	139 $\frac{1}{2}$	87	123 $\frac{1}{2}$	61	67 $\frac{1}{2}$	50 $\frac{1}{2}$
September.....	133	102 $\frac{1}{2}$	137 $\frac{1}{2}$	85 $\frac{1}{2}$	121	59 $\frac{1}{2}$	66	49 $\frac{1}{2}$
October.....	131 $\frac{1}{2}$	101 $\frac{1}{2}$	136	84 $\frac{1}{2}$	119 $\frac{1}{2}$	58 $\frac{1}{2}$	65	48 $\frac{1}{2}$
November.....	130 $\frac{1}{2}$	100 $\frac{1}{2}$	134 $\frac{1}{2}$	84	117 $\frac{1}{2}$	58 $\frac{1}{2}$	64 $\frac{1}{2}$	47 $\frac{1}{2}$
December.....	129 $\frac{1}{2}$	99 $\frac{1}{2}$	134 $\frac{1}{2}$	83 $\frac{1}{2}$	117 $\frac{1}{2}$	58	63 $\frac{1}{2}$	47 $\frac{1}{2}$
January, 1811..	129	99	133 $\frac{1}{2}$	82 $\frac{1}{2}$	116 $\frac{1}{2}$	57 $\frac{1}{2}$	63 $\frac{1}{2}$	47 $\frac{1}{2}$
February ⁶	130 $\frac{1}{2}$	100 $\frac{1}{2}$	135	84	118	57 $\frac{1}{2}$	65	47 $\frac{1}{2}$
March.....	127 $\frac{1}{2}$	98	132	81 $\frac{1}{2}$	115	57	62 $\frac{1}{2}$	47
April ⁷	127 $\frac{1}{2}$	97	132 $\frac{1}{2}$	83	116	58 $\frac{1}{2}$	64	46 $\frac{1}{2}$
May.....	125 $\frac{1}{2}$	96 $\frac{1}{2}$	130	80 $\frac{1}{2}$	113 $\frac{1}{2}$	56 $\frac{1}{2}$	61 $\frac{1}{2}$	46 $\frac{1}{2}$
June.....	124 $\frac{1}{2}$	95 $\frac{1}{2}$	129 $\frac{1}{2}$	79 $\frac{1}{2}$	112 $\frac{1}{2}$	55 $\frac{1}{2}$	61	46 $\frac{1}{2}$
July ⁸	124 $\frac{1}{2}$	96	129 $\frac{1}{2}$	80 $\frac{1}{2}$	112 $\frac{1}{2}$	55 $\frac{1}{2}$	62 $\frac{1}{2}$	46
August.....	121 $\frac{1}{2}$	93 $\frac{1}{2}$	127	78 $\frac{1}{2}$	109 $\frac{1}{2}$	54 $\frac{1}{2}$	60 $\frac{1}{2}$	45 $\frac{1}{2}$
September.....	122	92 $\frac{1}{2}$	127 $\frac{1}{2}$	79 $\frac{1}{2}$	109	56	59 $\frac{1}{2}$	45 $\frac{1}{2}$
October.....	119 $\frac{1}{2}$	92	125 $\frac{1}{2}$	77 $\frac{1}{2}$	108 $\frac{1}{2}$	54	59 $\frac{1}{2}$	45 $\frac{1}{2}$
November.....	121	93 $\frac{1}{2}$	126 $\frac{1}{2}$	77	110	56	59 $\frac{1}{2}$	45 $\frac{1}{2}$
December.....	119 $\frac{1}{2}$	91 $\frac{1}{2}$	125	77 $\frac{1}{2}$	108 $\frac{1}{2}$	54	59 $\frac{1}{2}$	45 $\frac{1}{2}$
January, 1812..	119	91 $\frac{1}{2}$	124 $\frac{1}{2}$	76 $\frac{1}{2}$	107 $\frac{1}{2}$	53 $\frac{1}{2}$	59	45 $\frac{1}{2}$
February.....	118 $\frac{1}{2}$	91 $\frac{1}{2}$	124	76 $\frac{1}{2}$	107 $\frac{1}{2}$	53 $\frac{1}{2}$	59	45 $\frac{1}{2}$
March.....	118 $\frac{1}{2}$	91	124	76 $\frac{1}{2}$	107 $\frac{1}{2}$	53 $\frac{1}{2}$	59 $\frac{1}{2}$	45 $\frac{1}{2}$
April.....	117 $\frac{1}{2}$	90 $\frac{1}{2}$	123	76 $\frac{1}{2}$	106 $\frac{1}{2}$	53 $\frac{1}{2}$	58 $\frac{1}{2}$	45 $\frac{1}{2}$
May.....	117 $\frac{1}{2}$	90 $\frac{1}{2}$	122 $\frac{1}{2}$	75 $\frac{1}{2}$	106 $\frac{1}{2}$	53	58 $\frac{1}{2}$	45 $\frac{1}{2}$
June.....	116 $\frac{1}{2}$	89 $\frac{1}{2}$	122	75 $\frac{1}{2}$	106	52 $\frac{1}{2}$	58	45 $\frac{1}{2}$
July.....	116	89 $\frac{1}{2}$	121 $\frac{1}{2}$	75 $\frac{1}{2}$	105 $\frac{1}{2}$	52 $\frac{1}{2}$	58 $\frac{1}{2}$	45 $\frac{1}{2}$
August, 1812...	115 $\frac{1}{2}$	89	121	74 $\frac{1}{2}$	105 $\frac{1}{2}$	52 $\frac{1}{2}$	58 $\frac{1}{2}$	45 $\frac{1}{2}$
August, 1813...	111 $\frac{1}{2}$	85 $\frac{1}{2}$	116 $\frac{1}{2}$	72 $\frac{1}{2}$	103 $\frac{1}{2}$	51 $\frac{1}{2}$	57 $\frac{1}{2}$	45
August, 1814...	108 $\frac{1}{2}$	85	114 $\frac{1}{2}$	71 $\frac{1}{2}$	103	51	57 $\frac{1}{2}$	45 $\frac{1}{2}$
August, 1815...	106 $\frac{1}{2}$	84 $\frac{1}{2}$	112 $\frac{1}{2}$	70 $\frac{1}{2}$	102 $\frac{1}{2}$	51 $\frac{1}{2}$	57 $\frac{1}{2}$	45

⁶ Very much rain since last weighed.

⁷ Rained several days previous to weighing.

⁸ Constant rain for two days previous to weighing.

The loss of weight in the preceding experiments was more rapid than in the similar experiments of Du Hamel: but much depends upon the nature of the soil in which the trees grow, as the timber of moist land loses more of its weight in seasoning than that which is the produce of a drier and better soil.

7. The process of seasoning may be facilitated by boiling, steaming, &c., as appears from the following experiments of Mr. Hookey. The three pieces marked Nos. 1, 2, and 3, were English oak, each four feet long, and three inches square; all cut from the same timber. No. 1 was placed in the steam kiln for twelve hours; No. 2 was boiled for the same time in fresh water; and No. 3 was left in its natural state. The weights of the three pieces, previous to the experiment, and at the end of each month for half a year afterwards, were as stated below.

Times of Weighing.	No. 1. Steamed.	No. 2. Boiled.	No. 3. Natural State.
	Weight. lbs. oz.	Weight. lbs. oz.	Weight. lbs. oz.
Previous to the experiment .	16 12½	16 15	16 14
After ditto	16 6	16 14	16 14
June	15 1	15 10	16 5
July	14 2	14 12	15 14
August	13 13	14 0	15 5
September	12 10	13 6	15 0
October	12 5	12 10	14 12
November	11 10	12 5	14 8

Each of the pieces was placed in the same place, in the open air, and in the same position, (i. e. ver-

tically,) after the experiment, and were continued so during the six months that their weights were taken.

From the above it appears that the process of seasoning went on more rapidly in the piece that was steamed than in that which was boiled; but that in the latter, the process was carried on much quicker than in the piece which was left in its natural state:

The first had its specific gravity reduced from 1050 to 744.

The second from 1084 to 788.

And the third from 1080 to 928.

We must look to the philosopher for a satisfactory solution of the problem presented in these results. Mr. Hookey⁹ accounts for the facts by supposing, that the process of boiling or steaming dissolves the *pithy* substance contained in the air tubes, by which means the latter fluid circulates more freely, and that the seasoning thereby proceeds with greater rapidity.

8. From the several experiments above given, and from others found in Du Hamel's work above referred to, it appears,

1. That the density of the same species of timber, and in the same climate, but on different soils, will vary as much as in the ratio of seven to five; and

⁹ To this gentleman is due the ingenious idea of bending large ship timbers.—See 'Transactions of the Society of Arts,' vol. xxxii.

that the strength of the same will be, both before and after seasoning, in nearly the ratio of five to four.

2. In healthy trees, or those which have not already passed their prime, the density of the butt is in some cases to that of the top in about the ratio of four to three, and that of the centre to the circumference, as seven to five.

3. The contrary occurs when the tree is left standing after it has acquired full maturity; viz. the butt will in this case be specifically lighter than the top, and the centre than the outward part of the trunk within the bark.

4. That oak, in seasoning, loses at least one-third of its original weight; and this process is much facilitated by steaming or boiling.

On these subjects, as well as a variety of others, relative to the quality of timber, &c., which do not properly fall within the plan of this work, the reader is referred to the Treatise of Du Hamel above mentioned, where he will find much useful and important information.

*Experiments made for determining the Strength of
Direct Cohesion of different Woods.*

9. It has been before remarked, that notwithstanding the mechanical operation in this kind of fracture is by far the most simple of the four alluded to, yet

it is the most difficult to submit to actual experiment in wood; and it was not till after some consideration, and one or two failures, that we were led to adopt the apparatus exhibited in Plate I.

Here AB, fig. 1, represents one of the pieces whose strength is to be determined, its whole length being 12 inches; the length of each square end $3\frac{1}{2}$ inches, and the side of the square end $1\frac{1}{2}$ inch: the intermediate part of 5 inches was turned in an excellent instrument, by a very correct workman,¹⁰ and brought down in the centre to $\frac{1}{8}$ rd or $\frac{1}{4}$ th of an inch in diameter;¹¹ but the other cylindrical parts were made each $\frac{3}{4}$ inch in diameter. CC, DD, fig. 2, represent two strong iron bars, brought to the form shown in the Plate; GG are two screws which are passed through the holes HH, in the bar DD, and are there screwed fast by the nuts I, I; E, E, are two semicircular collars, riveted one to each bar, which, when the two are fixed together, form a circular plate, as represented in fig. 4. The circular hollow parts *e, e*, are $\frac{3}{4}$ inch in diameter, so as to fit exactly the larger part of the cylinder shown in fig. 1. These bars, after being screwed together, were rested on their supports, as in fig. 4, and, as the workmen

¹⁰ Mr. Short, modeller to the Royal Military Academy.

¹¹ As it was difficult to measure very exactly the diameter of the small cylinder, it was found by winding a fine thread of silk ten times about it, and then dividing its length by the number of volutions, in order to get the mean circumference, and hence the diameter.

express it, *brought out of winding*, and accurately adjusted to a horizontal position by a spirit level.

The two iron boxes MNO, M'N'O', fig. 3, were made exactly to fit the square head B, of fig. 1, having also two semicircular holes at top, correctly fitted to the larger part of the cylinder: these were shut by passing the bolts through the holes N, M, and were thus secured by the two shears shown in fig. 4.

Having thus described the separate parts of the apparatus, the reader will perceive at once the manner in which they were employed in the experiment; viz. the head A, of fig. 1, was placed above the collar EE, fig. 2, the upper larger cylindrical part of fig. 1 being placed in the hollow parts *e, e*, of fig. 2, when the two parts were securely fixed together by the nuts and screws I, G; I, G. In the same manner the lower end B, of fig. 1, was enclosed in the two iron boxes MNO, M'N'O', fig. 3, and fastened in that position by means of the bolts, seen in fig. 4, and the shears above described. The whole was then rested on the props, fig. 4; and the hook of the scale being inserted in the circular hole formed by O, O', fig. 3, the whole was ready for the experiment, as shown at large in the former figure.

Every thing being thus prepared, the wedges shown in the Plate were introduced under the scale, to keep it steady, while the larger weights were put in; the former were then removed, and smaller

weights added in succession till the fracture took place.

The weights were 10-inch, 8-inch, and $5\frac{1}{2}$ -inch shells, loaded each with as many musket balls as brought them respectively to 100 lbs., 50 lbs., and 15 lbs. A few common weights of 7 lbs., 4 lbs., 2 lbs., &c., were also employed toward the conclusion of an experiment, where it was necessary to increase the weight by small degrees.

It should also be observed, that as a slight vibration of the scale might cause a fracture in the small cylinder submitted to the operation of the weight, four small braces were made use of, one at each corner of the scale, to prevent any such motion. These were attached to the four inward legs of the stand, which are omitted in the Plate, to avoid a complication of parts.

The results of these experiments are exhibited in the following Table.

TABLE I.

10. *Experiments on the Direct Cohesion of different Woods.*

No. of experiments.	Names of the woods.	Specific gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion on a squ. inch.
1	Fir	600	1.05	1140	12993	} 12857
2	do.	600	1.10	1260	13073	
3	do.	600	1.10	1191	12037	
4	do.	600	1.05	1160	13220	
5	do.	600	1.11	1213	12371	
6	do.	600	1.05	1180	13448	
7	do.	581	1.10	1059	11000	} 11549
8	do.	564	1.10	1201	12472	
9	do.	601	1.10	1094	11360	
10	do.	611	1.10	1130	11736	
11	do.	532	1.10	1076	11180	
12	do.	590	1.10	1112	11548	

The first six experiments were made upon the fragments of the 4 foot pieces (Art. 88), which were the same also as the triangular pieces, Nos. 3, 4, 7, and 8 (Art. 93), were cut from.

These pieces were all cut from a plank remarkably free from knots and irregularities, which throughout gave more uniform results than any other specimen.

No. 7, broke by a part of the fibres drawing out of the head of the piece: it was probably first broken by an accidental motion of the scale.

No. 9, broke by the whole of the middle cylinder drawing out of the head, to the length of about 2 inches, where there was a knot, which might break off the continuation of the fibres. The others were all complete fractures.

TABLE I.

11. Experiments on the Direct Cohesion of different Woods.

No. of experiments.	Names of the woods.	Specific gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion.
13	Ash	594	·8800	1100	17850	} 17207
14	do.	611	·9000	1096	17003	
15	do.	611	·8750	1024	16770	
16	do.	600	·8375	881	15784	} 16947
17	do.	600	·8625	1025	17315	
18	do.	600	·8750	1081	17742	
19	Beech	712	·880	716	11626	} 11467
20	do.	694	·890	721	11437	
21	do.	700	·900	731	11338	
22	Oak	770	1·10	856	8889	} 9198
23	do.	770	1·10	887	9211	
24	do.	770	1·10	908	9494	
25	do.	920	·8800	740	12008	} 11580
26	do.	920	·8750	712	11660	
27	do.	920	·8900	698	11072	

Nothing remarkable happened in the course of these experiments, except that No. 4 of the ash, viz. No. 16 above, was observed to twist, during the action of the weight, about $7\frac{1}{2}^{\circ}$, but the fracture took place in the small part of the cylinder: as this piece, however, bore less weight than any other of the ash, it is probably to be attributed to the above circumstance; a similar effect was observed in the specimens of mahogany, as stated in the following page.

It is proper to observe, that Nos. 13, 14, and 15 were made from the fragments of the 2-inch square ash pieces, Art. 98; those of the beech from the fragments of the similar pieces, Art. 99.

The first three oak pieces were off the same plank as the several battens, Art. 96. It was a very fine piece of English oak, which had been a considerable time in store, and was perfectly dry: the other specimen, viz. Nos. 25, 26, and 27, appears, from its specific gravity, to have been more recently felled: it was also of a closer texture.

TABLE I.

12. Experiments on the Direct Cohesion of different Woods.

No. of experiments.	Names of the woods.	Specific gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion.
28	Teak	860	·8625	868	14662	} 15090
29	do.	860	·8625	900	15203	
30	do.	860	·8625	912	15405	
31	Box	960	·8625	1168	19730	} 19891
32	do.	960	·8625	1160	19595	
33	do.	1024	·8625	1200	20348	
34	Pear	646	·8625	683	11537	} 9822
35	do.	646	·8500	523	9096	
36	do.	646	·8625	523	8834	
37	Mahogany	637	1·1125	783	7950	} 8041
38	do.	637	1·1125	783	7950	
39	do.	637	1·1125	810	8224	

Nos. 28, 29, and 30 were from a piece of teak which had been taken from an old ship. Some other specimens were tried, but the results were so irregular, that it would be useless to give them; and exactly the same occurred in the first experiments on the transverse strain of this wood.

In the first two experiments on box, the small part of the cylinder drew out of the head, which was $5\frac{1}{4}$ inches in length, but not so perfectly as in the fir piece already mentioned; the part that drew out being very tapering, so that we could but barely see through the hole thus formed. It is therefore obvious that, although the mean strength amounts to nearly 20,000 lbs. upon a square inch, this is still short of the absolute strength of direct cohesion of this wood.

The same may be observed with regard to the mahogany, but it proceeded from a different cause; viz. the twisting of the pieces, which, in all the experiments, wrenched the fibres asunder, instead of drawing them apart. The effect seems to have been exactly the same as would happen to a weight suspended to a rope, which would have a tendency to untwist; and it is highly probable that the fibres of the tree had acquired, in their growth, a similar situation with regard to each other as the component fibres of the rope, but of course in a much smaller degree.

13. *Experiments on the Lateral Adhesion of Fir.*

It is stated in a few of the preceding experiments, that the fibres, instead of breaking, as was intended, in some instances drew out, either wholly or in part, from the head of the pieces, notwithstanding these were, in one instance, more than 5 inches in length. This circumstance suggested the following experiments, in which the head of the piece was bored down very accurately to the distances stated in the third column, viz. to the insertion of the smaller cylinder into the greater part; the several pieces were then suspended, as in the foregoing experiments, and the weights put on as usual, till the separation took place; that is, till the small part was drawn out, or broken.

TABLE II.

No. of experiments.	Names of the woods.	Length drawn out.	Circumference.	Weight in lbs.	Weight reduced to one inch surface.	Mean value of lateral cohesion on one inch surface.
1	Fir.	1·625	1·1	996	556	} 592
2	do.	1·625	1·15	1187	621	
3	do.	1·625	1·15	1117	584	
4	do.	1·375	1·15	1066	634	
5	do.	1·500	1·15	1000	578	
6	do.	1·500	1·15	1000	578	

Nos. 1, 3, and 5 were drawn out very completely; the part which came out being nearly as perfect a cylinder as that which was turned: the other three were more or less irregular.

Nos. 2 and 4 twisted at least 10° before the separation took place.

It appears from the above, that the lateral adhesion is not more than one-twentieth of the direct cohesion in fir. With the other woods we did not attempt any experiments.

14. From a mean derived from the preceding experiments, and employing only the nearest whole numbers, it appears that the strength of direct cohesion on a square inch of

	lbs.
Box, is about	20,000
Ash	17,000
Teak	15,000
Fir	12,000
Beech	11,500
Oak	10,000
Pear	9,800
Mahogany	8,000

Also, that the strength of the lateral adhesion of

the fibres in fir is about equal to 600 lbs. on a square inch.

Some of these numbers differ considerably from those given by Musschenbroeck, as is stated in Art. 3; on which head it will be sufficient to observe, that the preceding experiments, from which the above results are drawn, were made with every possible care that the delicacy of the operation required.

15. *Practical Rule.*—Since the strength of direct cohesion must necessarily be proportional to the number of fibres, or to the area of the section, it follows, that the strength of any rod will be found by multiplying the number of square inches in its section by the corresponding tabular number, as given above.

This, however, gives the absolute strength, or rather the weight that would destroy the bar; and practical men assert, that not more than one-fourth of this ought to be employed. I have, however, left more than three-fourths of the whole weight hanging for twenty-four or forty-eight hours, without perceiving the least change in the state of the fibres, or any diminution of their ultimate strength.

On the Transverse Strength of Timber.

16. By the *transverse strength of timber* is to be understood the resistance which this material op-

poses to a force or weight acting upon it transversely to its length, either perpendicularly or obliquely; and it naturally divides itself into three distinct considerations, viz.:

1st. The mechanical strain which a given force acting in a given direction produces on the section of fracture.

2dly. The nature of the mechanical action of the fibres to resist this strain.

3dly. The actual strength of the fibres when thus excited; which of course varies considerably in woods of different kinds.

The two former are merely questions relating to theoretical mechanics and geometry, while the latter is wholly experimental.

Mechanism of the Transverse Strain.

17. A beam of timber ACIF, fig. 1, Plate II., fixed with one end in a wall, and loaded with a weight W at the other, will be deflected from its first horizontal position AH, into an oblique direction AF, fig. 2, supposing it for the present inflexible in every point, except in the section of fracture AC. And this deflection, as we shall see, takes place about a line denoted by *n* in the figure (called the *neutral axis*) within the centre of fracture, which it is very important to determine, when we are considering the nature of the resisting forces of the fibres; but at present our object is merely to estimate the

exciting or straining force, which is obviously the product of the weight into the effective length of the lever $n F$; that is, analytically denoting the strain by f ;

$$f = n F \cdot \cos n F B \cdot W, \text{ or}$$

$$f = l \cos \Delta \cdot W,$$

denoting $n F$ by l , the weight by W , and the angle $n F B$ of deflection by Δ .

It will be observed, that $n F$ is not the length of the beam, but the distance of the neutral axis from the point on which the weight is suspended; nor is the angle $n F B$ actually the angle of deflection of the beam; but as the depth of beams is generally small in comparison of their length, and the depth of the neutral axis still smaller, we shall in what follows, except the contrary be expressed, consider l as the length of the beam, and Δ as the angle of deflection, as it will simplify the investigation, and can produce no sensible error.

When a beam, instead of being fixed at one end into a wall, is merely rested on a support at its middle point, and loaded at each end, the tension of the upper fibre is still the same as in the former case; the length of the beam in the latter instance being supposed double what it is in the former; that is, supposing the beam FF' , fig. 3, to be double AF , fig. 2, then the three weights being equal, the tension of the fibre Ab , in both cases, will be the same; excepting only so much of it as depends upon the cosine of the angle of deflection,

which in fig. 3 will be only half that in fig. 2: the same general expression, however, will apply in both cases, by merely changing l in the former into $\frac{1}{2} l$ in the latter; so that we shall have in this case

$$f = \frac{1}{2} l \cos \Delta W.$$

18. Now, a beam resting on a fulcrum C, in the middle of its length, as in fig. 3, and acted upon by two weights W, W' , has commonly been considered in the same state, with regard to the strain upon it, as the equal beam FF' , fig. 4, which is rested on the two props FF' , and loaded with a double weight P , at its centre: and this is sufficiently correct in all common cases, although not strictly so when the deflection of the beam is considerable, as may be demonstrated as follows.

In the first place, it is obvious that the resistance of the props is not made in a direction parallel to that of the vertical weight P , but perpendicular to the arms of the lever $Fn, F'n$; and therefore, that the beam is, with regard to its strain, kept in equilibrio by the action of the three forces, $FO, F'O$, and OR ; the former, $FO, F'O$, being supposed perpendicular to $Fn, F'n$.

The reaction of the fulcrums F, F' , will therefore be to the weight P , as FO to half OR , or OC ; or as radius to the cosine of the angle FOn , or $nF'C$; that is, as radius to the cosine of the angle of deflection.

Hence, when a beam is rested upon two fixed props, and loaded at its middle point by any weight

P, the strain upon that middle point, arising from the reaction of the props, will be found by the following proportion, as

$$OC : OF :: \frac{1}{2} P : \frac{FO \cdot P}{2 OC}, \text{ or}$$

$$\cos \Delta : \text{rad} :: \frac{1}{2} P : \frac{P \text{ rad}}{2 \cos \Delta}, = \text{or } \frac{P}{2 \cos \Delta},$$

taking radius equal to unity ; or if we call $\frac{1}{2}P=W$, then, according to our former notation,

$$f = \frac{\frac{1}{2} l \cdot W}{\cos \Delta} = \frac{l \cdot P}{4 \cos \Delta}.$$

This supposes the arms of the lever Fn , $F'n$, to remain of the same length ; but it is obvious that this is also an erroneous hypothesis ; for the props, or fulcrums, being fixed, these arms, either by the stretching of the fibres, or by the piece of wood slipping between the points of support, are more and more lengthened as the piece descends ; viz., the length of the lever is to half the distance of the props, as rad to $\cos \Delta$; and, consequently, the strain on this account is again increased in the ratio of $\frac{\text{rad}}{\cos \Delta}$ or $\frac{1}{\cos \Delta}$ to radius 1 ; whence, by introducing this consideration, our former expression becomes

$$f = \frac{l P}{4 \cos^2 \Delta} = \frac{l P \sec^2 \Delta}{4}.$$

19. In all practical cases the angle of deflection Δ is so small that the secant may be considered as unity ; but in extreme cases of experimental fracture

it is considerable ; and as attending to this circumstance may serve to explain what has hitherto been considered as an anomaly in the experiments of Buffon and others, it may not be amiss to examine the question a little more particularly, especially as it seems to have escaped the attention of other authors.

Let, then, $A C B$, Plate III. fig. 1, represent a beam of timber, or simply a lever, which, in the first place, we will suppose to be kept in equilibrio by the two equal weights W, W' , and the resistance of the fulcrum C , or by a weight P , acting in an opposite direction CQ ; then it is obvious that the weight P must be exactly equal to the two weights W, W' , or $P = 2 W$, the lever being supposed void of gravity. But the effect of the weights W, W' , on the two levers AC, BC , as they relate to any strain at C , may be produced by two less weights w, w' , acting perpendicularly to the latter; and these less weights, from the nature of the composition and resolution of forces, are to the two given weights W, W' , in the ratio of OB , or OA , to OC .

If, therefore, the lever AB be kept in equilibrio by the weights w, w' , in the directions AO, BO , the reaction of the fulcrum, that is, the weight P , must be reduced in the ratio of $OC^2 : OB^2$; for the weights themselves are less in the simple ratio of these lines, and their perpendicular action is also less in the same proportion; and, consequently, the resistance at the fulcrum, or the weight P , will be decreased in the





upon the centre of a beam which is loaded at that point, having each of its ends fixed in a wall or other immoveable mass.

Here it is obvious, that the whole weight is not employed in producing the strain and consequent fracture of the middle section, a part of it being required to produce the strain and deflection at the points of fixing; consequently, beside the weight necessary to cause fracture, or to produce any given deflection in a beam merely supported, so much additional weight must be added, when the beam is fixed at each end, as will deflect the two half-lengths to the same degree; that is, referring to fig. 8, Plate III., the weight W must be greater than would be required to deflect the supported beam, by as much as it is necessary to deflect the two half-beams. But it is shown (Art. 53) that it requires four times the weight to produce the same deflection in a beam supported at each end, as is requisite to produce the same quantity in a beam of half the length; consequently, if we suppose the weight W , in the present instance, to be divided into six equal parts, four of these will be exerted in producing the deflection of the middle point, and one of each of the remaining two in producing the deflections at the points of fixing; therefore, only $\frac{2}{3}$ of the whole weight is employed in producing the centre deflection. The strain, therefore, *on the centre of the beam*, when fixed at each end, is to the strain arising from the same weight when it is merely supported as 2 : 3 ;

and consequently the weight necessary to produce the fracture will be as 3 to 2, which accords very accurately with experiment.

The formula, therefore, in this case, will be $f = \frac{1}{6} l W$, or more accurately, $f = \frac{1}{6} l W, \sec^2 \Delta$.

Most authors who have considered this case have stated this ratio as 4 to 2, instead of 3 to 2, supposing the beam to be equally liable to fracture at the ends as in the middle; but a mere inspection of the figure, with a mental reference to the actual experiment, is sufficient to show the fallacy of such an hypothesis. In fact, in every experiment that I made, after the complete fracture in the middle, the two fragments had been so little strained at the points of fixing, that they soon after recovered their correct rectilinear form.

If the beam, instead of being fixed at each end, were merely rested on two props, and extended beyond them on each side equal to half their distance; and if weights w, w' , fig. 9, were suspended from these latter points, each equal to one-fourth the weight W , then this would be double of that which would be necessary to produce the fracture in the common case: for, dividing the weight W into four equal parts, we may conceive two of these parts employed in producing the strain or fracture at E , and one of each of the other parts as acting in opposition to w and w' , and by these means tending to produce the fractures at F and F' .

This is the case which has been erroneously con-

founded with the former, but the distinction between them is sufficiently obvious; because here the tension of the fibres, in the places where the strains are excited, are all equal, whereas in the former the middle one was double of each of the other two.

Parent and Belidor, in their experiments, and indeed all experimentalists except Musschenbroeck, make the strength of their beams, when fixed at the ends, to the same when merely supported, in the ratio 3 to 2; but theorists have always made the ratio that of 4 to 2, as above stated, which, however, is obviously erroneous.

21. At present we have considered the load as being placed upon the middle of the beam; let us now endeavour to ascertain what strain will be excited in it, when the weight is placed in any other part than the centre, as at C, fig. 2, Plate III.

Here, since the tension of the fibre A B is the same, whether it be estimated towards F or F', we may suppose the weight, W, to be divided into two weights, which shall have to each other the ratio of I C to I' C; that is,

$$\text{as } I I' : I C :: W : \frac{I C \cdot W}{I I'},$$

$$I I' : I' C :: W : \frac{I' C \cdot W}{I I'}.$$

Then it is obvious, that whether we consider the first of these weights as acting at the point C of the lever C I', or the latter as acting at the point C of

the lever CI , or both of them as acting at the point C of the beam, or compound lever, II' , the strain or tension of the fibre AB will be the same, and will be expressed by

$$f = \frac{I'C \cdot W}{II'} \times IC = \frac{IC \cdot I'C \cdot W}{II'}; \text{ or}$$

$$f = \frac{IC \cdot W}{II'} \times I'C = \frac{I'C \cdot IC \cdot W}{II'}.$$

Hence, if l be taken to denote the length of the beam II' , and m and n , the two distances IC , $I'C$, then

$$f = \frac{mn}{m+n} W = \frac{mn}{l} W.$$

That is, the strain varies as the rectangle of the two parts into which the beam is divided by the point of suspension: and hence it follows, that the strain will be the greatest when the rectangle is the greatest; that is, when the weight acts at the centre.

22. Let us now take the case of two weights suspended from any two points of a beam, to determine the strain upon the beam at any given point.

Conceive $FII'F'$, Plate III. fig. 3, to be a beam resting on the two props FF' , and having two weights, equal or unequal, suspended from the two points D , E ; then, from the preceding formula, it appears that the strain at D is

$$f = \frac{ID \cdot DI'}{II'} \cdot W; \text{ and the strain at } E \text{ is}$$

$$f = \frac{IE \cdot EI'}{II'} \cdot W'.$$

Now, in order to find the strain at any other point, C, we have only to make the following proportions, viz.

$$DI' : CI' :: \frac{ID \cdot DI'}{II'} W : \frac{ID \cdot CI'}{II'} W = \text{the strain at}$$

C, as arising from that at D; and again,

$$EI : CI :: \frac{IE \cdot I'E}{II'} W' : \frac{I'E \cdot CI}{II'} W' = \text{the strain at C,}$$

as arising from that at E.

Consequently, the whole strain at C, arising from both weights, will be expressed by

$$f = \frac{ID \cdot CI' \cdot W + I'E \cdot CI \cdot W'}{II'}.$$

23. From this general formula may readily be deduced that for any particular case: for example,

1st. Suppose the beams uniformly loaded throughout, and the stress at any point C required.

In this case, D and E will be the centres of gravity of the two parts IC, and CI'; consequently,

$$ID = \frac{1}{2} IC \text{ and } I'E = \frac{1}{2} CI';$$

whence the expression becomes

$$f = \frac{(\frac{1}{2} IC \cdot I'C \cdot W) + (\frac{1}{2} I'C \cdot IC \cdot W')}{II'}; \text{ or}$$

$$f = \frac{IC \cdot I'C \cdot (W + W')}{2 II'}.$$

Where $(W + W')$ and II' being constant, it follows that f varies as the rectangle $IC \cdot I'C$; that is, in this case, the strain at any point C varies as the

rectangle of the two parts into which the beam is divided by that point.

2dly. Suppose, as another example, that the weights, W , W' , are equal to each other, and that C is the centre of the beam; then, since

$$IC = IC = \frac{1}{2} II', \text{ and } W = W';$$

the general expression becomes, in this particular case,

$$f = \frac{(ID + IE) \cdot IC \cdot W}{II'} = \frac{ID + IE}{2} \times W.$$

And if we further suppose $ID = IE$, then it becomes simply

$$f = ID \cdot W.$$

Now, if both weights acted at the centre, it appears, from the preceding investigation, that

$$f = \frac{1}{2} II' \cdot (2W) = \frac{1}{2} II' \cdot W = IC \cdot W.$$

Whence the strain in the two cases will be to each other as ID to IC : and hence the following practical deduction; viz.

24. When a beam is loaded with a weight, and that weight is appended to an inflexible bar or bearing, as DE , fig. 4, Plate III., the strain upon the beam will vary as the distance ID , or as the difference between the length of the beam and the length of the bearing; for the bearing DE being inflexible, the strains will be exerted in the points D and E , exactly in the same manner as if the bearing was removed, and half the weight hung on

at each of these points. This remark may be worth the consideration of practical men in various architectural constructions.

25. In the same manner as in Art. 23, it may be shown, that if a beam be loaded with many weights, $W, W', W'', W''', \&c.$, as in fig. 5, Plate III., all equal to each other, and every two of which are equally distant from the centre, the strain excited on the middle point C will be expressed by

$$f = (ID + ID' + ID'' + \&c.) \cdot W.$$

Hence, if the length of the beam be l , and the number of equal weights m , and the sum of all the weights W , then the above becomes

$$f = \left(0 + \frac{l}{m} + \frac{2l}{m} + \frac{3l}{m} + \&c. \frac{\frac{1}{2}ml}{m}\right) \times \frac{W}{m}; \text{ or,}$$

$$f = \frac{lW}{m^2} \times (1 + 2 + 3 + 4, \&c. \frac{1}{2}m); \text{ or,}$$

$$f = \frac{lW}{m^2} \times \frac{(\frac{1}{2}m + 1) \frac{1}{2}m}{2} = \frac{\frac{1}{4}lWm^2 + \frac{1}{2}lWm}{2m^2} = \frac{1}{8}lW + \frac{lW}{4m}.$$

Hence, when the weight is uniformly distributed through the whole length, the number of points of suspension, m , becoming infinite, the last term of the preceding expression, $\frac{lW}{4m}$, vanishes; and there results

$$f = \frac{1}{8}lW,$$

for the strain on the centre of a beam, when the weight W is uniformly distributed throughout its

length; which is half what it would be if it were all suspended from its middle point.

26. At present the weight has been supposed to act in a direction perpendicular to the fibres; that is, the different deflections to which the beam may be exposed in consequence of the different positions of the weight have not been taken into consideration; and it has been before explained, that it is not necessary to introduce the latter datum while we are merely contemplating the comparative strengths and strains of beams for architectural and mechanical constructions, in which the deflections are always inconsiderable, but that they are essentially necessary in the comparison of experiments on the ultimate strength; and, therefore, when we treat of those comparisons, it may be necessary to modify some of the preceding results. I shall not, however, pursue the subject further in this place, except so far as relates to the strain on beams, when the direction of the fibres and the exciting forces are placed obliquely to each other.

27. When a beam $ACFI$, or $A'C'F'I'$, fig. 6, Plate III., is placed obliquely in a wall, whether it be descending, as in the former, or ascending, as in the latter, the strain excited by the equal weights, $W W'$, on the equal arms IC , $I'C'$, will be the same, being in both cases expressed by

$$f = l W \cos I,$$

where l is the length, W the weight, and I the angle of inclination.

For, let IW , in both cases, be taken to represent the perpendicular force of the weight W , and let this be resolved into two other forces; the one, IK , perpendicular to the lever CI , and the other KW , parallel to it; then it is obvious that KI will represent the only effective force to turn the lever about the point C ; that is, the exciting force will be to the weight W as $KI : IW$, or as radius : cosine of KIW ; but the angle $KIW = CIL =$ the angle of inclination $= I$; therefore,

$$1 : \cos I :: W : W \cos I = IK,$$

which, combined with the lever $CI = l$, gives for the strain at C ,

$$f = lW \cos I.^{12}$$

Therefore, while we omit the consideration of the

¹² It has been assumed by some writers on this subject, and strangely adopted by others, that not only is the exciting force diminished in the ratio of rad to cos, but also that the power of resistance is increased in the ratio, viz. of cos to rad, because they say the area of fracture CA is increased in the latter proportion; whence they conclude, that the weight necessary to break a beam in an inclined position is to the weight when it is horizontal, as $\text{rad}^2 : \cos^2$.

Nothing, however, can be more obviously false, than to suppose the power of resistance to be increased; for if the force or weight W , or W' , fig. 6, which is denoted by IW , be resolved into the two IK , KW ; it is evident, that the force IK will have the same effect upon this beam (and no other), as if the beam was placed horizontally, and loaded with a vertical weight, which should be to W as IK to IW .

There might be some plausibility for the above hypothesis in

quantity of deflection, the strain on the two beams (their lengths, weights, and inclinations being the same) will be exactly equal to each other: and this is true, as has been before observed, while we are merely considering the application of timber to architectural purposes, but fails entirely in determining the ultimate strengths.

For the deflection of the beam I C brings it nearer and nearer to a horizontal position, where the effect of the weight is the greatest; while the deflection of the descending beam I C brings it more and more towards a vertical, where the effect of the weight is the least.

Conformably to this, I have always found, of three equal and similar beams, of which the one inclined upwards at a certain angle, another downwards at the same angle, and the third horizontal, that which had its inclination upwards was the weakest; the one which declined, the strongest; and the strength of the horizontal one, about a mean between both. —(See “Experiments,” Art. 95.) It is obvious, indeed, that the ultimate strength of a beam does not depend upon its original position, but upon that which it has attained immediately before the fracture takes place.

It may be proper to observe, that in the preceding expression, $f = l W \cos I$, that force only is included

crystallized bodies, but it will certainly not apply to fibrous ones, the number of fibres on which the resistance depends being still the same.

which has a tendency to turn the beam about the point C: there is, however, also another exciting force, but which does not act at any mechanical advantage, that is, the force represented by KW , which in the declining position of the beam $AFCI$ acts by tension, and in the ascending position of $A'F'C'I'$ by pressure: the entire expression, therefore, for the exciting force, is

$$f = l W \cos I + W \sin I.$$

But in most practical cases this latter force is very inconsiderable; first, because it does not act at any mechanical advantage through the intervention of the lever; and, secondly, because it acts equally upon the compressed and extended fibres; and, consequently, while it increases the one of these forces, it diminishes the other, and, therefore, in a certain degree, neutralizes its effect on both, on which account it may in most cases be omitted: and we must necessarily omit it in this place, because its real effect depends upon the proportionality between the area of compression and that of tension, and the comparative value of these two resisting forces, the determination of which will form the subject of experiment in a following section. It will, therefore, in this place, be sufficient to observe, that in the cases where the beam is vertical, and consequently $\cos I = 0$, and $\sin I = 1$, the former part of the expression disappears, and we have simply $F = W$; where, as in the suspended beam,

W must be equal to the force of direct cohesion in the area of fracture, and in the other case it will represent the weight necessary to crush the beam with a vertical pressure.

28. At present we have only considered the strain a beam is exposed to by being charged at any point with a given weight, without making any reference to the resistance to which it is opposed. Now, this resistance obviously depends upon the figure and area of the section of the beam at the breaking point, and all theories and experiments make this resistance vary in rectangular beams as the breadth and square of the depth. That the strength or resistance is as the breadth, is obvious; because, whatever resistance any given beam offers to fracture, two, three, or more such beams will offer two, three, or more times that resistance: and this is in fact the same as a beam of two, three, &c., times the breadth. And with regard to the depth, the resistance will be, in the first place, as the number of fibres; that is, as the depth: and, secondly, it varies as the length of the lever by which those fibres act; that is, as the distance of the several fibres from the centre about which the beam turns, wherever that point may be, which is also obviously as the depth; and hence, by combining the two causes, it will vary as the square of the depth when the breadth is the same: and therefore, generally, the resistance opposed to fracture by rectangular

beams is as the product of the breadth and square of the depth.

If we represent the breadth of a beam of any given wood by a , its depth by d , its length by l , all in inches, its angle of deflection by Δ , and the weight necessary to break it in lbs. by W ; also, the resistance of a rod an inch square by S : then $a d^2 S$ will be the resistance of the beam whose breadth is a and depth d . Now, in the instant before breaking, there must be an equilibrium between the strain and the resistance; and hence we obtain the following equations, viz.

1. *When the beam is fixed at one end, and loaded at the other,*

$$l W \cos \Delta = a d^2 S, \text{ or } \frac{l W \cos \Delta}{a d^2} = S, \text{ a constant quantity.}$$

2. *When the beam is supported at each end, and loaded in the middle,*

$$\frac{1}{4} l W \sec^2 \Delta = a d^2 S, \text{ or } \frac{l W \sec^2 \Delta}{4 a d^2} = S, \text{ constant.}$$

3. *When the beam is fixed at each end, and loaded in the middle,*

$$\frac{1}{8} l W \sec^2 \Delta = a d^2 S, \text{ or } \frac{l W \sec^2 \Delta}{6 a d^2} = S, \text{ constant.}$$

4. *When the beam fixed as in either of the last two cases is loaded at any other point than the centre,*

We shall have in the former case, by denoting the two unequal lengths by m and n ,

$$\frac{m n W}{l} \sec^2 \Delta = a d^2 S, \text{ or, } \frac{m n W \sec^2 \Delta}{l a d^2} = S :$$

and in the second,

$$\frac{2 m n W}{3 l} \sec^2 \Delta = a d^2 S, \text{ or } \frac{2 m n W \sec^2 \Delta}{3 l a d^2} = S,$$

still the same constant quantity.

The first formula will also apply to a beam fixed at any given angle of inclination; observing only, that the angle Δ , in this case, will represent the angle of the beam's inclination, increased or diminished by the angle of its deflection, according as its first position is ascending or descending; or rather, it will denote the angle of the beam's inclination at the moment of fracture.

In all these cases, as have been before stated, when it is only intended to apply the results to the common application of timber to architectural and other purposes, the angle of deflection may be omitted, and the equations then become simply,

$$\begin{array}{ll} 1. \frac{l W}{a d^2} = S, & 2. \frac{l W}{4 a d^2} = S, \\ 3. \frac{l W}{6 a d^2} = S, & 4. \frac{m n W}{l a d^2} = S, \\ 5. \frac{2 m n W}{3 l a d^2} = S. \end{array}$$

But in the comparison of the ultimate strength, under different circumstances, the angle of deflection must be retained; and it remains to show how far the introduction of this datum will explain what has hitherto been considered as paradoxical in the best conducted experiments.

29. One of the most remarkable discrepancies

between theory and experiment, is that already explained (Art. 20); viz. that the strength of a beam fixed at the ends is to that of a like beam merely supported, in the ratio of 3 to 2.

The next anomaly, or what has hitherto been considered as such, is that in which the strength has been observed to decrease in a higher ratio than that of the inverse of the lengths; or, which is more correct, that the strain increases in a higher ratio than the direct ratio of the lengths. Now, it appears from the preceding formulæ, that this is what ought to be the case; for the strain being denoted by

$$f = \frac{1}{4} l W \sec \Delta^2;$$

and as the ultimate deflection, *in quantity*, varies as the square of the length, (see Art. 54,) the *angle* Δ will vary as the length; and consequently, if the length of one beam be supposed l , and the other any number of times the same length, as $m l$, then the strain in the two cases will be as

$$\frac{1}{4} l W \sec^2 \Delta, \text{ to } \frac{1}{4} m l W \sec^2 m \Delta;$$

and therefore, where the resistance to be overcome is the same, W' will be to W as $\sec^2 \Delta : m \sec^2 m \Delta$ instead of being in the simple ratio of $1 : m$, as stated by most writers on this subject. This defalcation of strength was observed by Buffon in his experiments, and has been considered as an inexplicable paradox. Some of the reasons assigned by Dr. Robison may probably have their effect; but it is singular that the above explanation escaped so keen

a mathematician: it may not, perhaps, account for the whole discrepancy observed in the results, but it will certainly tend considerably towards reconciling them with each other. The case in which a beam is fixed at one end and loaded at the other, presents a deviation from the commonly established ratio of an opposite kind; for it has been seen (Art. 28), that the strain in this case is $l W \cos \Delta$; and since the angle Δ , varies as the length, the strain upon a beam of n times the length will be $n l W \cos n \Delta$; and hence, when the resistances are the same, we shall have

$$W : W' :: n \cos n \Delta : \cos \Delta,$$

instead of the simple ratio of $n : 1$; and, consequently, the strength will not decrease so rapidly as in the inverse ratio of the lengths.

The only experiments that I am aware of bearing on this point are those of M. Parent, the results of which are published in the 'Academy of Sciences' for 1707 and 1708, from which the author concludes that the weight necessary to break a beam fixed at one end and loaded at the other, and that of a beam of double the length supported at each end and loaded in the middle, and another equal to the latter, but fixed at each end, were as the Nos. 4, 6, and 10, and the preceding deductions (Art. 28) give the values of those weights

$$\frac{f}{l \cos \Delta} \quad \frac{4f}{2l \sec^2 \Delta} \quad \frac{6f}{2l \sec^2 \Delta}$$

observing, that $2l$, in the two latter expressions, is substituted for l in the formulæ referred to, because the beams are of double length: these ratios are the same as

$$3. \frac{1}{\cos \Delta}, \quad 6. \frac{1}{\sec^2 \Delta}, \quad \text{and } 9. \frac{1}{\sec^2 \Delta},$$

which, if the angle be considerable, will approximate towards the above numbers; but in the references I have seen to these experiments, neither the dimensions of the beams nor the amount of their deflection are stated.

Of the Mechanical Action of the Fibres to resist Fracture.

30. This is a subject which has engaged the attention of several very able mathematicians, whose results have differed very considerably from each other; and although the subject is now properly understood, and all writers adopt the same general view of the theory, yet it will not be uninteresting to take a rapid sketch of the doctrines which have been advanced in support of different hypotheses, by the writers alluded to.

31. Galileo, to whom the physical sciences are so much indebted, was the first who connected this subject with geometry, and endeavoured to compute the strength of different beams upon pure mathematical principles, by tracing the proportional strengths

which different bodies possessed, as depending upon their length, breadth, depth, form, and position.

It appears that this philosopher was led to these investigations, in consequence of a visit which he made to the arsenal and dockyards of Venice, and that they were first published in his 'Dialogues' in 1633. He considered solid bodies as being made up of numerous small fibres applied parallel to each other; and sought, or assumed, at first, the force with which they resisted the action of a power to separate them when applied parallel to their length; and thence readily deduced, that their resistance in this direction was directly as the area of the transverse perpendicular section; that is, as the number of fibres of which the body is composed.

He next considered in what manner the same fibres would oppose a force applied perpendicularly to their length, and ultimately came to the following conclusion: "that when a beam is fixed solidly in a horizontal position in a wall, or other immoveable mass, the resistance of the integrant fibres is proportional to their sum, multiplied into the distance of the centre of gravity of the area of fracture from the lowest point."

32. In order to illustrate this theory a little more explicitly, let RSTV, fig. 1, Plate II., represent a solid wall, or other immoveable mass, into which the beam CG is inserted, and let W be a weight suspended from its other extremity: then supposing

the beam to be insuperably strong in every part except in the vertical section A B C D, the fracture must necessarily take place in that section only; and, *according to the hypothesis of this author*, it will turn about the line C D, whereby the fracture will commence in the line A B, and terminate in the former C D. Galileo also further supposes, that the fibres forming the several horizontal plates, or laminæ, from C D to A B, act with equal force in resisting the fracture, and therefore differ in their energy only as they act at a greater or less distance from the supposed quiescent line, or *centre of motion*, C D.

Now, from the known property of the lever, it is obvious, that the equal forces acting at the several distances, $o1$, $o2$, $o3$, $o4$, &c., of the lever oe , will oppose resistances proportional to their respective distances; and therefore that their sum, that is, the constant force of each particle into its respective distance, is the force which must be overcome by the weight W , acting as on a lever, at the distance oK .

33. This will perhaps be better understood from the illustration given by M. Girard, in his '*Traité Analytique de la Résistance des Solides*,' which is as follows:

Let A C I F, fig. 7, Plate II., represent a longitudinal section of the beam C G, and w' , w'' , w''' , &c., so many small equal weights passing over pins or pulleys, at r' , r'' , r''' , r'''' , &c., acting at the several

distances, Cm' , Cm'' , Cm''' , &c., each weight being supposed equal to the cohesion of its respective lamina; then denoting each of these weights by the constant quantity f , the sum of all their energies, or resistances, will be expressed by the formula:

$$Cm'.f + Cm''.f + Cm'''.f + Cm'''.f + \&c. = \\ f \times (Cm' + Cm'' + Cm''' + Cm'''' + \&c.)$$

This, however, supposes the section to be rectangular, or that the number of fibres in each horizontal lamina is the same. When the beam is triangular, cylindrical, or has any other than a rectangular section, the several small weights must be made proportional to the breadth of the section at the point where each is supposed to act: the illustration, however, is equally obvious.

Since, then, the whole resistance to fracture is made up of the sum of the resistance of every particle or fibre, acting at different distances on the lever CA, which is supposed to turn upon C as a fulcrum, there must necessarily be some point in that lever, in which, if all the several forces were united, their resistance to the weight W would be exactly the same as in the actual operation; *and this point is the centre of gravity of the section represented by AC.*

For let ABC, fig. 10, represent the section of any formed beam whatever, FH, any variable absciss, $=x$, and DE, the corresponding double ordinate, $=y$; then, by what is stated above, the energy or

force of all the particles in the line DE will be as DE . HF, or as xy ; and consequently, the differential of that force will be $yx dx$, and the sum of all these forces will, therefore, be denoted by $\int yx dx$. Now the area of the section may be expressed by $\int y dx$; and, assuming G as the centre of energy sought, we shall have

$$FG \cdot \int y dx = \int yx dx.$$

$$\text{Whence } FG = \frac{\int yx dx}{\int y dx},$$

which is the well-known expression for the centre of gravity.

34. From these considerations, or at least from others tantamount to them, Galileo deduces his general theorem for the resistances of solids; which, from what is above stated, is obviously as follows: viz.

When a beam is solidly fixed with one end in a wall, or other immoveable mass, the weight necessary to produce the fracture, is to the force of direct cohesion of all its fibres, as the distance of the centre of gravity of the section of fracture, from the lowest point of that section, to the length of the beam, or the distance at which the weight acts from the same point.

From other investigations, which it is unnecessary to exhibit in this place, the author endeavours to show, that whatever weight is sufficient to break a beam, fixed as above, double that weight will be

necessary to break a beam of equal breadth and depth, and of twice the length, when supported at each end on two props; and four times the same weight, when the latter is fixed with each end solidly in a wall, &c., &c.

35. Nothing can be desired more simple than the results obtained by this theory; but, unfortunately, it is founded on hypotheses which have nothing equivalent to them in nature. In the first place, it assumes the beam to be inflexible, and insuperably strong, except at the section of fracture: secondly, that the fibres are inextensible and incompressible: and, thirdly, that the beam turns about its lowest point when fixed at one end, or its upper when supported at both, and therefore, that every fibre in the section is exerting its force in resisting extension: and, lastly, if this be not implied in the former objection, that every fibre acts with equal energy, whatever may be the tension to which it is exposed.

With regard to the first of these suppositions, it is obvious that no beam of timber, or any other body with which we are acquainted, is perfectly inflexible; nor any (and more particularly timber) whose fibres are not both extensible and compressible; and consequently, a beam of such matter will not turn about its lowest point, as a fulcrum: and, lastly, the supposition of every fibre exerting a constant resistance, is now known to be decidedly erroneous.

The theory of Galileo having these radical defects, it necessarily happened, as soon as it was attempted to compare its results with experiments, (which the author himself had never done,) that it was found defective. The first person, we believe, who did this, was Mariotte, a member of the French Academy, who, having soon discovered its inaccuracy, proposed to substitute another theory in its place, which was published in 1680, in his '*Traité du Mouvement des Eaux*;' and here we find the first notice of extensible and compressible parts of the section of fracture, the neutral axis, &c. This attracted the attention of Leibnitz, who, after examining the theory of Galileo and the experiments of Mariotte, published his own thoughts on the subject in a Memoir which appeared in the '*Leipsic Acts*,' in 1684.

36. He stated that every body before breaking was subject to a certain degree of deflection, which could not have place if the fibres were, as Galileo had supposed, inextensible; and thence, assuming the principle first suggested by Dr. Hooke, viz., *ut tensio sic vis*, or that the tension varies as the force, he concluded that every fibre, instead of acting with an equal force, exerted a power of resistance proportional to its quantity of extension; or, which is the same, proportional to its distance from the line about which the beam was supposed to turn: but he still considered the fibres to be incompressible, or

at least, what amounts to the same, that the beam turned about its lowest or highest point, accordingly as it was fixed at one end, or supported at both.

Thus, to use a similar illustration in this case to that we have done in the former, instead of the fracture being opposed by the action of the equal forces or weights w' , w'' , w''' , &c., fig. 7, the resistance is supposed to be equal to the decreasing weights w' , w'' , w''' , &c., fig. 8, these being to each other in the proportion of their respective distances from the axis of rotation.

The only alteration which this hypothesis introduced into the final results, was the removal of the centre of energy G, to another point I, fig. 10, nearer or further from the centre of motion, according to the figure of the transverse section of the beam: and this new point is found to be distant from that axis, *by a quantity equal to the product of the distances of the centres of gravity and oscillation from the axis of motion, divided by the depth of the section.*

For, let ABC, fig. 10, represent, as before, the area of fracture of any beam, FH = x any variable absciss, and DE = y , the corresponding double ordinate. Also, make CF = d , and let f represent the absolute and ultimate force of a fibre at C, at the instant of rupture: then, since the resistance opposed by each fibre is supposed to vary as its tension,

or as its distance from F, we have $d : x :: f : \frac{fx}{d} =$ the

force of a fibre at H; and the number of fibres acting at this distance being y , we shall have $\frac{fxy}{d}$, for the sum of the resistances of all the fibres or particles in the line DE: but this force, acting upon the lever at the distance HF, its resistance will be expressed by $\frac{fx^2y}{d}$; and hence the sum of all the resistances of every fibre in the section will be $= \int \frac{fyx^2dx}{d}$.

Now this is to be equal to the direct cohesion of all the fibres acting at some required distance FI; that is,

$$FI \cdot \int y dx \cdot f = \frac{f}{d} \times \int yx^2 dx, \text{ or}$$

$$FI = \frac{1}{d} \cdot \frac{\int yx^2 dx}{\int y dx}.$$

The variable part of this expression is exactly equivalent to the general formula for the centre of oscillation of a surface, multiplied by the former expression for the centre of gravity, that is, using $\frac{1}{d}$ as a coefficient,

$$FI = \frac{1}{d} \cdot \frac{\int yx^2 dx}{\int y dx} = \frac{\int yx dx}{\int y dx} \times \frac{\int yx^2 dx}{\int yx dx} \cdot \frac{1}{d},$$

as is obvious. And since these centres are generally known in most of the figures which fall under consideration in the present inquiry, we may avail ourselves of them, independently of calculation, in determining what may properly be termed the *centre of energy*, or *centre of tension*: but, in other

cases, recourse must be had to the general differential expression

$$FI = \frac{1}{d} \times \frac{\int y x^2 dx}{\int y dx}.$$

37. Referring again to the formula previously found for the centre of energy on the Galilean hypothesis, and denoting the absolute strength of cohesion on a square inch by f ; also writing d for the depth of the beam in inches, a the area of fracture, and l the length likewise in inches: then the general expression for the ultimate strength of any beam, fixed with one end in a wall, would be on the hypothesis of

$$\text{Galileo, } S = \frac{\int y x dx}{\int y dx} \cdot \frac{af}{l}.$$

$$\text{Leibnitz, } S = \frac{\int y x^2 dx}{\int y dx} \cdot \frac{af}{l}.$$

When the beam is *supported* at both ends, these must be each multiplied by four; and when *fixed* at both ends, by eight.

This being the case, both theories give the same results, so far as relates to the comparison of similar-formed beams, but of different dimensions: thus, for example, it appears from both, that when the breadth and depth are the same, the strength varies inversely as the length: when the length and depth are the same, the strength varies directly as the breadth; and when the length and breadth are the same, the strength varies as the square of the depth:

deductions which have been found to agree very nearly with experiment.

38. There are other conditions, however, resulting from the same formulæ, in which the two theories are totally irreconcilable with each other, and in which neither will agree with actual experiment.

In the first place, although the proportions are the same, the absolute strength in the one case is to that in the other as two to three in rectangular beams; and in triangular ones the disagreement is still more striking. Again, according to Galileo, the strength of a triangular beam with its edge upwards, when fixed by one end in a wall, or with its base upwards, when supported at both ends, is to the strength of the same beam in the reversed position, as one to two; and according to Leibnitz, as one to three: whereas, experiment shows it to be neither the one nor the other.

So also, square beams fixed, in one instance with the side vertical, and in the other with the diagonal vertical, have their strengths, according to Galileo, in the ratio of 1 to $\sqrt{2}$; and according to Leibnitz, in that of $\frac{4}{3} : \sqrt{2}$; whereas, experiments show the beam to *be stronger in the former position than in the latter*.

In both theories, also, the strength of hollow cylinders, not bored through the axis, but nearer one side than the other, varies according as the boring is nearer the upper or lower surface, and is

greatest of all when the cylinder is infinitely thin on that side about which it is supposed to turn ; whereas, experiment shows the very reverse of this, and that the beam is absolutely weakest, when, according to both these writers, it ought to be the strongest.

39. The subject was afterwards taken up by James Bernoulli, who observed, that the instant before a body is broken across with a transverse strain, such as we have been considering, a part of the fibres only are in a state of tension, and a part in a state of compression,—a circumstance that had not before been introduced into the conditions of this problem (except perhaps by Mariotte) ; and he moreover doubted of the justness of the principle, *ut tensio sic vis*, employed by Leibnitz, and made some experiments, whereby he proved that, at least, this is not a universal law of nature. But he unfortunately stopped at this point, contenting himself with showing the inadequacy of the theory he had been examining ; but without substituting any new one in its place, except so far as his theory of the *elastic curve* (a problem which arose out of the present question) may be considered as applicable to this subject. Had he pursued the idea he seems first to have promulgated, of a part of the fibres being stretched, and a part compressed—and, consequently, that the line about which the beam turns, is somewhere within the area of the section of fracture—we might have expected, from his extra-

ordinary talents, a complete solution of this interesting problem: instead of which, he contented himself with stating a few general observations, and with pointing out the difficulty of determining the neutral axis, or of that line which suffers neither compression nor extension; which is the principal desideratum for establishing a correct theory.

40. The next important step in this inquiry was made by Dr. Robison, under the article **STRENGTH**, in the ‘Encyclopædia Britannica;’ and here for the first time the position of the neutral axis, or that line in a beam which suffers neither extension nor compression, is introduced as a necessary datum. The position of this line was not, however, determined by Dr. Robison, nor had it been attempted to be found, to the best of my knowledge, when I made the experiments on which I founded my ‘Essay on the Strength of Timber.’ In these, I found its position in two or three different kinds of wood experimentally, and thence endeavoured to determine the law of action of the fibres at different distances from the neutral axis, and arrived at a conclusion, “that, however difficult it might be to account for the fact, the theory of resistance assumed by Galileo was nearer the truth than the generally admitted law, *ut tensio sic vis*.” But in this investigation, I had fallen into an error, by assuming the momenta of the forces on each side the neutral line to be equal to each other, instead

•

of the forces themselves,—an error which was first pointed out by Mr. Eaton Hodgkinson, in a very able Paper on this subject, in vol. iv. of the ‘Manchester Memoirs,’ New Series. This correction being made, the agreement is greatly in favour of the latter hypothesis, the truth of which is not now, I believe, doubted by any one,—making, of course, great allowance for the very variable force of the fibres in different kinds of wood, and even of the fibres in the same section, when the latter is of considerable area.

Fortunately it is seldom that the strength of timber is of great importance, except in the form of rectangular or square beams; and its strength in these forms is deducible from experiments on similar-formed beams, without any reference to the exact position of the neutral axis; but still, as a point of theory, and wherever the question relates to beams of other figures, it is essential that we should have reference to it. Without, therefore, pursuing our historical sketch to a greater length, we shall proceed at once to illustrate the nature of the neutral axis, and the consequent mechanical action of the fibres in resisting fracture.

41. It has been before remarked, that when a beam is submitted to a transverse strain, being either supported at its two extremities and loaded in the middle, or fixed at one end in a wall and loaded at the other, it will not, as was formerly assumed by

Galileo and Leibnitz, turn about its upper or lower surface, but about a line within the area of fracture; which line is what is denominated the *neutral line*, or *neutral axis of rotation*.

If the fibres of a beam (referring, for instance, to fig. 1, Plate II.) were wholly incompressible, there is no doubt that the beam, when loaded at the end I, would turn about the line CD; and every fibre of it, from C to A, would be in a state of tension.

And, on the contrary, if the fibres were wholly inextensible, then, if the beam turned at all, it must be about the line AB, and every fibre from A to C would be in a state of compression.

But we know of no bodies in nature that are either inextensible or incompressible; and, therefore, the rotation of the beam will neither take place about A nor C, but on an intermediate point or line, n ; and all the fibres above that line will be in a state of tension, and those below it in a state of compression; while those which are situated so as exactly to coincide with its plane, will be neither extended nor compressed, but be in a state perfectly neutral with regard to both.

42. It is obvious, that the fibres submitted to tension are more and more extended as they are situated further from the point n , and at A their extension is the greatest. The same has also place with the fibres submitted to compression, this being greatest at C; and, whatever may be the law of the

forces necessary for producing these several degrees of tension and compression, or whatever may be the law of the resistances which they offer after they are produced, we may conceive some point situated between A and n , into which, if all the resistances to tension were united, and some point between n and C, into which, if all the resistances to compression were condensed, the reaction arising from these two aggregate forces would be the same as in the actual operation; and these points are what are designated the *centres of tension and compression*.

43. With regard to the situation of the neutral axis, we have nothing to guide us in the determination but experiments; and these seem to indicate, that in rectangular fir beams it is at about $\frac{5}{8}$ ths of the depth of the section of fracture when the beam is broken on two supports; or, at $\frac{3}{8}$ ths of the same when it is broken by having one end fixed in a wall, and loaded at the other;—that is, in both cases the number of fibres exposed to compression are to those submitted to tension in about the ratio of 5 to 3.

This was pointed out very unequivocally in several of the experiments stated in the following pages; the beams in most cases showing very distinctly, after the fracture, what part of the section had been compressed, and what had experienced tension; the compressed fibres always breaking very short, having been first crippled by the pressure to

which they had been exposed, while the lower part was drawn out in long fibres, frequently 5 or 6 inches in length.

Another criterion was found in the external appearance of the side of the beam exposed to pressure before the fracture took place: this always exhibited itself in a wedge-like form, the lower point of which, when the beam was broken on two props, was commonly found to divide the depth in about the ratio above stated.

It should be observed, however, that Mr. Hodgkinson, in the experiments he has described in the article above referred to, finds the ratio to be nearly 4 to 4, instead of 3 to 5; and unquestionably there must be considerable irregularities in the position of this line in different specimens of timber, even of the same kind, and much more in woods of different kinds. Without, therefore, attempting to determine this point, we may at all events assume, from what has been above stated, that there is necessarily such a point in the area of fracture in all beams; and this is sufficient for our present purpose, as it is intended in the first instance to speak here only of rectangular beams.

44. Referring to fig. 2, Plate II., let n denote the neutral axis of the rectangular beam $ACIF$, bAn representing the part suffering extension, and nCd that submitted to compression. Let also t denote the amount of tension of the extreme fibre bA , and C

the compression of the extreme fibre Cd . Then, assuming that the resistance to tension of a fibre is proportional to the quantity of tension, or to its distance from the neutral axis, if we call the whole depth of extension $An=d'$, and denote any variable distance from n by x , we have $d' : t :: x : \frac{tx}{d'}$, the tension of a fibre at that part. Consequently, the sum of all the tensions will be expressed by

$$\int \frac{tx dx}{d'} = \frac{1}{2} t d' \text{ (when } x = d' \text{)};$$

and in the same way, assuming the same law of compression, the sum of all the compressions will be expressed by

$$\int \frac{cx dx}{d''} = \frac{1}{2} c d'' \text{ (when } x = d'' \text{)},$$

d'' denoting the depth of compression; which two forces are equal to each other; for it is this equality which determines the motion to take place about the line n : therefore $\frac{1}{2} t d' = \frac{1}{2} c d''$, or $t d' = c d''$.

45. It may be proper to observe, that c here is not intended to represent the force requisite to compress a fibre the same quantity that the force t extends it, but simply the force of compression at C , corresponding to that of the tension at A .

46. Now, to estimate the effect of those forces, it will be seen that the tension of any fibre at the variable distance x being $\frac{tx}{d'}$, and this acting at the

distance x , the effect will be $\frac{t x^2}{d'}$, and the sum of all the effects

$$\int \frac{t x^2 d x}{d'} = \frac{1}{3} d' t^2 \text{ (when } x = d') ;$$

and in the same way the sum of the compressing forces will be

$$\int \frac{c x^2 d x}{d''} = \frac{1}{3} d'' c^2 \text{ (when } x = d'') ;$$

and, therefore, the whole sum of both species of resistances will be

$$\frac{1}{3} d'' c^2 + \frac{1}{3} d' t^2 ;$$

and, since $d'' c = d' t$, this sum becomes

$$\frac{1}{3} (d'' + d') d' t ;$$

or, taking $d'' + d' = d$, the whole depth, it becomes

$$\frac{1}{3} d d' t.$$

That is, in rectangular beams the resistance is equal to the product of one-third of the whole depth into the depth of tension, and into the force of tension on the extreme fibre.

If, therefore, we knew in all cases the depth of tension, or the relative depth of tension and compression, and the force of direct cohesion, we might compute the transverse strength of rectangular beams, independently of any other data; but these being both very precarious, the best method of determining the strength of beams of wood is by comparative experiments on other beams; for, since the resistance is expressed by $\frac{1}{3} d d' t$, and d' is always proportional to d in the same material, it follows that

the whole resistance is as the square of the depth, as is stated Art. 28; and the resistance being also necessarily as the breadth, it follows that in all rectangular beams the resistance is as the breadth and square of the depth; and we have seen that the strain is as the length into the weight: consequently, calling the breadth b , the depth d , the length l , and the breaking weight w , we ought to have $\frac{lw}{bd^2} = S$, a constant quantity for materials of the same kind, when fixed or supported in the same manner; and when they are fixed or supported in different ways, the formulæ investigated Art. 16 et seq. will enable us still to make the requisite reductions.

The principal data, therefore, that a practical man requires for determining the requisite dimensions of beams, rafters, &c., are such as give this constant quantity S , for all variety of woods; and such will be found in a subsequent part of this Treatise.

On the Deflection of Beams.

47. Hitherto we have considered a beam of timber as inflexible in every part except at its point of fracture, which served to simplify the investigations and the conception of the subject, without in any way affecting the accuracy of the result, the strain at the point of fracture being the same in both cases; but it is frequently very important to know the amount

of deflection a given weight will produce, and the law of action which obtains in these cases.

48. In order to this investigation, let $ABCD$, fig. 5, Plate II., represent a beam fixed into a solid wall, and in its natural horizontal position, its weight being supposed nothing, or inconsiderable with regard to that with which it is loaded: and let us suppose it to be made up of the several parts $AB\ ab, ab\ a'b', a'b'\ a''b'',$ &c., each of which is considered to be subject to compression and extension: then, when the beam is loaded with a weight W , it will be brought into the curvilinear form shown in the second position in the figure. Draw the several tangents $Am, an, a'o, a''p,$ &c.; and admitting that the quantity of extension and compression is proportional to the extending and compressing forces, we shall have the several angles $mAn, na'o, oa''p, pa'''d,$ (see fig. 6,) proportional to the distances $CF, Cf, Cf', Cf'',$ &c., these being the effective lengths of the levers, by means of which the force or weight W is exerted at those several points: and the same will have place if we suppose the number of laminæ to be indefinitely great, and therefore the thickness of each indefinitely small: and hence we see the fundamental property of the curve which a beam thus fixed and loaded will assume; viz. "that the curvature at every point is as the distance of that point from the line of direction of the weight," which is, in fact, the *elastic curve*, first proposed by Galileo,

but the correct investigation of which we owe to James Bernoulli, who published it in the 'Memoirs of the Academy of Sciences' for 1703. Other investigations of it have since been given by John Bernoulli, 'Opera Omnia,' tom. iv. p. 242; as also in his Essay on the 'Theory and Manœuvres of Ships;' and particularly by Euler, in the Appendix to his celebrated work, 'Methodus inveniendi Lineas Curvas.'

49. It is to be observed, however, that the supposition of the extension and compression being exactly proportional to the exciting forces, is only a particular and very limited case of the elastic curve; for if that extension were as any function of those forces, it would still not wholly change, although it would modify, the fundamental property of it: but its investigation under this general character would carry us far beyond our present purpose, and, at the same time, would be of no use in our future investigation; for it appears from experiment, that the quantity of extension, in consequence of the imperfect elasticity of the fibres, is very irregular, and that after a certain deflection has been obtained, it seems subject to no determinate law; a circumstance which we have endeavoured to illustrate in a subsequent article: but during the early part of the experiment, that is, while the weight is considerably less than that which is required to produce the ultimate fracture,

the law of the deflections is nearly uniform, and proportional to the exciting force; it will, therefore, be sufficient to consider the elastic curve under this particular case, being the only one that is applicable to the present inquiry.

50. Let, then, AB , fig. 10, Plate III., represent a thin elastic lamina, without weight, and in its first natural horizontal position; AC the position of it after being loaded with any given weight W : at any point in the curve R draw the tangent RT , and conceive the curve to be divided into an indefinite number of equal small parts, Aa , Rr ; and since, by the hypothesis, the extension of each fibre is proportional to the force by which it is excited; if rs and ba be drawn perpendicular to the curve at a and r , the former may be taken to denote the extension of the particle Aa , and the latter that of the particle Rr ; and we shall have $rs : ab :: \text{force in } R : \text{force in } A$, or $:: CL \times W : CG \times W$. Let AF and RX be the radii of curvature at the points A and R , then the triangles Aab and AaF , as also Rsr and RrX , are similar; and, therefore, since $Aa = Rr$, we have

$$rs : Rr :: Rr : RX$$

$$ab : Aa :: Aa : AF;$$

$$\text{therefore } rs : ab :: AF : RX;$$

$$\text{but } rs : ab :: CL : CG,$$

$$\text{and consequently, } CL : CG :: AF : RX,$$

whence again,

$$CL \cdot RX = CG \cdot AF, \text{ a constant quantity} = A.$$

In order now to trace the property of the curve, let $CL=x$, $RL=y$, and $RC=z$; then, as is shown by writers on the differential calculus, the radius of curvature

$$RX = \frac{dx^3}{-dx \cdot d^2y} = \frac{(dx^2 + dy^2)^{\frac{3}{2}}}{-dx \cdot d^2y};$$

and consequently

$$\frac{x dx^3}{-dx d^2y} = A, \text{ or } \frac{x(dx^2 + dy^2)^{\frac{3}{2}}}{-dx \cdot d^2y} = A.$$

In its present form this equation is not integrable, but we may accommodate it to our purpose, without any sensible error, while the deflections are small, by supposing $dx=dz$, in which case it becomes

$$\frac{x dx^2}{-d^2y} = A, \text{ or } x dx = A \frac{d^2y}{-dx}.$$

Or assuming dx as constant, and taking the integral

$$\frac{1}{2} x^2 + C = \frac{-A dy}{dx}, \text{ C being the correction.}$$

Now, when $x=l$, $\frac{1}{2} l^2 + C = 0$, $\frac{dy}{dx}$ being in that case $= 0$, therefore the correct integral is

$$\frac{1}{2} (x^2 - l^2) = -A \frac{dy}{dx}.$$

Multiplying now by dx , we have

$$\frac{1}{2} dx (x^2 - l^2) = -A dy,$$

and taking the integral

$$\frac{1}{6} l^2 x - \frac{1}{6} x^3 = A y,$$

which requires no correction.

By means of this equation, the curve may be constructed while the deflections are small with regard

to the length of the laminæ; but it will obviously apply to no other case, because it is obtained on a supposition of dx being equal to dz , which is in no case strictly true; although the difference, while the deflections are small, is inconsiderable, and may be admitted without any sensible error.

Writing l for x and b for y , the above becomes

$$\frac{l^3}{3b} = A, \text{ or } \frac{l^3}{3b} = CG \cdot AF:$$

or, since in the case here supposed $CG = l$, *very nearly*, this equation may be still further reduced to

$$\frac{l^2}{3b} = AF:$$

and hence it follows, that while AF remains constant, or the curvature at A is the same; that is, while the strain upon the beam at that point is constant, the deflection b must vary as the square of the length.

But the strain (the weight remaining the same) is as l ; or AF is reciprocally as l : and, therefore, while the weight is the same,

$$\frac{l^2}{3b} = \frac{AF}{l} \text{ or } \frac{l^3}{3b} = AF:$$

consequently, while the weight remains the same, the deflection b is as the cube of the length: but we have seen that, *cæteris paribus*, the deflection is as the weight; therefore, generally

$$\frac{W l^3}{3b} = E \text{ a constant quantity;}$$

that is, the deflection is as the weight into the cube of the length.

51. This deduction being contrary to the experimental results of M. Girard, ought to be examined with caution: we propose, therefore, investigating the nature of the curve on different principles, and on such as will probably be more intelligible to many readers.

It has been shown above, that an approximation to the actual state of the curve is all that can be obtained; and this approximation may be obtained perhaps more satisfactorily as follows.

Let ABCD, figs. 5, 6, Plate II., represent the deflected beam, and let it be divided as above supposed (Art. 48) into any number of equal inflexible parts, $ABab$, $ab a' b'$, &c., and let ad , $a' d'$, $a'' d''$, &c., drawn perpendicular to the respective tangents at A, a , a' , &c., represent the deflections at those points, which, from what has been above shown, will be proportional to CF, Cf, Cf', &c.; and as the investigation is only intended to apply to small deflections, let us consider these several lines, ad , $a' d'$, &c., instead of being perpendicular each to its respective tangent, to be all parallel to each other, and perpendicular to Am; let us also denote the first of these ad by d , which may be denominated the *element of deflection*, and let the number of parts or laminæ into which the beam is divided be denoted by m , then we shall have

$$\frac{m}{m} d = ad$$

$$m : m - 1 :: d : \frac{m - 1}{m} d = a' d'$$

$$m : m - 2 :: d : \frac{m - 2}{m} d = a' d''$$

$$m : m - 3 :: d : \frac{m - 3}{m} d = a''' d'''$$

&c. &c. &c.

Also, according to our supposition,

$$n m = m \times a d = \frac{m^2}{m} d$$

$$n o = (m - 1) a' d' = \frac{(m - 1)^2}{m} d$$

$$o p = (m - 2) a'' d'' = \frac{(m - 2)^2}{m} d$$

&c. &c. &c.

Whence the whole deflection $m D$ will be expressed by the series

$$m D = \frac{d}{m} \left\{ m^2 + (m - 1)^2 + (m - 2)^2 + \&c. 1^2 \right\} \dots (1)$$

or by the summation of the series,

$$m D = \frac{d}{m} \left\{ \frac{m^3}{3} + \frac{m^2}{2} + \frac{m}{6} \right\}, \text{ or}$$

$$m D = d \left\{ \frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right\}.$$

That is, while the number of parts m are supposed finite, $m D$ varies as $\left(\frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right) d$; but when m is infinite, then the two latter terms vanish, as being inconsiderable with regard to the first; and we have $m D = \frac{m^2 d}{3}$.

In the same manner, if l' were the length of any other beam, of which the number of parts were m' , but the parts individually in length equal to the

former, and the element of deflection d' , we should have

$$m' D' = \frac{m'^2 d'}{3};$$

Whence

$$m D : m' D' :: m^2 d : m'^2 d'; \text{ but } m : m' :: l : l';$$

therefore,

$$m D \text{ varies as } \frac{l^2 d^*}{3};$$

that is, the deflection varies as the square of the length, and the element of deflection; but the element d obviously varies as the strain; that is, as $l W$: therefore, again, the deflection varies as $\frac{l^3 W}{3}$; or denoting the deflection $m D$ by b , we have $\frac{l^3 W}{3 b} = E$, a constant quantity, the same result as before.

52. The same may be otherwise demonstrated as follows:

In the above investigation it is shown that $D m$, which is supposed to represent the deflection, is expressed by the equation

$$m D = d \left\{ \frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right\},$$

* We have used the above process for the convenience of those who may not be acquainted with the fluxional or differential calculus: those who are, will see immediately that the summation, expressed in equation (1), is equal to $\frac{d}{m}$ times the integral of $x^2 dx$; that is,

$$\frac{d}{m} \int x^2 dx = \frac{dx^3}{3m} = \frac{dm^2}{3} \text{ when } dx = m.$$

and that in any other beam, of which the number of parts are m' , the deflection is also

$$m' D' = d' \left\{ \frac{m'^2}{3} + \frac{m'}{2} + \frac{1}{6} \right\},$$

from which we conclude, that when m is infinite, the deflections are as

$$d m^2 : d' m'^2; \text{ or as } d l^2 : d' l'^2;$$

where l and l' denote the two lengths. If this should not appear to involve all that precision and accuracy that may be desired, it may be considered under a point of view somewhat different to the former, and will probably carry more conviction with it to some of our readers:

Supposing, therefore, the equation

$$m D = d \left\{ \frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right\}$$

to be established; and calling l the length of the beam, and λ the length of each of the equal sides of the polygon, we shall have $\frac{l}{\lambda} = m$; and substituting this for m in the preceding equation, we obtain

$$m D = d \left\{ \frac{l^2}{3 \lambda^2} + \frac{l}{2 \lambda} + \frac{1}{6} \right\}, \text{ or}$$

$$m D = d \left\{ \frac{2 l^2 + 3 l \lambda + \lambda^2}{6 \lambda^2} \right\};$$

and in the same manner, if the length of another beam is l' , and $m' D'$ denotes its deflection, we find

$$m' D' = d' \left\{ \frac{2 l'^2 + 3 l' \lambda + \lambda^2}{6 \lambda^2} \right\};$$

λ , or the length of each side of the polygon, being,

by the supposition, the same in both cases; we shall have, therefore,

$$m D : m' D' :: d \{2 l^2 + 3 l \lambda + \lambda^2\} : d' \{2 l'^2 + 3 l' \lambda + \lambda^2\}.$$

This result is wholly independent of any particular value of λ , and therefore is true, when λ becomes indefinitely small; that is, in the case of a continued curve. But here, as λ is indefinitely small, the last two terms of each of the third and fourth members of the above ratio vanish, and that ratio then becomes simply

$$m D : m' D' :: d l^2 : d' l'^2;$$

that is, the deflection varies as the element of deflection into the square of the length; or, as the element of deflection into the square of the length divided by 3, as we have found it in the article in question.

53. In a similar manner we may investigate the law of deflection when the weight, instead of being all applied at the extremity of the beam, is equally distributed throughout its whole length, or when it is divided into equal portions, and suspended at equal distances, as at the points a' , a'' , a''' , &c., fig. 5, Plate II.

For, calling d' , as before, the element of deflection $= a d$, it is obvious that the successive deflections, instead of decreasing as before, in the simple ratio of the length, will now decrease as the square of the length, because both the weight and the length of

lever decrease in the same manner. Our successive deflections therefore, in this case, will be

$$\begin{aligned}\frac{m^3}{m^2} d' &= a d \\ m^2 : (m-1)^2 :: d' : \frac{(m-1)^2}{m^2} d' &= d' d' \\ m^2 : (m-2)^2 :: d' : \frac{(m-2)^2}{m^2} d' &= d'' d'' \\ m^2 : (m-3)^2 :: d' : \frac{(m-3)^2}{m^2} d' &= d''' d''' \\ &\&c. \quad \&c. \quad \&c.\end{aligned}$$

Also, according to the same supposition as that above adopted, we shall have

$$\begin{aligned}n m &= m . a d = \frac{m^3}{m^2} d' \\ n o &= (m-1) d' d' = \frac{(m-1)^3}{m^2} d' \\ o p &= (m-2) d'' d'' = \frac{(m-2)^3}{m^2} d' \\ &\&c. \quad \&c. \quad \&c.\end{aligned}$$

Whence the whole deflection $m D$ will now be expressed by the series

$$m D = \frac{d'}{m^2} \left\{ m^3 + (m-1)^3 + (m-2)^3 + \&c. 1^3 \right\}$$

or by summation,

$$\begin{aligned}m D &= \frac{d'}{m^2} \left\{ \frac{m^4}{4} + \frac{m^3}{2} + \frac{m^2}{2} \right\}; \text{ or,} \\ m D &= d' \left\{ \frac{m^2}{4} + \frac{m}{2} + \frac{1}{4} \right\}:\end{aligned}$$

which expression is analogous to that in Article 51, and shows that in this case also, when m is infinite, that is, when the weight is uniformly distributed,

the deflection is as the weight and cube of the length, or as the square of the length and element of deflection, because the expression then becomes

$$m D = \frac{m^2}{4} d.$$

But in order to compare the real quantity of deflection in this case with that of the former, it must be observed, that the weight being the same, the strain on the beam will, in the first instance, be double what it is in the second; and the element d in the former will be double d' in the latter, or $d' = \frac{1}{2} d$. Substituting, therefore, $\frac{1}{2} d$ for d' , our expression

$$m D = \frac{m^2}{4} d', \text{ becomes } \frac{m^2}{8} d;$$

whereas in the former case it is $\frac{m^2}{3} d$; therefore the beams being of the same length, the deflection, when the weight is all collected at the extremity, is to that of the beam equally loaded throughout its length with the same weight, as

$$\frac{m^2}{3} d : \frac{m^2}{8} d, \text{ or as 8 to 3.}$$

The expression for the elasticity in this case will therefore be $\frac{l^3 W}{8 b} = E$, the same constant quantity as before.

The principles of investigation given in Art. 52 are equally applicable in this case.

54. In the preceding investigations the deflections have only been considered with reference to beams

fixed at one end: let us now endeavour to investigate the same, on a supposition of their being supported at both ends. In order to which, it may be observed, in the first place, that whatever weight is just sufficient to break a beam fixed by one end in a wall, the same weight may be borne at the other end of it, (the arms or levers being supposed of equal length,) if the wall were removed, and the beam merely supported on a fulcrum, or prop, in its middle point, as in fig. 3, Plate II., the tension in both cases being the same; just as a line passing over a pulley, and loaded at each end with an equal weight, has the same tension as a single fixed line loaded with only one of those weights; and what is here stated of the ultimate degree of tension, is obviously true of any quantity of it: that is, whatever tension the fibres may have in the former case, they will have precisely the same in the latter. But it is not the same with the deflections under these two circumstances of equal strains, the element of deflection being, in one case, double that in the other. For the extension of the fibres $A b$, $A b$, fig. 2 and fig. 3, Plate II., being equal by the supposition, the angles $A n b$, in both figures, will be equal: but as in one case, fig. 2, the line $n b$ is vertical, and in the other, fig. 3, it declines equally from the vertical with the line $n A$, the deflection of the beam, (supposing it, for simplicity, to remain inflexible in every part, except in the section $A n C$,) in the latter case, from the line $H H$, will be only half

that of the half-beam fixed as in fig. 2; that is, the element of deflection in the former instances will be only half that in the latter; and consequently, as we have shown that the deflections are, *cæteris paribus*, as the element of deflection, it follows, that the successive and ultimate deflections in the two cases will have the same ratio; that is, they will be to each other as 2 to 1.

Again, the beam $FI F' I'$, fig. 3, is similarly situated, at least as far as our present question is concerned, with regard to the strain upon it, and therefore to its deflections, as the equal beam $FIF'I'$, fig. 4; whether we consider the latter to rest against a fulcrum at C , and to be strained by the two weights W, W' passing over the pulleys Q, Q' ; or, as being supported on two fulcrums, F, F' , and loaded in the middle with the weight P , equal to the two weights W, W' .

Hence, then, we conclude, that the deflection of a beam fixed at one end in a wall, and loaded at the other, is double that of a beam of twice the length, supported at both ends, and loaded in the middle with a double weight; that is, the strain being the same in both cases: consequently, when the weights are the same, the deflection in the first instance is to that in the second as 4 : 1.

And when the length and weight are both the same, the deflections will be to each other as 1 : 32.

For the strain will be four times greater on the beam fixed at one end than on that supported at

both; and, therefore, all other things being the same, the element of deflection would also be four times greater: but we have seen, that with the same strain the element of deflection is double in one case what it is in the other: it will, therefore, by combining the two effects, be eight times greater: also, the entire deflection is as the element of deflection into the square of the length; and, according to our supposition, the length is double; whence, upon the whole, it appears that the deflection in the one case is to that in the other as $1 : 8 \times 4$, or as 1 to 32.

The same formula will, therefore, apply in this case as in Art. 50; viz. $\frac{l^3 W}{3 b} = E$, a constant quantity; observing only, that the value of E is here thirty-two times greater than in the former.

55. When the weight is distributed throughout the length of the beam, instead of being all collected in the middle, it is a known mechanical principle, that the strain on the centre will be the same as it would be with half the entire weight collected in that point; and, consequently, the element of deflection in the same place will also be one-half of what it would be if the whole weight was collected there.

But now, in order to compare the strain and consequent deflection at any other point, D, fig. 9, Plate II., we must first observe, that the resistance

of the fulcrum at B is constant; and therefore, that the strain at D, as arising from that resistance, will be found as follows; viz. $CB : DB :: d : \frac{DB d'}{CB}$ = the element of deflection at D, as arising from the resistance at B; d' denoting the deflection at C.

But the point D has a further strain to sustain, and consequently a further deflection, arising from the weight of the part between C and D. Now this weight will be to the whole weight W, as CD to AB, or 2CB; that is,

$$2CB : CD :: W : \frac{CD \cdot W}{2CB}.$$

Consequently, the deflection arising from this strain, as referred towards B, will be

$$CB^2 : CD \times BD :: d' : \frac{CD \cdot BD}{CB^2} d'.$$

Whence the entire deflection from the tangent of the curve at the point D will be

$$\frac{DB}{BC} d' + \frac{CD \cdot DB}{BC^2} d' = \frac{(CB + CD) DB}{BC^2} d'.$$

Which deflection, referred to the perpendicular BF, will be

$$\frac{(CB + CD) DB^2 d'}{BC^2}.$$

If, now, we denote CB by m , and DB by n , in which case $CD = m - n$, the above will become

$$\frac{(2m - n) n^2}{m^2} d' = \frac{2m n^2 - n^3}{m^2} d'.$$

And, by giving to n the successive values, 1, 2, 3, &c.,

as in our preceding investigations, and summing the resulting series, or by finding the value of

$$\int \frac{2mx^2 - x^3}{m^2} d' dx,$$

when $x = m$, we shall have for the entire deflection,

$$BC = \frac{5m^2}{12} d.$$

But it has been shown, that in the former case, where the weight is all collected in the middle, the deflection is $\frac{m^2}{3} d$; and, therefore, since $d' = \frac{1}{2} d$, the deflections in the two cases will be as $\frac{1}{3} : \frac{5}{24}$, or 8 to 5.

Now it has been seen, that when a beam or rod is fixed only at one end, the deflection, when the weight is uniformly distributed, is to the same when that weight is collected at the extremity, as 3 to 8: whereas we have found above, that when the beam is supported at its ends, the deflections in the like cases are to each other as 5 to 8.

Whence, if a long rod or plank is, in the first instance, supported in the middle, and the ends be deflected; and, in the second, the ends are supported, and the middle left to descend, the deflection in the latter case is to that in the former as 5 to 3.

Of the Deflection as depending on the Breadth and Depth.

56. In the preceding investigations we have supposed the beams, although of different lengths, to be

all of the same breadth and depth ; or, as opposing equal resistance : when these dimensions are not the same, the resistance is as the breadth and square of the depth, Art. 46 ; and, therefore, when the weight is increased in that proportion, the quantity of extension will, by hypothesis, be the same, the length being here supposed constant ; but, by a reference to fig. 2, Plate II., it will appear, that the extension of the fibre bA being supposed constant, the angle bnA , or HAF , (which is equivalent to what we have denominated the element of deflection,) will be inversely as nA , or CA , the depth of the beam.

Hence with the same weight the deflection will be inversely as the breadth and square of the depth into the element of deflection, which is itself inversely as the depth. Hence, every thing else being the same, the deflection will vary inversely as the breadth and cube of the depth ; but we have seen that when the breadth and depth are constant the deflections are as the weight and cube of the length, therefore generally, if l denote the length of a beam, b its breadth, and d its depth, also W the weight with which it is loaded, the deflection will vary as $\frac{l^3 \cdot W}{b \cdot d^3}$; and if, therefore, we denote the deflection by δ ,

$$\frac{l^3 W}{b d^3 \delta} = E, \text{ a constant quantity.}$$

57. This is a conclusion which necessarily arises out of the above investigation, but being at variance

with the experiments of M. Girard, which are very numerous, I was a little surprised at the result thus obtained, and re-examined my investigations, under an impression that some error had crept in, and escaped my observation. At length, not being able to discover any, I referred to the experimental results, the greater part of which were in favour of my own theoretical deductions: still, however, as these were different beams, and many of the deflections considerable, while the investigation was supposed to apply only to those cases in which it was very small, I was still doubtful, and therefore procured three pieces of fir, each 6 feet 6 inches in length, and 2 inches in depth, by $1\frac{1}{2}$ inch in breadth, and of very uniform texture: these pieces were rested on two props, as represented in Plate IV.; first at the distance of 3 feet, and then at 6 feet.

If, therefore, the deflections varied as the square of the length, according to the results of M. Girard, the deflections ought to be, in the second case, four times what they were in the first; but if the deflections were as the cubes of the lengths, as they should be according to my deduction, then the deflection would be eight times as much. I accordingly made the experiments with great care; and the following are the results that were obtained.

No. 1.

Feet long.	Inches deep.	Breadth.	Weight, lbs.	Deflection.
3	2	1½	120	·09
3	2	1½	180	·12
6	2	1½	120	·68
6	2	1½	180	1·00

The same piece.

3	1½	2	120	·19
3	1½	2	180	·28
6	1½	2	120	1·38
6	1½	2	180	1·91

No. 2.

3	2	1½	120	·10
3	2	1½	180	·15
6	2	1½	120	·72
6	2	1½	180	1·05

The same piece.

3	1½	2	120	·18
3	1½	2	180	·28
6	1½	2	120	1·30
6	1½	2	180	2·00

No. 3.

3	2	1½	120	·07
3	2	1½	180	·11
6	2	1½	120	·65
6	2	1½	180	·96

The same piece.

3	1½	2	120	·16
3	1½	2	180	·24
6	1½	2	120	1·25
6	1½	2	180	1·85

58. It was impossible, after these experiments, any longer to doubt the correctness of the preceding investigations; the deflection of the 6-feet beams answering so very nearly to the cube, or to eight times that of the same at 3 feet. With regard to the deflection being inversely as the cube of the depth into the breadth, that is, inversely, as $b d^3 : b^3 d$, or as $b^2 : d^2$, in the above experiments; this also is confirmed as far as the comparison can be made, but the difference in these two dimensions is not so great as in the lengths, and therefore the results, perhaps, not so conclusive.

M. Girard makes the deflections inversely, as $b d^2 : b^2 d$; that is, in the above cases, as $b : d$, which by no means agrees with the above results: the discrepancy will, however, be best seen by computing the deflections; first of the long beam from that of the short one being given, and comparing them with those determined from experiment; and then computing the deflections of the beams in the direction of their least depth, from those given for their greater.

	Feet.		Lbs.		Deflection, computed according to M. Girard.		Deflection from pre- ceding formulae.		Deflection from expe- riments.
No. 1.	6	...	120	...	·36	...	·72	...	·68
	6	...	180	...	·48	...	·96	...	1·00
No. 1.	6	...	120	...	·76	...	1·52	...	1·38
	6	...	180	...	1·12	...	2·34	...	1·91
No. 2.	6	...	120	...	·40	...	·80	...	·72
	6	...	180	...	·60	...	1·20	...	1·05
No. 2.	6	...	120	...	·72	...	1·44	...	1·30
	6	...	180	...	1·12	...	2·24	...	2·00
No. 3.	6	...	120	...	·28	...	·56	...	·65
	6	...	180	...	·44	...	·88	...	·96
No. 3.	6	...	120	...	·64	...	1·28	...	1·25
	6	...	180	...	·94	...	1·92	...	1·81

It only requires a comparison to be made between the last column and the other two, to decide which of the two formulæ best agrees with the actual state of the beam's deflection.

59. The above are obtained from a comparison of the lengths of the beam : let us now make a similar comparison, as depending upon their depth and breadth.

				Deflection accord- ing to M. Girard.		Deflection from the formulæ.		Deflection from experi- ment.	
Feet.		lbs.		Defl. $\propto \frac{1}{b d^3}$		Defl. $\propto \frac{1}{b d^3}$			
No. 1.	3	...	120	...	·12	...	·16	...	·19
	3	...	180	...	·16	...	·21	...	·28
No. 1.	6	...	120	...	·91	...	1·21	...	1·38
	6	...	180	...	1·33	...	1·77	...	1·91
No. 2.	3	...	120	...	1·33	...	1·77	...	1·80
	3	...	180	...	·20	...	·27	...	·28
No. 2.	6	...	120	...	·96	...	1·28	...	1·30
	6	...	180	...	1·40	...	1·87	...	2·00
No. 3.	3	...	120	...	·10	...	1·43	...	·16
	3	...	180	...	·157	...	2·24	...	·24
No. 3.	6	...	120	...	·928	...	1·32	...	1·25
	6	...	180	...	1·371	...	1·95	...	1·85

Here, again, the agreement between the last column and the preceding one is so near, in comparison with that computed according to M. Girard's principle, as to leave no doubt concerning the legitimacy of our formulæ.

Still, however, I was desirous of further proof, and therefore procured three pieces of very clean fir, free from knots, 10 feet 6 inches long, 3 inches deep, and $1\frac{1}{2}$ inch in thickness; and an ivory scale very accurately graduated into 40ths of an inch, which was now fixed to the batten, instead of the scale of 10ths of inches hitherto employed: by which means the deflections could be accurately observed to within about $\frac{1}{80}$ th of an inch.

One of the beams was laid on with the props 9 feet apart, and the weights gradually added till the deflection was 27 of the equal parts on the scale: I then unloaded it, and set the props 6 feet

asunder, and applied again the same weights, and the deflection was exactly eight divisions.

Now, in case of the deflections being as the square of the length, we ought to have had

$$9^2 : 6^2 :: 27 : 12$$

for the deflection at 6 feet. But if the deflections were as the cubes,

$$9^3 : 6^3 :: 27 : 8,$$

precisely the same as it was found to be by the experiment.

The props were then brought to the distance of 3 feet; and the same weights being used, the deflection was exactly $\frac{1}{40}$ th of an inch, or one division: whereas it ought, according to M. Girard, to have been $\frac{3}{40}$ ths, or three divisions.

The second batten was now laid on at 9 feet, and brought to a deflection of $40\frac{1}{2}$ divisions; the same weights brought it at 6 feet to $12\frac{1}{2}$ divisions, and at 3 feet to $1\frac{1}{2}$; whereas if the deflections had been as the squares, they ought to have been 18 and $4\frac{1}{2}$ respectively.

60. The third beam was deflected to 54 divisions at 9 feet, and the same weights brought it to $16\frac{1}{2}$ at 6 feet, and to 2 divisions at 3 feet, instead of 24 and 6, as required by the law which M. Girard had deduced from his experiments.

I next tried each of the pieces again at the distance of 6 feet, laid in the contrary way, viz. with

their least thickness vertical; and placing on each the same weights as had been before employed, the deflections were, for

No. 1	32 divisions.
No. 2	48 ditto.
No. 3	64 ditto.

Which show that the deflections were also as the cubes of the depth into the breadth, and not as the squares; for had that law obtained, these deflections would have been 16, 24, and 32.

61. After the preceding experiments were gone through, I made the following series on the same battens, and have computed, in every case, the value of the constant quantity, which we may call the elasticity, E , from the formula $\frac{l^3 W}{b d^3 \delta} = E$, the reduced mean of which is $E = 5317610$, whence we have $\frac{l^3 W}{b d^3 \delta} = 5317610$, from which any one of these five quantities may be found, when the other four are given.

63. As a further confirmation of the preceding deductions, the following, from M. Dupin's experiments, may be added, which I had not seen when the above was written. The pieces on which M. Dupin's experiments were made, were 2 metres in length, and of various lateral dimensions, viz. 1, 2, and 3, &c. centimetres, to a decimeter in the squareage; they were performed with care, and conducted with great ability.

64. The following are some of the principal theorems which this author has drawn from his experiments and investigations, as connected with this part of our inquiry; viz.

1. The deflections of the same beam resting on props at each end, and loaded in the middle with small weights, are as those weights.

2. When the same piece is rested on props at the same distance, and loaded at its middle point with different small weights; these weights are reciprocally proportional to the radius of curvature at that point; and the curvature itself is consequently proportional to the weights.

3. The deflection is, *cæteris paribus*, inversely as the cube of the depth; also the depth being the same, the deflection is inversely as the breadth.

4. The deflection is, therefore, *cæteris paribus*, directly as the cube of the length.

From which it necessarily follows, agreeably to

the preceding deductions, that $\frac{l^3 W}{b d^3 \delta} = \text{a constant quantity.}$

5. M. Dupin also demonstrates experimentally, the ratio which has been stated between the deflection of beams supported at each end, and loaded in the middle, and the deflection of the same when the weight is uniformly spread; at least his experiments give results approximating towards that ratio, viz. experimentally he has found it to be as 19 : 30, while the theory required the ratio of 5 to 8; or reducing both to the same antecedent, the first is as 95 to 150, and the second as 95 to 152, which is as nearly correct as it is possible to expect, considering, in the first place, that it is impossible practically to distribute the weights so as to have them perfectly uniform; and in the second, that the investigation belongs only to infinitely small deflections; while experimentally they are rendered sufficiently obvious to be submitted to actual measurement. The same author has found various other interesting results; but we cannot allow any further abstracts in this place.

65. It is important to observe, before concluding this chapter, that all the foregoing investigations have been made exclusively with reference to rectangular beams, and that they must only be considered as being applicable to that form; for, notwithstanding we have throughout made our deduc-

tions from a comparison of the depths, breadths, &c., it is obviously not the depth of the whole beam, but that of its neutral axis, on which the deflection depends; but as the latter, in rectangular beams, is always as the whole depth, we may use the one for the other indifferently, and we made choice of the latter for the sake of simplicity.

Practical Deductions.

66. The following practical deductions flow immediately from the preceding investigations, and with them we shall conclude this chapter.

1. It has been shown, that the successive deflections are directly as the weight and cube of the length, and reciprocally, as the breadth and cube of the depth, or that when the beam is fixed at one end, and loaded at the other,

$$\frac{l^3 W}{b d^3 \delta} = E,^* \text{ is a constant quantity.}$$

When fixed at one end uniformly loaded, (see Art. 50,)

$$\frac{3 l^3 W}{8 b d^3 \delta} = E, \text{ the same constant.}$$

* It may be proper to observe, that the original expression is $\frac{l^3 W}{3 b d^3 \delta} = E$, a constant; of course $\frac{l^3 W}{b d^3 \delta} = E$, is constant also; and we prefer the latter expression, for the sake of simplicity.

When supported at both ends, and loaded in the middle,

$$\frac{l^3 W}{32 b d^3 \delta} = E, \text{ the same constant.}$$

2. And hence it follows, that in order to preserve the same stiffness in beams, the depth must be increased in the same proportion as the length, the breadth remaining constant.

3. In square beams of different lengths, the stiffness will be the same, when s^4 is as l , s being the side of the square, and l the length.

4. If the depth is given, the stiffness will be the same when b is as l^3 , or when $b^{\frac{1}{3}}$ is as l .

5. The deflection of different beams arising from their own weight, having their several dimensions proportional, will be as the square of either of their like lineal dimensions. For it has been seen that in all these cases $\frac{l^3 W}{b d^3 \delta} = E$, a constant quantity: and if, therefore, we suppose each of these dimensions to be increased m times; then the weight W will be increased m^3 times, and we shall, therefore, have

$$\frac{l^3 m^3 W m^3}{b m d^3 m^3 \delta'} = E, \text{ or } \frac{l^3 W m^2}{b d^3 \delta'} = E;$$

consequently, since E is the same in both, δ' must have varied as m^2 .

The same will apply to beams loaded throughout proportional to the dimensions; and it is a fact which ought to be kept constantly in view in the

construction of models on a small scale, of works intended to be executed on a large one.

6. With regard to the ultimate deflection of beams before their rupture, the same relations do not obtain; for it is obvious, from what has been already stated, (Art. 54,) that the depth being the same, the element of deflection will, in the breaking state of the beam, be constant; and, consequently, the ultimate deflection will in this case be as the square of the length, and it will be inversely as the depth when the length is the same; and if both these dimensions remain constant, the last deflections will be constant also, whatever may be the breadth of the beam.

The formula, therefore, applicable to this case, is $\frac{l^2}{d\Delta} = U$, a constant quantity, where Δ is the last deflection, l the length, and d the depth of the beam.

But little dependence, however, can be placed on this last deduction, because the law of deflections becomes very uncertain, after the elasticity has ceased to be perfect; which is some time before the rupture takes place.

Experiments on the Transverse Strength of Timber.

67. Reference has already been made to the experiments of Galileo, Mariotte, Muschenbroeck, and others, which had been made in the earlier stages

of this inquiry; but unfortunately, from one cause or other, the results obtained from them are little accordant with each other; and we think it useless to embarrass the reader, by giving a long detail of labours on which no dependence can be placed: passing over, therefore, many early experiments, we come to those of M. Buffon, by far the most valuable, both as respects the number of them, and the size of the pieces of timber on which they were made; many of them having been from 20 to 28 feet in length, and from 4 to 8 inches square. This philosopher was furnished by the French Government with ample funds, and every necessary means for carrying on his experiments on a grand scale; and he discharged the duty thus imposed upon him in a manner highly creditable to himself, and to the satisfaction of the Academy; but he did not, perhaps, possess the mathematical knowledge necessary for making the best use of his results. His experiments, however, are not the less valuable; as they are no doubt faithfully related, and furnish a sound foundation for the establishment of a correct theory.

He commenced his operations, with Du Hamel, on pieces of small dimensions; and tried them in succession from the heart to the bark of the tree, and from the root upwards. From these experiments, it was found, that the heart was the densest, that the density decreased from hence to the circumference, and that the strength decreased also in nearly the same proportion.

He also made trial of the proportional strength of battens, accordingly as they were laid, with the annual layers, vertical or horizontal, and found a difference in the strength, in these two cases, nearly in the ratio of 8 to 7; the difference, no doubt, arising from the cohesion of the layers with each other being considerably less than that between the fibres themselves. Some experiments have been referred to, in Art. 13, to show the quantity of this lateral cohesion, although it must be allowed to be rather a subject of curiosity than utility; for large beams, whose strength it is the most important to be acquainted with, commonly occupy the whole, or nearly the whole, section of the tree.

M. Buffon found likewise, that oak timber lost much of its strength in the course of drying, or seasoning; and therefore, in order to secure uniformity, his trees were all felled in the same season of the year, were squared the day after, and experimented on the third day. Trying them in this green state, gave him an opportunity of observing a very curious phenomenon; namely, that when the weights were laid briskly on, nearly sufficient to break the log, a very sensible smoke was observed to issue from the two ends, with a sharp hissing noise, which continued all the time the tree was bending or cracking.

This philosopher, as above stated, drew no important conclusions from his experiments: he seems to have had in view no favourite theory, either of

his own or of any other writer, and was therefore free from any bias, or any desire to accommodate his experiments to a particular hypothesis: besides, his beams were too large for him to deceive himself in this respect, as there is reason to believe has been the case with some authors. Upon the whole, these are certainly the most valuable experiments that had yet been made upon the transverse strength and strain of oak timber, whether they be considered as the means of furnishing practical precedent, or theoretical data: the following Table of this author's results will therefore, it is presumed, be acceptable to the English engineer, for whose convenience the several results are reduced to English weights and measures.

68. *Table of the Results of Buffon's Experiments on the Transverse Strength of Square Oak Beams.*

No.	Side of square.		Length.		Weight of the pieces.		Weights which broke the pieces.		Deflection before cracking.	
	In inches.	In metres.	In feet and inches.	In metres.	In lbs.	In kilogr.	In lbs.	In kilogr.	In inches.	In metres.
1	4.28	.1082	7 6	2.2732	64.56	29.34	5756	2616	3.47	.0946
2			7 6	2.2732	60.25	27.39	5676	2580	4.82	.1217
3			8 6.8	2.5979	73.17	33.26	4950	2250	4.01	.1013
4			8 6.8	2.5979	67.79	30.81	4842	2201	5.00	.1262
5			9 7.7	2.9227	83.85	37.66	4401	2005	5.17	.1307
6			9 7.7	2.9227	76.39	34.73	4250	1932	5.89	.1488
7			10 8.5	3.2473	90.37	41.09	3900	1773	6.25	.1578
8			10 8.5	3.2473	88.23	40.11	3884	1760	6.96	.1758
9			12 10.3	3.8969	107.60	48.91	3281	1491	7.50	.1894
10			12 10.3	3.8969	105.45	47.93	3174	1430	7.50	.1894
11			7 6	2.2732	101.14	45.98	12670	5759	2.67	.0676
12			7 6	2.2732	95.32	43.33	12133	5515	2.67	.0676
13			8 6.8	2.5979	111.91	50.87	10653	4842	2.85	.0721
14			8 6.8	2.5979	109.76	49.89	10411	4732	3.12	.0789
15			9 7.7	2.9227	126.97	57.72	9038	4108	3.21	.0811
16			9 7.7	2.9227	124.82	56.74	8958	4072	3.51	.0878
17			9 7.7	2.9227	173.75	56.25	8822	4011	3.75	.0946
18	5.35	.1353	10 8.5	3.2473	142.04	64.69	7774	3534	3.39	.0856
19			10 8.5	3.2473	139.91	63.60	7586	3448	3.74	.0946
20			10 8.5	3.2473	138.28	62.85	7639	3472	4.28	.1082
21			12 10.3	3.8969	167.87	76.30	6510	2959	5.89	.1488
22			12 10.3	3.8969	165.71	75.32	6563	2983	6.16	.1556
23			15 0	4.5464	191.54	87.06	5811	2641	8.57	.2164
24			15 0	4.5464	189.38	86.09	5595	2543	8.83	.2231
25			17 1.7	5.1959	224.90	102.23	4861	2164	8.65	.2186
26			17 1.7	5.1959	220.59	100.27	4600	2091	8.74	.2209
27			19 3.4	5.8453	249.65	113.48	4034	1834	8.57	.2164
28			19 3.4	5.8453	248.57	113.00	3248	1799	8.74	.2209
29			21 5.1	6.4947	284.12	129.64	3523	1601	9.46	.2389
30			21 5.1	6.4947	278.71	126.69	3416	1553	10.71	.2706
31			25 8.6	7.7939	333.59	151.63	2367	1076	11.88	.2976
32			25 8.6	7.7939	330.36	150.16	2286	1039	12.05	.3652
33			30 0	9.0928	391.69	178.04	1936	880	19.78	.4870
34			30 0	9.0928	365.40	166.09	1882	855	23.57	.5952

EXPERIMENTS ON TRANSVERSE STRENGTH. 103

TABLE—(CONTINUED.)

No.	Side of square.		Length.		Weight of the pieces.		Weights which broke the pieces.		Deflection before cracking.	
	In inches.	In metres.	In feet and inches.	In metres.	In lbs.	In kilogr.	In lbs.	In kilogr.	In inches.	In metres.
35	6.43	.1624	7 6	2.2732	138.27	62.85	20715	9416		
36			7 6	2.2732	129.66	58.93	20069	9122		
37			8 6.8	2.5979	160.33	72.88	16894	7679	2.50	.0631
38			8 6.8	2.5979	157.11	71.41	16517	7508	2.58	.0653
39			9 7.7	2.9227	178.63	81.19	14473	6579	2.67	.0676
40			9 7.7	2.9227	177.02	80.46	13607	6285	2.83	.0766
41			10 8.5	3.2473	202.30	91.96	12347	5612	3.21	.0811
42			10 8.5	3.2473	200.18	91.00	11863	5392	3.74	.0946
43			12 10.3	3.8969	241.04	109.56	9900	4500	4.28	.1082
44			12 10.3	3.8969	237.82	108.10	9684	4402	4.38	.1107
45			12 0	4.5464	274.40	124.73	8016	3644	4.82	.1217
46			12 0	4.5464	273.33	124.24	8070	3668	4.38	.1107
47			17 1.7	5.1959	316.37	143.80	6725	3057	5.89	.1488
48			17 1.7	5.1959	315.29	143.32	6967	3167	6.25	.1578
49			19 3.4	5.8453	359.42	163.37	6052	2751	7.94	.2014
50			19 3.4	5.8453	356.18	161.90	5918	2690	9.10	.2299
51			21 5.1	6.4946	405.69	184.40	5406	2457	10.17	.2570
52			21 5.1	6.4946	403.53	183.43	5246	2384	9.46	.2389
53	7.5	.1894	8 6.8	2.5979	219.52	100.00	28140	12791	2.94	.0743
54			8 6.8	2.5979	219.52	100.00	27926	12693	2.68	.0676
55			9 7.7	2.9227	244.28	111.03	24535	11152	3.31	.0836
56			9 7.7	2.9227	242.12	110.05	23562	10712	3.12	.0789
57			10 8.5	3.2473	273.33	124.24	21145	9611	2.73	.0689
58			10 8.5	3.2473	271.15	123.25	20769	9440	3.21	.0811
59			12 10.3	3.8969	321.98	147.72	18078	8217	3.12	.0789
60			12 10.3	3.8969	323.90	147.23	16733	7606	3.56	.0900
61			15 0	4.5464	382.01	173.64	14634	6652	4.46	.1127
62			15 0	4.5464	377.72	171.69	13828	6285	4.01	.1013
63			17 1.7	5.1959	436.90	198.59	11944	5429	5.17	.1307
64			17 1.7	5.1959	433.67	197.12	11729	5331	5.62	.1420
65			19 3.4	5.8453	488.55	222.07	10163	4622	5.89	.1488
66			19 3.4	5.8453	488.55	222.07	10113	4597	6.25	.1578
67			21 5.1	6.4946	653.45	247.01	9200	4182		.2119
68			21 5.1	6.4946	654.55	247.50	8608	3913	9.10	.2299
69	8.57	.2165	10 8.5	3.2473	356.19	161.90	29916	13598	3.21	.0811
70			10 8.5	3.2473	356.19	161.90	28709	13049	2.41	.0611
71			12 10.3	3.8969	427.22	194.19	24619	11190	3.21	.0811
72			12 10.3	3.8969	425.60	193.45	23654	10750	3.12	.0789
73			15 0	4.5464	496.08	225.49	21575	9807	4.10	.1036
74			15 0	4.5464	493.93	224.51	20894	9538	3.39	.0856
75			17 1.7	5.1959	564.96	266.80	18078	8217	6.53	.1398
76			17 1.7	5.1959	563.88	266.31	17163	7801	4.01	.1013
77			19 3.4	5.8453	639.21	290.55	14526	6603	4.82	.1217
78			19 3.4	5.8453	638.53	290.06	13801		4.37	.1104
79			21 5.1	6.4946	712.34	323.79	12670	5759	6.96	.1758
80			21 5.1	6.4946	710.23	322.83	13128	5967	6.43	.1623

It has been observed, that the preceding Table may be considered as furnishing the most useful results relative to the transverse strength of oak beams, of any hitherto made public; both as they regard practical precedent and theoretical data; but, with reference to the former, the engineer must bear well in mind the green state of the wood when the experiments were performed, which adds much to its strength, on account of the fibres in that state offering a much greater resistance to compression, than when the timber has been well dried and seasoned.

We come now to more recent experiments.

69. A knowledge of the strength and elasticity of timber being subjects of the highest importance in the constructions of ships, &c., the surveyors of His Majesty's navy have, at different times, ordered experiments to be made, directed to this object; and they have in the most handsome manner supplied me with every information they were in possession of, relative to those inquiries; a favour for which I am equally indebted to the liberal views of those gentlemen, and to the friendly interference and recommendation of John Knowles, Esq., secretary to that Board, through whom it was solicited.

The following Table contains the results of experiments carried on in the dockyard at Deptford, by Colonel Beaufoy, on English and Dantzic oak, Riga fir, and pitch pine. The several pieces

were each 5 feet long and 2 inches square, fixed at one end in a mortise to the length of 1 foot, so that the part projecting was 4 feet; and the weight was hung on at that distance from the fulcrum. The twenty-five pieces of Dantzic oak were cut from the same tree, of which the mean specific gravity was 854. The several pieces of Riga fir were also all from one tree, of which the mean specific gravity was 537.; as were those of pitch pine, but the specific gravity is not stated. Of the English oak, the first six pieces were from one tree, of which the specific gravity was 922, and the other thirteen from another; the latter very irregular and cross-grained, but its weight is not given: nor do I find any indication of the particular weight of each piece, nor the situation it occupied with regard to its distance from the heart or centre. It is simply stated, that the last piece of oak was the heart of the tree, and that it was the weakest.

The deflections were measured in degrees and minutes, on a graduated arc of the same radius as the beam, viz. 4 feet, and were taken as every 14 lbs. were put on: we have given, however, only the mean, the last weights, and the corresponding deflections. It appears from all these experiments, that the deflections are very nearly in the ratio of the weights, till about one-half, or a little less than one-half the weight, is laid on, after which they become more rapid, and very irregular.

70. Table of Experiments carried on in the Dockyard, Deptford, on Beams of different woods, fixed at one end: by Col. Beaufoy.

No. of experiment.	Danish oak, 25 pieces, 4 ft. long, 3 in. square.	Deflection in degrees, &c.	Riga fir, 25 pieces, 4 ft. long, 3 inches square.	Deflection in degrees, &c.	Pitch pine, 24 pieces, 4 ft. long, 2 inches square.	Deflection in degrees, &c.	English oak, 19 pieces, 4 ft. long, 3 inches square.	Deflection in degrees, &c.
1	98	2 3	98	1 24	98	1 15	98	1 15
2	98	2 6	98	1 21	98	1 18	98	1 14
3	98	2 21	98	1 14	98	1 20	98	1 12
4	98	2 36	98	1 12	98	1 6	98	1 10
5	98	2 48	98	1 23	98	1 8	98	1 19
6	98	2 32	98	1 26	98	1 6	98	1 14
7	98	2 24	98	1 36	98	1 0	98	1 17
8	98	2 9	98	1 25	98	1 6	98	1 16
9	98	1 54	98	1 20	98	1 20	98	1 20
10	98	1 46	98	1 16	98	1 24	98	1 32
11	98	1 58	98	1 25	98	1 10	98	1 47
12	98	1 51	98	1 39	98	1 8	98	1 26
13	98	2 24	98	1 15	98	1 0	98	1 32
14	98	2 4	98	1 26	98	1 15	98	1 20
15	98	2 23	98	1 21	98	1 10	98	1 28
16	98	2 37	98	1 14	98	1 15	98	1 44
17	98	2 1	98	1 20	98	2 12	98	1 34
18	98	1 57	98	1 30	98	1 12	98	1 30
19	98	1 40	98	1 38	98	0 54	98	1 46
20	98	1 54	98	1 31	98	1 2		
21	98	2 19	98	1 20	98	1 8		
22	98	1 57	98	1 18	98	1 12	1st Sir, Sum 1551	
23	98	2 36	98	1 16	98	1 12	Mean 258 lbs.	
24	98	2 30	98	1 24	98	0 54	13 following. Sum 2740	
25	98	3 18	98	1 16			Mean 211 lbs.	

These experiments furnish the absolute and comparative strength of the four following woods, viz. :

Length 4 feet, 2 inches square.	{	Dantzic oak . 167 lbs. . . .	Sp. gr. 854
		Riga fir ¹³ . . 202 lbs. . . .	Sp. gr. 537
		Pitch pine . . 272 lbs. . . .	Sp. gr.
		English oak . 258 lbs. . . .	Sp. gr. 922
		Ditto	211 lbs.

Other experiments were made by the same gentleman on battens of $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, and 3 inches square, and of the same length. The particulars are not stated ; but it appeared from them, that the ratio of the strengths a little exceeded the ratio of the cubes of the sides.

71. Other experiments were also made upon pieces of the same dimensions, spliced and fixed in different ways : the *scarph* in all of them was 12 inches long, and 13 inches from the end, viz. about an inch from the fulcrum. The results were as follow :

<i>Scarph up and down . .</i>	{	No. 1, broke in the splice	112 lbs.
		No. 2, ditto	109 lbs.
<i>Scarph flatwise, large end uppermost, and towards the fulcrum</i>	{	No. 1, nails drew through the small end of the scarph	104 lbs.
		No. 2, ditto	98 lbs.
<i>Scarph flatwise, small end towards the fulcrum</i>	{	No. 1, broke in the thick part of the scarph .	84 lbs.
		No. 2, ditto	90 lbs.

¹³ It may be proper to observe, that No. 13, in Col. Beaufoy's Report of the Riga fir, was very irregular, having been broken

From these experiments it is inferred, that the two former positions of spliced pieces are preferable to the last.

72. The following experiments were made under the same authority, by Messrs. Peake and Barrallier.

It is necessary, in order that the reader may properly understand the results contained in the fourth, fifth, and sixth columns of the following Table, to explain the nature of the apparatus by which these several pieces were submitted to experiment. An oak pillar, 12 inches square, had a hole of 2 inches square cut in it, for the purpose of receiving the end of the batten, the pillar itself being securely fixed, between the principal floor-joist and the tye-beam, in the mould-loft in Woolwich dockyard; and a semicircular piece of oak, of 6 inches radius, was well fixed to the principal pillar, to prevent the batten from crippling at its lower side. This semicircle was divided into inches and parts, and as the weights were successively applied, the batten was deflected, and in some measure bent over this arc; and the numbers in the columns above mentioned show to what extent this bending took place.

As to the numbers in the other columns, they will be readily understood, from the description given at

with only 98 lbs.: this experiment is therefore rejected, and its place is supplied with experiment No. 26. It may also be further stated, that the above mean weights are obtained by dividing the sum of all the breaking weights by the number of them.

their heads in the Table;—the first showing the number of the experiments; the second, the number of years the pieces had been in store; the third, the specific gravity; the fourth and fifth, the part of the arc which came in contact with the batten, with 56 lbs. and 112 lbs. respectively; the sixth, the contact which remained after removing the last weight; the seventh column shows the whole curvature; the eighth, the weight under which the piece crippled; the ninth, the weight under which it broke; and the tenth contains sundry remarks.

73. Table of Experiments on Riga Fir Battens, 2 inches square, fixed at one end, and the weight acting at 5 feet from the fulcrum.

Note.—These pieces were all kept dry.

No. of experiment.	Years in store.	Specific gravity.	Arc received by the battens under the weight of		Arc remaining after the last weight was removed.	Total curvature.	Weight under which the beam was crippled.	Weight under which it broke.	REMARKS.
			55 lbs.	112 lbs.					
1	13	474	3 $\frac{1}{2}$	7 $\frac{1}{2}$	I	12	112	144	Part of a jib-boom.
2	6	693	3 $\frac{1}{2}$	6 $\frac{1}{2}$	I	16	202	220	
3	11	474	4 $\frac{1}{2}$	7 $\frac{1}{2}$	I	11	112	144	Same as No. 1.
4	13	513	3 $\frac{1}{2}$	5 $\frac{1}{2}$	0 $\frac{1}{2}$	13	167	194	Ditto.
5	6	758	4 $\frac{1}{2}$	12 $\frac{1}{2}$	2 $\frac{1}{2}$		112	112	Sound batten.
6	6	804	3 $\frac{1}{2}$	7	0 $\frac{1}{2}$		126	111	Ditto.
7	6	756	3 $\frac{1}{2}$	7	0 $\frac{1}{2}$		126	127	Ditto.
8	6	696	3 $\frac{1}{2}$	7 $\frac{1}{2}$	0 $\frac{1}{2}$		133	141	Ditto.
9	6	720	3 $\frac{1}{2}$	7 $\frac{1}{2}$	0 $\frac{1}{2}$		126	126	Broke short.
10	6	726	4	7 $\frac{1}{2}$	0 $\frac{1}{2}$		137	137	Ditto.
11	6	756					77	77	Very shaky.
12	6	726	3 $\frac{1}{2}$	8 $\frac{1}{2}$	0 $\frac{1}{2}$		126	126	Sound.
13	6	720	2 $\frac{1}{2}$	5 $\frac{1}{2}$	0 $\frac{1}{2}$		127	138	Ditto.
14	6	720	3	5 $\frac{1}{2}$	0 $\frac{1}{2}$		147	147	Ditto.
15	6	700	2 $\frac{1}{2}$	6	0 $\frac{1}{2}$		147	147	Ditto.
16	6	622	3 $\frac{1}{2}$	7 $\frac{1}{2}$	0 $\frac{1}{2}$		133	135	Very dry.
17	10	558	3 $\frac{1}{2}$	7 $\frac{1}{2}$	0 $\frac{1}{2}$		133	133	Broke short.
18	10	564	3 $\frac{1}{2}$	6 $\frac{1}{2}$	0 $\frac{1}{2}$		140	149	Ditto.
19	10	522	3	5 $\frac{1}{2}$	0 $\frac{1}{2}$		140	140	Fine texture.
20	8	546	3 $\frac{1}{2}$	6 $\frac{1}{2}$	0 $\frac{1}{2}$		133	140	Ditto.
21	8	558	2 $\frac{1}{2}$	5 $\frac{1}{2}$	0 $\frac{1}{2}$	13 $\frac{1}{2}$	140	147	Broke short.
22	3	828	3 $\frac{1}{2}$	6 $\frac{1}{2}$	0 $\frac{1}{2}$		161	161	Ditto.
23	6	693	3 $\frac{1}{2}$	6 $\frac{1}{2}$	I	16	202	220	Same as No. 2.
24	6	705	3 $\frac{1}{2}$	6 $\frac{1}{2}$	0 $\frac{1}{2}$	13 $\frac{1}{2}$	168	182	Broke sudden.
25	13	486	3 $\frac{1}{2}$	7 $\frac{1}{2}$	I	12	112	116	
26	10	513	3 $\frac{1}{2}$	11 $\frac{1}{2}$	0 $\frac{1}{2}$	13 $\frac{1}{2}$	167	194	
27	8	546	3 $\frac{1}{2}$	5 $\frac{1}{2}$	0	15	168	202	
28	8	561	2 $\frac{1}{2}$	4 $\frac{1}{2}$	0 $\frac{1}{2}$	9 $\frac{1}{2}$	168	191	
27) 17856			Sum, rejecting No. 11.			27) 4132			
Mean, 633						Mean, 153			

The preceding Table, by Colonel Beaufoy, reckoning the strength to be inversely as the length, gives 5 : 4 :: 202 : 161 lbs. for the mean ; which is in defect only 1 lb. ; the mean of the former being 162 lbs. at 4 feet.

TABLE—(CONTINUED).

Experiments in every respect similar to the last, except that the several pieces were kept wet.

No. of experiment.	Years in store.	Specific gravity.	Are received by the battens under the weight of		Are remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	REMARKS.
			56 lbs.	112 lbs.					
			inches.	inches.	inches.	inches.	lbs.	lbs.	
1	29	639	3	5	0	12½	193	207	
2	6	615	2½	4½	0½	13½	248	261	
3	13	534	3½	6	0½	14	126	158	
4	13	555	2½	5	0	15	153	208	
5	29	639	■	5	0	12½	193	207	
6	6	876	2½	5½	1½		136	136	Very shaky.
7	6	666	2½	4½	0½		140	199	Sound.
8	6	666	2½	4½	0½		158	190	Ditto.
9	6	696	2½	4½	0½		154	172	Ditto.
10	6	782	2½	4½	0½		168	180	Ditto.
11	6	690	2½	4½	0½		168	168	Little shaky.
12	6	720	2	■	0½		168	203	Very sound.
13	6	690	2	3½	0½		176	186	Sound.
14	6	708	■	4½	0		128	128	Very cross-grained.
15	6	726	2½	3½	0½		209	214	Sound.
16	6	702	2½	4	0½		214	214	Ditto. [grained.
17	10	606	1½	10½	0		133	133	Very shaky and cross-
18	10	720	3½	12	1		112	112	} These broke very slowly.
19	10	■	2½	5½	0½		159	159	
20	10	666	2½	6½	0½		■	132	
21	10	540	3½	8½	0½		117	117	Ditto shaky.
22	10	528	3½	9½	0½		132	132	Coarse-grained.
23	8	648	4½	12			112	112	Cross-grained.
24	8	552	3½	6½	0½		151	153	Coarse-grained.
25	29	738	2½	4½	0½	13	146	160	
26	10	684	3½	10½			112	135	
27	10	684		12½	1½			137	
28	6	615	2½	4½	0½	13½	248	261	
29	10	492	4½	■	1½	13½	128	135	Very shaky.
30	6	594	2½	4½	0½	12½	224	233	
31	8	564	4½	7½	1	13½	140	161	
32	13	534	3½	6	0½	14	■	158	Coarse soft grain.
33	13	495	4½	11½	1½	16½	112	149	Shaky and knotty.
34	29	639	3	5	0	12½	197	207	
35	8	600	3½	5½	0½	14	168	199	
36	13	■	3½	5	0½	15	147	147	
37	13	555	2½	■		15	153	208	
Sum, 37)23390						Sum, 37)6371			
Mean, 632 wet.						Mean, 175 wet.			
Mean, 633 dry.						Mean, 153 dry.			
Mean, 537 of both.						Mean, 161 Colonel Beaufoy.			
3)1802						3)486			
Mean, 600						Mean of the three, 162			

TABLE—(CONTINUED).

Containing similar Experiments on Battens of the same Dimensions, of different kinds of Wood.

No. of experiment.	Years in store.	Specific gravity.	Arc received by the battens under the weight of		Arc remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	REMARKS.
			55 lbs.	112 lbs.					
VIRGINIA YELLOW PINE.									
1	Time unknown.	564	4½	10	98	98	Dry and defective.
2		720	2¾	4¾	0¾	16½	246	251	Ditto.
3		498	6	15½	233	233	Ditto.
4		618	4½	3½	0¾	26¾	206	234	Ditto.
5		498	3½	6¾	0¾	..	126	135	Part of old topmast.
6		522	3½	8¾	0¾	11¾	133	133	Dry.
7		492	3½	6½	0¾	..	140	147	Ditto.
PITCH PINE.									
8	do.	816	2	3¾	0¼	9¼	196	203	Dry.
9	do.	816	1½	2¾	0½	..	336	365	Ditto.
10	do.	996	2½	3¾	0¼	12½	238	244	From Lukin's kiln.
11	do.	738	2	4	0½	12½	224	332	Dry.
12	do.	732	2	3½	0½	11¾	308	308	Ditto.
13	do.	696	2½	3¾	0½	14	287	303	Ditto.
14	do.	708	2½	4½	0½	17	273	293	Ditto.
15	do.	720	2½	4½	0½	..	140	..	Defective.
CANADIAN WHITE PINE.									
16	1	648	4½	14	98	123	Wet.
17	10	672	4¾	14	98	119	Ditto.
18	8	714	4	14	84	103	Ditto.
19	8	660	5¾	14	84	108	Ditto.
20	4	720	3¾	14	84	91	Ditto.
21	4	714	3¾	10½	84	96	Ditto.
22	8	618	3¾	10¾	1¾	18½	116	122	Dry.
LARCH.									
23	4	526	7¾	16½	4	34	..	170	Dry.
24	4	540	3½	7½	0¾	14½	133	133	Ditto.
25	4	570	5½	10½	1	15	..	137	Ditto.
26	4	526	3¾	6¾	0¾	16½	160	162	Ditto.
DANTZIC FIR.									
27	4	690	2¾	4¾	0½	..	158	158	Wet.
28	4	648	2½	4¾	0¼	12½	140	140	Ditto.
29	4	630	2½	4¾	0½	12½	140	140	Ditto.
30	3	624	3¾	6½	0¾	11¾	186	192	Ditto.
ASH.									
31	1	858	2	4½	0¾	16	224	239	Quite green.
32	1	828	2½	4¾	3¾	18½	..	217	Ditto.
33	..	660	3¾	6½	0½	12¾	..	196	Old capstan bar.
TEAK.									
34	2	672	2¾	4¾	..	16½	224	271	} Old bowsprit.
35	2	606	2¾	4½	..	12½	224	257	

74. The preceding Table furnishes the following means, viz.:—each bar being

5 feet long, and 2 inches square. }	Riga	{ Dry 153 Wet 172 }	162	mean	sp. gr.	633
	Virginia yellow pine	189	.	.	.	558
	Pitch pine	256	.	.	.	777
	Canadian white pine	109	.	.	.	678
	Larch	150	.	.	.	540
	Dantzic ditto	156	.	.	.	648
	Ash	217	.	.	.	782
	Teak	264	.	.	.	639

It may be remarked, that the strength of pitch pine, according to these experiments, exceeds very considerably what was found by Colonel Beaufoy; while that of the Riga fir, taking a mean between the wet and dry, is exactly the same in both: but it is to be observed, that in the experiments by Messrs. Peake and Barrallier, the bending of the pieces over the arc, as above described, shortens the ultimate radius; and therefore they ought to be stronger than with the uniform radius of 5 feet: consequently the specimens of Riga fir in these experiments were really weaker than those of Colonel Beaufoy, although they apparently agree with each other.

*Experiments on Triangular Oak Beams, &c., by
Mr. Couch.*

75. In a preceding chapter, we have given the detail of several valuable experiments by Mr.

Couch, of Plymouth Dockyard; and the two following Tables are due to the same gentleman. They exhibit the detail and results of experiments on the lateral or transverse strength of triangular prisms of Canadian oak, the sections of which were equilateral triangles, the sides being 3 inches; and also on some pieces reduced to the form of trapezoids, by cutting off the vertex, or upper angle, to one-third of the depth.

The short pieces, viz. those 3 feet 3 inches, Table I., were fixed by one end horizontally in a 3-inch mortise; the others, as given in Table II., which were 6 feet 6 inches, were fixed at each end into 3-inch mortises, so as to prevent the ends from rising; and in both cases they were so well fitted as to require slight blows of the mallet to drive them in.

These experiments were made in order to obtain data connected with mast-making, and to ascertain how far the commonly received notion was correct—namely, that if the vertex, or upper edge of a triangular prismatic beam, be cut off to one-third of the depth, the pieces will be stronger than before; or, in other words, that a part opposes more resistance than the whole;—which assertion, as anticipated, was satisfactorily contradicted by the following results.

These experiments are also very conclusive on another point, viz. that the strength of triangular prisms does not follow the law laid down either by

Leibnitz or Galileo; for, according to the former, the weights required to break a beam of this kind, with its base upwards, ought to be three times greater than in the reverse position; and according to the latter, it ought to be double. Now, the mean of the first seven experiments is 306, and of the next four 348; which is very far from the weight required in either of the above theories.

TABLE I.

78. Experiments on Triangular Oak Beams, by Mr. B. Couch. Pieces 3 feet 3 inches long, fixed by one end horizontally into a pillar; 3 feet beyond the prop.

Weight placed on the end.

Order of the experiments.	Position, form, &c.	Deflection below the first position.	Weight in lbs. supported.	Weight of the pieces.	The same pieces placed end for end, after altering their position or form.	Deflection below the first position.	Weight in lbs. supported.	Weight of the pieces.
				lbs. oz.				lbs. oz.
1	Angle upward.	9	290	3 1	Reduced to trapezoids, narrow end upward.	9	261	2 13
2		9	313	3 3½				
3		9	290	3 3				
4		9	333	3 6				
5		9	309	3 6				
6		9	308	3 5				
7		10	298	3 4				
8	Angle downward.	16	332	3 10	Angle upward.	9	286	3 7
9		11	349	3 7				
10		11	351	3 3				
11		11	360	3 4				
12	Trapezoid, narrow end up.	8	283	3 4				
13		11	285	3 1½				

Sum of the first seven weights = 2141

Sum of the next four = 1392

7)2141	4)1392
306 mean.	348 mean.

Sum of the six trapezoids = 1618

6)1618

269 mean.

TABLE II.

77. *Experiments, by Mr. Couch, on pieces 6 feet 6 inches long, each end fixed into pillars horizontally; 6 feet between the props.*

Weights placed on the middle.

Order of the experiments.	Position, form, &c.	Deflection below the first position.	Weight in lbs. supported.	Weight of the pieces.	REMARKS.
1	Angle upward.	6	980	lbs. oz. 7 5	
2		6	896	6 9	
3		6	1008	7 3	
4		5	1116	6 14	
5		6	1288	6 15	
6	Angle downward.	3	1056	6 14	Fractured $\frac{3}{8}$ inch on the angle.
		4	1166		Ditto 1 inch on the angle.
		7	1257		Broke nearly off.
7		2	870	7 2	Sprung a little on the angle.
		3	947		Broke nearly off.
8		3	1003	7 3	Sprung $\frac{1}{2}$ inch on angle, and continued breaking with the addition of every $\frac{1}{2}$ cwt., fibre after fibre, $\frac{3}{8}$ inch at a time, till all gave way.
		5	1366		
9		2 $\frac{1}{2}$	1285	7 14	Sprung without giving warning, from angle to half the depth.
10		2 $\frac{1}{2}$	1395	9 2	Sprung $\frac{1}{2}$ an inch on angle.
		3 $\frac{1}{2}$	1686		
11	Trapezoid, narrow face upward.	6	1319	8 6	Coarse, strong grain.
12		7	1099	6 0	Fine, weak grain.

78. The following Table exhibits the detail and results of experiments carried on also by Mr. Couch, on the lateral or transverse strength of Riga, Norway, and Halifax spars; as also on pieces of timber wrought to the shape of the said spars, (viz. frustrum of cones,) converted from large logs of red pine, yellow pine, and oak, all the growth of Canada.

The spars and other pieces were all of the same dimensions, viz. 27 feet long, $3\frac{1}{4}$ inches diameter at the butt, and to the distance of 5 feet from the butt: the upper end was $1\frac{1}{4}$ inch in diameter.

They were fixed by the greater end horizontally into a mortise, the prop or fulcrum being 5 feet from the butt; and the weights were placed 1 foot from the smaller end, leaving a lever or purchase of 21 feet.

TABLE III.

79. Experiments on Riga, Norway, and Halifax Spars, Red and Yellow Pine, &c., by Mr. Couch.

Order of experiments.	Species of wood.	Weight of each spar.	Deflection.	Weights which broke them.	REMARKS.
		lbs.	feet.		
1	Riga spar	29½	11	130	
2	Riga spar	29½	11	137	Upset or compressed, very much broken.
		..	12½	144	
3	Norway spar ..	32	12	168	Upset, lower part.
		..	13½	172	
4	Norway spar ..	36½	11	180	Upset, very much.
		..	12½	206	
5	Halifax spar	37½	11½	115	
6	Halifax spar	34½	12½	188	The tension of the fibres of this spar was much increased by being placed near a large fire for several days.
7	Red pine timber..	40½	14	150	
8	Red pine timber..	42½	14	180	
9	Red pine timber..	42½	14	202	
10	Yellow pine timber	33½	{ rapid deflection }	56	Fibres undulated.
11	Yellow pine timber	32	14	146	
12	Yellow pine timber	33½	{ rapid deflection }	56	Fibres undulated.
13	Oak timber.	52½	16	231	
14	Oak timber.	53½	18	254	

The experiments which have been now detailed relative to the transverse strains, are, it is presumed, all that are historically deserving of any particular notice in this place ; we shall, therefore, now proceed to describe the experiments from which the data given in a subsequent part of this work have been obtained.

Experiments made at the Royal Military Academy.

80. The foregoing were the principal experiments which had been made on the strength of timber, when I undertook to enter upon an investigation of this subject. They each furnished certain results; but there was no attempt at generalizing and connecting one set of results with another, by certain rules. Some rules, indeed, were to be found in different authors; but they differed in most cases the one from the other, not only numerically, but in principle. My object, therefore, has been to endeavour to examine these points of difference by independent and distinct experiments, and ultimately to furnish such practical rules as might be had recourse to by practical men.

An Explanation of the Method of making the Experiments on the Transverse Stress and Strength of Battens of different Woods, with a Description of the Apparatus, &c.

81. These experiments may be divided into four classes, viz. 1st, When the battens were supported on two props, as shown in Plate IV. 2dly, When they were fixed horizontally, with one end in a wall, as in fig. 3, Plate V. 3dly, When they were fixed at any given angle, as shown in figs. 1 and 2,

Plate V.; and, lastly, When both ends were firmly fixed, as in fig. 4 of the same Plate.

Plate IV. represents an experiment on a fir batten, A B, 7 feet in length, and 2 inches square, resting on the two props C D, E F, 6 feet asunder: the two weights P P are 1 lb. each, and were used to keep the fine silk line, to which they were attached, stretched in a horizontal position between the props: to facilitate which, the line was made to pass over two small brass rollers, one of which is shown at G. By means of this line, and the several small scales, s s s, &c., each divided into 10ths of inches, the deflection of the batten might be observed with great accuracy; and in this manner those given in the detail of the experiments were taken.

The number of these scales were varied at pleasure; commonly there was only one in the centre; at other times we had from three to ten, or even more; and in some few cases a board was placed against the batten, and the curve traced upon it with a pencil.

The small ivory scale at H was intended to measure the successive lengthening or stretching of the lower fibres, and was thus adjusted:

A fine silk line was fixed at the end A of the batten A B, and brought under the whole length of A to B: the scale, which had two fine steel points attached to it, was fixed by them into the under side of the batten, as shown at H: at the top of the scale was a small brass wheel or roller, over which

the silk passed, and to the end of this was hung a small semicylindrical brass weight, with its flat side towards the scale: two fine grooves were also cut, one in each of the brass plates, with which the tops of the props C D, E F, were defended, in order to allow the silk line to pass freely in them under the piece.

The batten thus furnished was now rested on the two props, with the line placed so as to pass in the two grooves above mentioned; and by means of a screw, by which the line was attached to the piece at A, the weight at H was adjusted to 0, on the same scale, which was divided from 0 upwards into 40ths of inches.

It is obvious now, that after the weights begin to give the batten any deflection, the small weight at H will be raised along the scale by a quantity exactly equal to the difference between the original length of the bottom fibres, and the length to which they are stretched at the time of making the observation; and in this manner the stretching of the fibres at several different degrees of deflection was measured in a few experiments: but as it did not appear that any useful application of this datum could be made in the theory, and as it required a longer time to adjust, &c., it was employed, comparatively, but in a few cases.

It would be useless to enter more minutely into an explanation of these experiments, as the process will be obvious from an inspection of the Plate: it

will be therefore sufficient, merely to observe further, that the artist has chosen to represent the apparatus as if the experiments were performed in the open air; and the consequence is, that the props do not appear sufficiently steady: they were, however, performed under cover, on a substantial floor; and the trestles or props were made to slide in grooves, and were firmly fixed in them, so as to render the whole perfectly secure and steady: and to prevent any momentum in loading the scale, this was always made stationary by wedges, when the larger weights were introduced.

82. In order to make the experiments on those pieces which were fixed by one end in a wall, the following means were employed. A block of hard wood, ABCD, fig. 3, Plate V., about 18 inches long, and 12 inches in breadth and depth, was cut through at about 5 inches from each end, as at *abcd*, for the convenience of forming a hole 2 inches in breadth and depth, or rather more; the one with the side of the square vertical, and the other with the diagonal vertical, as shown in the figure. The parts of the block were then screw-bolted together; and an iron socket, exactly 2 inches square on the inside, was made to fit these holes very accurately, but so that it might be taken out and put in at pleasure: a hole was then cut out of a very heavy solid wall, a little larger than the block, into which the latter was fixed by means of

inverted wedges, whereby the whole was rendered perfectly firm and immoveable.

The pieces of timber on which the experiments were made were 2 inches square, and therefore fitted tight into the iron sockets above mentioned, the edges of which are shown in the figure; the under side being made slightly curving, to prevent the cutting of the lower face of the piece after the weight was hung on: and as the deflection would have rendered the scale liable to slip off, an iron plate with two studs riveted to it, was screwed on the end of the batten, as shown at E and F, the former being bent into a right angle to fit its upper edge.

In the same manner the blocks of figs. 1 and 2 were made and fixed, differing from the former in nothing except the hole being made to form an angle of 26° with the horizon; the first ascending, and the other descending.

Those of fig. 4 were precisely the same as the lower part of fig. 3, and were fixed into two walls exactly 6 feet asunder.

Every thing being thus adjusted, the scale was hung on, as shown in Plate IV., but which, for simplicity, is merely represented, in Plate V., by a single ball W.

83. It may not be amiss to add, that the walls in which the blocks were fixed were not less than 40 feet high, although in the Plate they are repre-

sented as if they were not above 6 feet: it being thought useless to show them in their full height.

Such were the means employed for assuring accuracy in the results, and which it has been thought right to explain at length, in order that the reader may judge of the degree of confidence to which these experiments are entitled. This has been commonly omitted by preceding authors, and has been the subject of just complaint by those who would have wished to avail themselves of their data for the purpose of theoretical investigation; so that in cases where a disagreement was found to have place between the theoretical and practical results, it was always doubtful to which the error belonged, and was therefore attributed to either, as best suited the views of the writer.

The following are the results of the different experiments made on the transverse strain, arranged according to their dimensions.

TABLE I.

84. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Weight reduced to sp. gr. 600.	Mean wt. corresponding to sp. gr. 600.
1	15	1	1	504	360	428	439
2				533	388	436	
3				564	418	444	
4				646	453	421	
5				588	453	462	
6				600	441	441	
7	18	1	1	552	318	346	342
8				647	364	338	
9				724	436	371	
10				719	404	337	
11				648	353	327	
12				672	376	336	

The above experiments were made principally in order to determine what relation there might be between the ultimate strength and the specific gravity of the rods: they were therefore selected out of those which had been the same time in store, and that differed the most from each other in their specific gravity, and principally from the fragments of those that had been broken in preceding experiments, of which the detail is given in the subsequent pages.

The reduced weight in the seventh column above is found on a supposition that the strength is as the specific gravity: a reduction which is adopted throughout.

We can see no physical reason for the circumstance of the strength being so nearly proportional to the specific gravity. It ought rather, one would have supposed, to have been as the $\frac{2}{3}$ rd power; for, supposing the number of particles to be as the specific gravity, the number of them in any section would be as the $\frac{1}{3}$ rd power of the latter. Upon the whole, however, the simple ratio of the strengths being as the specific gravities, seems to answer better than any other.

TABLE—(CONTINUED).

85. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches	Breadth in inches.	Deflection.	Specific gravity	Weight in lbs.	Weight reduced to sp. gr. 600.	Mean Weight sp. gr. 600.
1	24	1	1	1·25	..	270	..	265
2				1·25	..	262	..	
3				1·25	..	262	..	
4				..	560	261	279	288
5				..	560	283	303	
6				..	540	256	284	
7	30	1	1	1·80	..	242	..	237
8				1·80	..	234	..	
9				1·80	..	235	..	
10	36	1	1	1·85	577	229	237	196
11				3·12	505	162	192	
12				3·00	505	148	160	
13				2·2	553	181	196	
14				3·2	553	181	196	
15				2·2	553	181	196	

The specific gravities of Nos. 1, 2, and 3 were not observed, nor the deflections of 3, 4, and 5. The deflections of 1, 2, and 3 were all the same, viz. for 220 lbs. $\frac{1}{8}$ inch; for 250 lbs. one inch; for 260 lbs. $1\frac{1}{2}$ inch.

The specific gravities of Nos. 7, 8, and 9, not observed; these, with Nos. 1, 2, and 3, were broken before it was thought necessary to introduce that consideration.

Nos. 11 and 12 were both off a very light plank, and were very elastic.

Nos. 13, 14, and 15 were very uniform rods. Nos. 13 and 15 were bound to two pieces of the same thickness as themselves, but each half the whole length, to prevent any curving; and No. 14 was broken as usual. It seems, therefore, that the curving of the batten does not weaken it, although it increases the deflection.

TABLE—(CONTINUED).

86. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific gravity.	Weight in lbs.	Reduced to sp. gr. 600.	Mean Weight sp. gr. 600.
1	24	1½	¾	..	646	420	390	397
2				..	646	424	393	
3				..	646	441	409	
4				·70	746	557	448	435
5				·70	709	501	424	
6				·70	734	531	434	
7	30	1½	¾	1·12	733	412	337	336
8				1·12	733	411	336	
9				..	646	360	334	

No. 1 was a very complete fracture, showing very distinctly the part of the section which had been compressed, and that which had acted by tension ; the latter rather exceeded ⅓rd of the whole depth. In Nos. 2 and 3 the same appearance might be observed, but not so perfectly. No. 3 hung two hours and a half before breaking ; the others only ten minutes.

Nos. 4, 5, and 6 were remarkably sound pitch pine, full of turpentine. No. 5 would probably have borne as much as No. 4 or No. 6, but that the upper part, on which the weight hung, was more tender, and was much crippled in the experiment.

Nos. 7 and 8 were part of the same plank as Nos. 4, 5, and 6 ; and No. 9 was part of the specimen from which Nos. 1, 2, and 3 were made.

It appears from the first of the above set of experiments, that the strength is in a higher ratio than that of the specific gravities.

TABLE—(CONTINUED).

87. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific gravity.	Weight in lbs.	Reduced sp. gr. 600.	Mean weight sp. gr. 600.
1	24	2	1 {	·625	613	1190	1164	} 1119
2				..	563	1000	1066	
3				..	600	1128	1128	
4	30	2	1 {	..	586	882	903	} 900
5				..	581	871	901	
6				1·08	571	852	895	
7	36	2	1 {	1·00	..	600	..	} 600
8				1·12	..	622	..	
9				1·12	..	680	..	
10				1·12	..	595	..	
11				1·52	..	552	..	
12				1·50	..	550	..	
13	36	2	1 {	1·12	606	722	715	} 745
14				1·12	606	752	744	
15				1·12	564	730	776	

No. 1 was left for twenty-four hours, with 865lbs. hanging upon it, without any deflection beyond what it had acquired in a few minutes.

The successive deflections of No. 6 were

520 lbs. = $\frac{5}{16}$ inch, 620 lbs. = $\frac{6}{16}$, 720 lbs. = $\frac{7}{16}$.

Nos. 7, 8, and 12, were broken before it was thought necessary to introduce the specific gravities; they were lighter and weaker wood than the preceding; and Nos. 5 and 6 were obviously damaged, by being exposed to wet.

The successive deflections and stretching of Nos. 13 and 14 were as follows; viz.

220 lbs.	deflection	$\frac{1}{8}$	stretching	0
420	$\frac{4}{8}$	$\frac{1}{16}$
520	$\frac{5}{8}$	$\frac{3}{16}$
580	$\frac{6}{8}$	$\frac{1}{16}$

TABLE—(CONTINUED).

88. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Successive deflections.	Mean weight reduced to sp. gr. 600.
1	44	2	2	630	421 848 1054 1166 1211 1226 1288 1317	·175 ·266 ·300 ·350 ·566 ·660 ·450 ·700 ·900 ·530 ·900 1·025 ·600 1·00 1·15 ·650 1·10 1·30 ·900 1·57 1·95 2·35	1255
2	44	2	2	..	421 848 1054	·175 ·275 ·350 ·366 ·633 ·763 2·00	
3	48	2	2	601	421 711 920 1020 1125	·15 ·25 ·33 ·36 ·27 ·47 ·60 ·66 ·40 ·60 ·90 1·02 ·53 ·90 1·23 1·4 2·3	1116
4	48	2	2	601	1110	The same deflection.	

The deflections in the above experiments were measured by scales fixed on the pieces at equal distances, from one end to the middle, as explained in Art. 81.

It was remarked, in the experiment No. 1, that the deflection of the piece was very sensibly affected, after 1240 lbs. were on, by the addition and subtraction of a 7 lb. weight.

No. 2 was part of the same plank as No. 1, and only parted from it by the saw, although it was so much weaker; it was sappy and light, but the account of its specific gravity was lost, or not taken.

In Nos. 3 and 4 seven scales were used, placed at equal distances, viz. one at every six inches. The deflections are only given above from the middle to one end.

TABLE—(CONTINUED).

89. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches	Breadth in inches.	Specific gravity	Weight in lbs.	Successive Deflections and Lengthening.		Mean weight reduced to sp. gr. 600.
						Deflections.	Length.	
1	60	..	2	..	788			
2	60	2	2	..	421 521 711 811 711	·33 ·56 ·75 ·40 ·70 ·96 ·73 1·30 1·80 ·93 1·70 2·37 not observed	·087 ·125 ·162	770
3	60	2	2	..				
4	72	2	2	563	221 421 521 621 682 221 421 521 621 760	·35 ·60 ·75 ·70 1·2 1·45 ·90 1·55 1·87 1·30 2·30 2·80 4·30 ·30 ·53 ·65 ·60 1·03 1·20 ·76 1·33 1·50 1·00 1·70 2·00 3·50	·062 ·125 ·150 ·187 ·200 ·075 ·162 ·187 ·225 ·350	744
5	72	2	2	600				

But little dependence can be placed upon the experiments Nos. 1, 2, and 3. No. 1 was part of a weak plank; and Nos. 2 and 3 were cut from one piece, which was at first 8 feet 6 inches; after breaking it at 5 feet, the remnant, which was then 6 feet, was broken again at 5 feet, breaking with the weight stated in No. 3: the latter part was nearest the root end. The specific gravities were not taken.

Nothing particular was noticed in experiments 4 and 5. The lengthening of the piece was measured by means of the instrument described Art. 81. And, in order to protect the battens against the splintering which commonly happened in the preceding experiments, they were bound round with twine on each side of the place of fracture, leaving about two inches clear in the middle.

Observations relative to the preceding Experiments.

90. It is proper here to observe, that the preceding results must not be considered as furnishing any data that are applicable to fir in general; for as the object was principally to ascertain the law which takes place between the strength and the dimensions of the pieces, the greatest care was taken in selecting the best and most perfect specimens of the kind that could be procured: several of the planks had been in store for a considerable time, and were perfectly seasoned, which accounts for their specific gravities being less than is usually found for Riga fir and Christiana deals, of which the specimens principally consisted. By this means a greater uniformity was found in the results, and a greater strength than is generally due to this kind of wood; but the results were obviously so much the better adapted for eliciting a correct idea of the nature of the straining and resisting forces. The medium strength of Riga fir will be found in the general Table of Data.

TABLE II.

91. *Miscellaneous Experiments on Fir Beams, cross-cut in the centre, and supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	De- flection.	Mean weight reduced to sp. gr. 600.
1	30	2	1	581	808	1·00	856
2	30	2	1	581	220	·250	
					420	·440	
					520	·500	
					620	·625	
					780	·750	
					846	·875	
3	30	2	1	580	835	·875	{ Same deflec- tions as No. 2.

The preceding experiments having shown pretty clearly the situation of the neutral axis; viz. that it was at about $\frac{3}{8}$ ths of the depth of the section from the bottom; these bars, which were part of the same specimens as those of the same dimensions (Art. 87), were cut down $1\frac{1}{4}$ inch, or $\frac{3}{8}$ ths of the depth, and the saw-groove filled up by a thin slip of pear-tree, sufficiently tight to preserve the stiffness of the battens, but without straining them. They were then loaded as usual, and were broken with the weights above stated.

On examining the wedges, or slips of pear-tree, after the experiments, it was found that No. 1 was a little longer than No. 3, and No. 3 than No. 2; and the wedge of another batten, that broke with a considerable less weight, was $\frac{1}{16}$ th of an inch longer than any of them. The impression of the fibres was very distinctly marked on the wedges; strongest at top, and gradually weakening towards the bottom, where they could scarcely be distinguished.

These experiments seem to indicate that the neutral axis was very nearly at $\frac{3}{8}$ ths of the depth of the batten. The deflection of No. 1 exceeded that of Nos. 2 and 3 by $\frac{1}{8}$ th throughout.

TABLE II.—(CONTINUED.)

92. Miscellaneous Experiments on Fir Battens, grooved out in the centre, and supported at each end.

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	De- flection.	REMARKS.
1	36	2	1½	564	421 711 1095	·25 ·43 1·0	} Whole beam. } Groove down-wards ⅔rd in. deep, and ½ in. broad. } Groove upw ^{ds} . ⅓rd inch deep, & ½ in. broad.
2	36	2	1½	564	421 711 985	·300 ·566 1·10	
3	36	2	1½	538	421 621 780	·366 ·630 1·50	

These weights, reduced to specific gravity 600, gave No. 1, 1164; No. 2, 1047; No. 3, 870.

The experiments in the preceding page having nearly pointed out the position of the neutral axis, these experiments were made with a particular view. Nos. 2 and 3 were grooved out, in the centre of their breadth, from end to end; the former to ⅓rd of the depth, and the latter to ⅔rds, and each ½ an inch broad; viz. ⅓rd of the breadth. The idea was, that what No. 2 broke short of the weight required in the whole batten, would be the measure of ⅓rd of the tension; and what No. 3 broke short of the same, would be the measure of ⅓rd of the compression. This view of the subject was afterwards found to be erroneous; but the experiments were retained, on a supposition that they might still form some standard of comparison.

TABLE II.—(CONTINUED.)

93. *Miscellaneous Experiments on Triangular Fir Battens.*

No of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Position of the battens.	Mean weight reduced to sp. gr. 600.
1 2	24	$\frac{1}{2}\sqrt{2}$	$\sqrt{2}$ {	118 97	Base upwards. Do. downwards.	
3 4 5 6 7 8	24	$\sqrt{3}$	2 {	613 588 559 574	740 740 680 680	Base upwards. Do. do. Do. do. Do. do.	} 740 } 720
	24	$\sqrt{3}$	2 {	619 603	637 637	Base downwards. Do. do.	
9 10	20	$\sqrt{3}$	2 {	.. 630	907 843	Base upwards. Do. downwards.	

These pieces were made out of the fragments of the 2-inch square battens; viz.

- 3 and 8 out of No. 3, art. 88.
- 4 and 7 out of No. 4, art. 88.
- 5 and 6 out of No. 4, art. 89.
- 9 out of No. 2, art. 88.
- 10 out of No. 1, art. 88.

All these pieces, except Nos. 5 and 6, were rested in triangular saddles of hard wood, cut very exactly to the angle of the batten, when they were broken with their edge down; but when the edge was upward, a similar one was placed on the centre, in order that the weight might not break down its edge. This latter saddle was about half an inch thick.

Nos. 5 and 6 had pieces glued and screwed on at their ends, in order to render their bearings solid; but it did not appear to make any difference: they were weaker than Nos. 3 and 4; but the piece from which they were made, viz. No. 4, Art. 89, was itself comparatively weak, as appears by that experiment.

TABLE II.—(CONTINUED.)

94. *Experiments on Fir Battens, fixed at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	De- flection.	Weight in lbs.	Reduced to sp. gr. 600.	REMARKS.
1	72	2	2	581	.45 1.00 1.30 2.1	220 620 822 1024	1058	The whole time of the experiment 34 min.; after last weight 6 m.
2	72	2	2	581	.41 .95 1.25 2.1	220 620 822 1139	1174	Whole time 28 m.
3	72	2	2	611	.40 .87 1.35 2.2	220 620 822 1090	1070	Whole time 45 m.
4	72	2	2	600	.45 1.00 1.30 2.3	220 620 822 1120	1120	Whole time 18 m.
					Mean weight	1105		

Nothing remarkable occurred in making these experiments. We have before (Art. 82) explained the methods that were employed in order to ensure a permanent fixing of the two ends, which was done with the greater care, as experiment and theory differed very materially in the comparative strength of equal battens, when *fixed* at each end, and when only *supported*: all former theories make the strength in the two cases as two to one, while most experimentalists state it as in the ratio of 3 : 2. According to the former, the mean strength of these beams, as compared with those at Art. 89, ought to have been 1442 lbs., and according to the latter, 1116 lbs.: the mean is 1105 lbs.; which is consistent with what has been shown, Art. 20.

TABLE II.—(CONTINUED.)

95. *Experiments on Fir Battens, fixed at one end, at different angles of inclination and in different positions.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Deflection in inches.	Weight reduced to length 36, & sp. gr. 600.	Position of the beams, &c.
1	36	2	2	560	317	5·0	400	} Side parallel to the horiz ⁿ .
2	32	2	2	609	432	6·0	400	
3	32	2	2	571	417	6·0	389	
4	30	√8	√8	600	462	4·9	385	} Diagonal vertical.
5	30	√8	√8	613	469	4·7	391	
6	30	√8	√8	620	466	4·9	389	
7	24	2	1	620	279	4·1	180	} Horizontal.
8	24	2	1	600	276	3·9	184	
9	24	2	1	596	273	4·3	183	} Angle 26° upwards.
10	24	2	1	581	281	4·1	193	
11	24	2	1	600	294	3·9	196	} Angle 26° downwards.
12	24	2	1	601	290	4·0	193	

The first six of the above pieces were the fragments of the first two and last specimens of the preceding page; care having been taken, in those experiments, to prevent the weights from going quite down, which would have endangered the breaking of the pieces at the ends where they were fixed in the wall. By blocking the scale as soon as the fracture commenced in the middle, the ends were left perfectly whole, the parts recovering completely their original rectilinear form.

The first three of the above were broken in the same position; viz. with the sides parallel and perpendicular to the horizon; the next three angle-ways, viz. with the diagonal vertical.

Nos. 7 and 8 were fixed in the usual horizontal position; Nos. 9 and 10, which were the same two pieces inverted, or turned end for end, were fixed at an angle of inclination upwards of 26°; and Nos. 11 and 12 at the same angle downwards.

TABLE II.—(CONTINUED.)

96. *Experiments on Oak Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Reduced to sp. gr. 800.	Mean reduced weight.
1 2 3	18	1	1 {	767 768 768	323 353 339	337 368 368	} 358
4 5 6	24	1	1 {	764 774 774	266 251 260	278 260 268	} 269
7 8 9	30	1	1 {	777 777 777	196 196 196	202 202 202	} 202
10 11 12	36	1	1 {	2·95 4·20 ..	158 190 176		} 180*

Nos. 1, 2, 5, and 4, were all from one piece, near the root end, and rather cross-grained, particularly Nos. 1 and 5. Nos. 2 and 4 were cut from the ends of these. Nos. 7, 8, and 9, each bore 286 lbs. without any appearance of fracture; but each broke immediately with the addition of 15 lbs.: it was therefore only taken as 10 lbs.

No. 11 was remarkably elastic; and, just before its fracture, its curve was traced on a plane-board placed against it, and the ordinates carefully measured at every inch, and were found as follow:

Ordinates, ·26, ·53, ·85, 1·13, 1·4, 1·7, 1·93, 2·2, 2·45.

Abscisses, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Ordinates, 2·65, 2·87, 3·1, 3·3, 3·46, 3·63, 3·75, 3·82, 3·9.

Abscisses, 10, 11, 12, 13, 14, 15, 16, 17, 18.

* The specific gravities of 10, 11, 12, were not noted; the mean 180 is found by assuming them at 777, being part of the same plank as the above.

TABLE II.—(CONTINUED.)

97. *Experiments on Oak Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	De-flection.	Weight in lbs.	Reduced to sp. gr. 800.	Mean reduced weight.
1 2 3	24	1½	¾ {	768 784 777	1·1 1·1 1·1	387 408 395	403 416 406	} 408
4 5 6	30	1½	¾ {	777 784 768	1·5 1·5 1·5	316 327 300	325 333 311	} 326
7 8 9	30 30 30	2 2 2	1 1 1	777 {	1·4 1·4 1·4	721 736 736	742 758 758	} 753
10 11 12	36	2	1	764 {	598 607 612	626 635 641	} 634

The successive deflections of Nos. 1, 2, 3 were measured as follow, viz.

Weights.	No. 1.	Deflection of No. 2.	No. 3.
321	·65	·62	·65
366	·85	·72	·85
380	1·05	·95	1·05
387	1·10	1·05	1·08

The deflections of Nos. 7 and 9 were exactly equal, and were measured on three equidistant ordinates: the lengthening of the fibres was also in both cases equal: the particulars are as below, viz.

Weights.	Deflections.	Lengthened.
421 Nos. 7 and 9	·10 ·25 ·366	·075
521	·13 ·35 ·466	·100
621	·19 ·50 ·700	·125
671	·20 ·60 ·800	·150
721 1·400	

The deflections of No. 8 were not observed.

TABLE II.—(CONTINUED.)

98. *Experiments on Ash Battens, fixed at one end in a wall.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Weight reduced to 36 inches length, and 2 in. square.*	Position of the beams.
1	36	2	2	658	11½	436	436	Side parallel to the horizon.
2	36	2	2	730	14½	431	431	
3	30	√8	√8	658	5½	471	392	Diagonal vertical.
4	30	√8	√8	730	6½	466	388	
5	24	2	1	730	5	352	470	Fixed at an < 26° down.
6	24	2	1	730	not obs.	321	428	Ditto, ditto, upwards.
7	24	2	1	730	6	332	441	Horizontal.
8	24	2	1	730	6	321	428	Horizontal.
9	24	2	1	730	not obs.	302	403	Angle 26° upwards.

No. 1 was the same piece as No. 3.

No. 2 „ „ No. 4.

No. 5 „ „ No. 7.

No. 8 „ „ No. 9.

Nos. 1, 2, 5, and 8 were first broken at one end (but not so as to completely separate the parts); after which they were turned end for end, and broken again, as stated in Nos. 3, 4, 7, and 9. No. 6 was so fractured in the first experiment, that it could not be submitted to a second trial. The same thing always occurred when the beam was first broken at an angle upwards: it appeared, in these cases, to turn on a point about 6 inches from the wall where the strain and curvature seemed to be the greatest, and from which point the fracture commenced, splitting the piece through its whole length.

In the above experiment, No. 2, the neutral line was remarkably well defined, and appeared to be very nearly, or exactly, at ¼ths of the whole depth; the same as in fir.

* The reduction in column 8 is made on a supposition that the strength is inversely as the length.

TABLE II.—(CONTINUED.)

99. *Experiments on Beech Battens, fixed at one end in a wall, at different inclinations and in different positions.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Weights reduced to 26 inches length, and 2 in. square.	Position of the beams.
1	36	2	1	700	11	401	401	Side parallel to the horizon.
2	36	2	2	690	8	401	401	
3	36	2	2	700	11	401	401	
4	30	$\sqrt{8}$	$\sqrt{8}$	690	5	466	388	Diagonal vertical.
5	30	$\sqrt{8}$	$\sqrt{8}$	700	6	451	376	
6	24	2	1	740	$4\frac{1}{2}$	371	495	Fixed at an $< 26^\circ$ down.
7	24	2	1	740	5	352	469	Ditto $< 26^\circ$ upwards.
8	24	2	1	740	5	352	469	Horizontal.
9	24	2	1	740	5	352	469	Ditto.
10	24	2	1	740	$5\frac{1}{2}$	317	463	At an $< 26^\circ$ upwards.

No. 2 was the same piece as No. 4.

No. 3 " " No. 5.

No. 6 " " No. 8.

No. 9 " " No. 10.

Nos. 1 and 7 were so much splintered in the first experiments, that they could not be submitted to a second trial, as was done with Nos. 2, 3, 6, and 9. These, after being broke at one end, (without a total separation,) were turned end for end, and then broken with the weights indicated in Nos. 4, 5, 8, and 10.

It should be observed here, that the deflections were not, in these experiments, measured so accurately as in those that were supported at each end: the apparatus not being so convenient, we were generally satisfied with measuring it to the nearest $\frac{1}{4}$ of an inch: the successive deflections, however, seemed to follow, while the weights were small, the ratio of the weights, as was observed in the preceding experiments. The deflections from first to last were as follow:

121 lbs.	No. 1 =	$1\frac{1}{2}$	No. 2 =	$1\frac{1}{4}$	No. 3 =	$1\frac{1}{2}$
221 lbs.	=	$3\frac{1}{2}$	=	3	=	$3\frac{1}{2}$
271 lbs.	=	5	=	$3\frac{1}{2}$	=	$4\frac{1}{2}$
321 lbs.	=	7	=	5	=	$6\frac{1}{2}$
401 lbs.	=	11	=	8	=	11

TABLE II.—(CONTINUED.)

100. *Experiments on Solid and Hollow Cylinders, supported at each end.*

No. of experiments.	Names of woods.	Specific gravity.	Length in inches.	Diameter external.	Diameter internal.	Breaking weight.	Deflections in inches.
1	} Fir.	581	48	2	solid.	740	2·0
2		603	48	2	do.	796	2·1
3		580	48	2	do.	780	1·9
4	} Ash.	590	46	2	solid.	700	2·7
5		590	46	2	solid.	730	2·5
6		586	46	2	$\frac{1}{2}$ inch.	650	3·0
7		540	46	2	$\frac{1}{2}$ inch.	664	3·0
8		601	46	2	$\frac{3}{4}$ inch.	646	3·1
9		601	46	2	$\frac{3}{4}$ inch.	654	2·9
10		580	46	2	1 inch.	631	2·8
11		580	46	2	1 inch.	630	3·6

The fir pieces were part of the same plank as those of 4 feet, given in Art. 88, viz. Nos. 3 and 4, which was a very fine specimen of Christiana deal, and had been in store a considerable time.

The ash cylinders were obviously of a much weaker quality than those of which the detail is given at Art. 98; but the results were very uniform, and they, therefore, furnish a good comparison between the strength of solid and hollow cylinders amongst themselves, although we cannot compare them with our square battens, as they were of a much inferior quality to the preceding square pieces. The fir cylinders, on the contrary, furnish no comparison between solid and hollow cylinders; but they may be correctly compared with like pieces of the same dimension square, being, as stated above, precisely the same wood as Nos. 3 and 4, Art. 88.

101. Similar experiments to those last described were made on battens of elm and teak ; but the results of the latter were so irregular, that it would be useless to give the detail of them : it will be sufficient to observe, that one of the pieces of teak bore 478 lbs., which was more than equal to the load borne by the ash pieces of the same dimensions ; viz. 3 feet long by 2 inches square ; while the other two pieces broke with little more than 300 lbs., the deflection in each case being about 7 inches : and one piece 2 feet long, 2 inches deep, and 1 inch in breadth, fixed at one end, and at an angle of 26° upwards, broke with 422 lbs., which is considerably more than was found to be necessary for breaking an equal piece of ash.

The elm battens gave much more uniform results, although the pieces were found very weak in comparison with those of ash and beech. The mean weight which broke the three pieces 3 feet long and 2 inches square, was 216 lbs. ; and the mean of the same three pieces inverted and fixed diagonally, was 296 lbs., the latter being broken at 30 inches ; the mean specific gravity was 570.

Remark.—If the same reduction be made here as in the pieces of ash and beech, we shall have

$$36 : 30 :: 296 : 246,$$

which shows that the strength of elm is the same whether it be fixed direct or diagonally ; whereas it was found that ash and beech were both weakest in the latter position.

Determination of Practical Data.

102. It has been observed, that all the preceding specimens of wood were selected from deals, planks, and battens, which had been in store a considerable time, and that only the best, or those of the most uniform texture, were chosen for the purpose; the object of the experiments not having been to furnish practical data, but to compare, under the most favourable circumstances, the theoretical formulæ with experimental results. This having been effected, and the agreement having been found generally perfectly satisfactory, it became necessary to make another series of experiments on woods of more common quality, in order to furnish data for practical cases. The author therefore applied to the Admiralty, and obtained permission to select specimens for experiment, from all the timber in store in Woolwich Dockyard; in which selection he was kindly assisted by Mr. Hockey, assistant builder in that establishment.

It has been shown, (Art. 28,) that as regards the absolute strength of a beam, we ought to find,

When the beam is fixed at one end and loaded at the other,

$$\frac{lW}{ad^2} = S,$$

a constant quantity for all wood of the same quality, whatever may be the length l , the breadth a , or the

depth d ; consequently, S once determined, remains the same, and serves for computing the strength of any sized beam of the same wood, or the dimensions necessary to insure a given strength in a given direction. That is, of the four quantities l , a , d , W , any three being given, the fourth may be found thus,

$$\begin{aligned} W &= \frac{S a \cdot d^2}{l} \\ l &= \frac{S a d^2}{W} \\ a &= \frac{l W}{S d^2} \\ d &= \sqrt{\frac{l W}{a S}} \end{aligned} \left\{ \begin{array}{l} \text{In square beams } a = d = \sqrt[3]{\frac{l W}{S}}. \end{array} \right.$$

When supported at one end and loaded in middle,

$$\frac{l W}{4 a d^2} = S.$$

In this case, therefore,

$$\begin{aligned} W &= \frac{4 a d^2 S}{l} \\ l &= \frac{4 a d^2 S}{W} \\ a &= \frac{l W}{4 d^2 S} \\ d &= \sqrt{\frac{l W}{4 a S}} \end{aligned} \left\{ \begin{array}{l} \text{In square beams } a = d = \sqrt[3]{\frac{l W}{4 S}}. \end{array} \right.$$

When the beam is fixed at both ends and loaded in the middle,

$$\begin{aligned} W &= \frac{6 a d^2 S}{l} \\ l &= \frac{6 a d^2 S}{W} \\ a &= \frac{l W}{6 d^2 S} \\ d &= \sqrt{\frac{l W}{6 a S}} \end{aligned} \left. \vphantom{\begin{aligned} W &= \frac{6 a d^2 S}{l} \\ l &= \frac{6 a d^2 S}{W} \\ a &= \frac{l W}{6 d^2 S} \\ d &= \sqrt{\frac{l W}{6 a S}} \end{aligned}} \right\} \text{In square beams } a = d = \sqrt[3]{\frac{l W}{6 S}}.$$

When the beam is supported at both ends and loaded at an intermediate point,

$$\begin{aligned} W &= \frac{l a d^2 S}{m n} \\ l &= \frac{m n W}{a d^2 S} \\ a &= \frac{m n W}{l d^2 S} \\ d &= \sqrt{\frac{m n W}{l a S}} \end{aligned} \left. \vphantom{\begin{aligned} W &= \frac{l a d^2 S}{m n} \\ l &= \frac{m n W}{a d^2 S} \\ a &= \frac{m n W}{l d^2 S} \\ d &= \sqrt{\frac{m n W}{l a S}} \end{aligned}} \right\} \text{In square beams } a = d = \sqrt[3]{\frac{m n W}{l S}}.$$

When the beam is fixed at both ends and loaded at an intermediate point,

$$\begin{aligned} W &= \frac{3 l a d^2 S}{2 m n} \\ l &= \frac{2 m n W}{3 a d^2 S} \\ a &= \frac{2 m n W}{3 l d^2 S} \\ d &= \sqrt{\frac{2 m n W}{3 l a S}} \end{aligned} \left. \vphantom{\begin{aligned} W &= \frac{3 l a d^2 S}{2 m n} \\ l &= \frac{2 m n W}{3 a d^2 S} \\ a &= \frac{2 m n W}{3 l d^2 S} \\ d &= \sqrt{\frac{2 m n W}{3 l a S}} \end{aligned}} \right\} \text{In square beams } a = d = \sqrt[3]{\frac{2 m n W}{3 l S}}.$$

When the weight is uniformly distributed, the

same formulæ will apply; but W in this case will represent only half the required or given weight.

103. Again, it has been found, (Art. 66, &c.,) using a for b , in reference to elasticity and deflection, that

When a beam is fixed at one end and loaded at the other,

$$\frac{l^3 W}{a d^3 \delta} = E,$$

a constant quantity for all woods of the same quality.

When fixed at one end uniformly loaded,

$$\frac{3 l^3 W}{8 a d^3 \delta} = E.$$

When supported at each end and loaded in the middle,

$$\frac{l^3 W}{32 a d^3 \delta} = E.$$

When supported at each end and uniformly loaded,

$$\frac{5}{8} \times \frac{l^3 W}{32 a d^3 \delta} = E.$$

E therefore being determined for any given wood, the other quantities may be found by a proper inversion of these formulæ, as in the preceding cases of strength. These several values of S and E have been found experimentally on the several specimens as stated in the following Table.

(COPY OF A REPORT TRANSMITTED TO THE HON. THE PRINCIPAL OFFICERS
COMMISSIONERS OF HIS MAJESTY'S NAVY.)

TABLE OF DATA,

CONTAINING THE

(104.) *Results of Experiments on the Elasticity and Strength of vari
Species of Timber : selected from Woolwich Dockyard.*

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect.		Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{P W'}{32 a d^3 b}$	Value of from the form $S = \frac{I W}{4 a c}$	
1	Teak, 7 ft. by 2 in. sq.	742	{ 300	1.065 }	1020	4.75	1.2	301800	2462	
2		749	{ 300	1.093 }	975	4.20	1.2			
■		744	{ 300	1.150 }						
			{ 300	1.130 }						
			{ 300	1.276 }						
			{ 300	1.192 }						
Mean Results ..	745	300	1.151	938	4.32	1.2				
4	Poon, 7 ft. by 2 in. sq.	600	{ 150	.830 }	860	6.00	1.25	211200	2221	
5		570	{ 150	.780 }	848	5.75	..			
6		568	{ 150	.820 }						
			{ 150	.837 }						
			{ 150	.837 }						
			{ 150	.830 }						
Mean Results ..	579	150	.822	846	5.92	1.225				
7	English Oak, 1st specimen, 7 ft. by 2 in. sq.	986	{ 150	1.420 }	470	6.00	1.3	109200	1181	
8		998	{ 150	1.420 }	421	5.90	1.3			
			{ 150	1.700 }						
			{ 150	1.700 }						
			9	inferior specimen.	925	{ 150	1.650 }			460
{ 150		1.650 }								
Mean Results ..	969	150	1.590	450	5.90	1.3				

* NOTE.—For the sake of simplifying the calculations, the value of E is not carried on beyond the nearest fourth figure.

TABLE—(CONTINUED).

No. of specimens.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect.		Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{l^3 W'}{32 a d^3 \delta}$	Value of S, from the formula $S = \frac{l W}{4 a d^2}$
			Weight in lbs.	Deflect° in inches.					
10	English Oak, 2nd specimen, 6 ft. by 2 in. sq. reduced to 7 ft.	942	200	1.260	640	7.90	1.2		
11			200	1.280					
		900	200	1.290	623	8.30	1.2		
			200	1.290					
		960	200	1.275	649	8.10	1.2		
			200	1.285					
	Mean Results ..	934	200	1.280	637	8.10	1.2	181400	1672
13	Canadian Oak, 7 ft. by 2 in. sq.	865	225	1.150	660	5.70	1.1		
			225	1.150					
14		885	225	1.009	708	6.20	..		
			225	1.011					
15		867	225	1.070	651	6.10	1.15		
			225	1.070					
	Mean Results ..	872	225	1.080	673	6.00	1.125	268600	1766
16	Dantzic Oak, 7 ft. by 2 in. sq.	767	200	1.710	520	5.00	1.2		
			200	1.690					
17		787	200	1.260	580	4.10	1.2		
			200	1.300					
18		713	200	1.855	580	5.50	..		
			200	1.715					
	Mean Results ..	756	200	1.590	560	4.86	1.2	148900	1457
	Adriatic Oak, 7 ft. by 2 in. sq.	941	150	1.070	560	6.00	1.20		
			150	1.070					
20		948	150	1.550	500	5.50	1.25		
			150	1.450					
21		1090	150	1.720	520	5.70	1.15		
			150	1.720					
	Mean Results ..	993	150	1.430	526	5.73	1.2	121800	1383
22	Ash, 7 ft. by 2 in. sq.	760	225	1.270	777	9.00	1.35		
			225	1.250					
23		758	225	1.300	760	9.10	1.30		
			225	1.270					
24		762	225	1.240	780	8.66	1.25		
			225	1.270					
	Mean Results ..	760	225	1.266	772	8.92	1.3	205600	1920

TABLE—(CONTINUED).

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect.		Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{l^3 W'}{32 a d^3 \delta}$	Value α from the form $S = \frac{l^3}{4 a}$
			Weight in lbs.	Deflect ⁿ . in inches.					
25	Beech, 7 ft. by 2 in. sq.	712	{ 150	{ 1.075	565	6.00	1.2		
			{ 150	{ 1.025					
26		628	{ 150	{ 1.009	600	5.70	..		
			{ 150	{ 1.024					
27		688	{ 150	{ 1.025	615	5.50	1.2		
			{ 150	{ 1.000					
	Mean Results ..	696	150	1.026	593	5.73	1.2	169200	1556
28	Elm, 6 ft. by 2 in. sq. reduced to 7 feet.	583	{ 125	{ 1.620	368	7.00	1.2		
			{ 125	{ 1.610					
29		540	{ 125	{ 1.420	398	6.93	1.1		
			{ 125	{ 1.460					
		535	{ 125	{ 2.070	394	6.86	..		
			{ 125	{ 1.930					
	Mean Results ..	553	125	1.685	386	6.93	1.15	97480	1013
31	Pitch Pine, 7 ft. by 2 in. sq.	712	{ 150	{ 1.133	650	6.25	1.2		
			{ 150	{ 1.166					
32		628	{ 150	{ 1.140	595	5.75	1.2		
			{ 150	{ 1.110					
33		641	{ 150	{ 1.166	620	6.00	..		
			{ 150	{ 1.091					
	Mean Results ..	660	150	1.134	622	6.00	1.2	153200	1632
34	Red Pine, 7 ft. by 2 in. sq.	655	{ 150	{ .825	473	5.70	1.3		
			{ 150	{ .825					
35		667	{ 150	{ .700	530	5.83	1.25		
			{ 150	{ .725					
36		650	{ 150	{ .725	530	5.96	1.25		
			{ 150	{ .730					
	Mean Results ..	657	150	.755	511	5.83	1.264	230000	1341
37	New England Fir, 7 ft. by 2 in. sq.	560	{ 150	{ .862	446	4.50	1.36		
			{ 150	{ .862					
38		560	{ 150	{ .970	403	4.70	1.30		
			{ 150	{ .970					
39		540	{ 150	{ .960	411	4.78	1.33		
			{ 150	{ .960					
	Mean Results ..	553	150	.931	420	4.66	1.33	273900	1102

TABLE—(CONTINUED).

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect.		Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{l^3 W'}{32 a d^3 \delta}$	Value of S, from the formula $S = \frac{l W}{4 a d^2}$
			Weight in lbs.	Deflect ⁿ in inches.					
40	Riga Fir, 1st specimen, 7 ft. by 2 in. sq.	730	{ 125	{ .812	420	5.80	1.35	166100	1108
41		765	{ 125	{ .837	440	6.10	1.33		
42			{ 125	{ .912					
		763	{ 125	{ .937	406	6.10	..		
	Mean Results ..	753	125	.870	422	6.00	1.35		
43	Riga Fir, 2nd specimen, 6 ft. by 2 in. sq.	714	{ 150	{ .794	567	5.50	..	123800	1051
44		768	{ 150	{ .794	367	6.00	..		
45			{ 150	{ .907					
		732	{ 150	{ .909	467	6.50	..		
	Mean Results ..	738	150	.883	467	6.00	..		
46	Mar Forest Fir, 1st specimen, 7 ft. by 2 in. sq.	715	{ 125	{ 1.560	360	5.50	1.3	80670	1144
47		616	{ 125	{ 1.500	463	5.50	1.3		
48			{ 125	{ 1.370					
49			{ 125	{ 1.250					
		684	{ 125	{ 1.370	465	7.00	1.3		
	Mean Results ..	696	125	1.442	436	6.00	1.3		
50	Mar Forest, 2nd specimen, 6 ft. by 2 in. sq.	720	{ 150	{ 1.150	600	7.00	1.3	108700	1262
51		756	{ 150	{ 1.150	517	6.00	1.3		
52			{ 150	{ 1.250					
		603	{ 150	{ 0.675	567	6.25	..		
	Mean Results ..	693	150	1.006	561	6.42	1.3		
53	Mar Forest, 3rd specimen, 6 ft. by 2 in. sq.	700	{ 150	{ 1.150	561	6.5	1.3	108700	1262
54		710	{ 150	{ 1.150	570	6.5	1.3		
55			{ 150	{ 1.230					
		698	{ 150	{ 1.170	552	6.25	1.3		
	Mean Results ..	703	150	1.006	561	6.42	1.3		

TABLE—(CONTINUED).

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect.		Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{P W'}{32 a d^3 \delta}$	Value of S, from the formula $S = \frac{1}{4} \frac{W}{a d^2}$
			Weight in lbs.	Deflect ⁿ . in inches.					
56	Larch, 1st specimen, 7 ft. by 2 in. sq.	504	{ 125	{ 1.930	300	8.60	..		
			{ 125	{ 1.910					
57		576	{ 125	{ 1.740	340	8.60	..		
			{ 125	{ 1.760					
58		514	{ 125	{ 1.970	336	8.54	..		
			{ 125	{ 2.000					
	Mean Results ..	531	125	1.885	325	8.58	..	77040	853
59	Larch, 2nd specimen, 6 ft. by 2 in. sq.	552	{ 125	{ 0.750	412	6.00	..		
			{ 125	{ 0.750					
60		480	{ 125	{ 0.812	412	4.50	..		
			{ 125	{ 0.812					
61		534	{ 125	{ 0.875	398	4.50	..		
			{ 125	{					
	Mean Results ..	522	125	0.812	370	5.00	..	112200	832
62	Larch, 3rd specimen, 6 ft. by 2 in. sq.	546	{ 150	{ 0.750	417	4.70	1.25		
			{ 150	{ 0.750					
63		552	{ 150	{ 0.825	497	4.90	1.25		
			{ 150	{ 0.825					
64		552	{ 150	{ 0.750	537	5.00	1.20		
			{ 150	{ 0.750					
65		576	{ 150	{ 1.050	552	5.40	1.20		
			{ 150	{ .950					
	Mean Results ..	556	150	0.831	501	5.00	1.225	131600	1127
66	Larch, 4th specimen, 6 ft. by 2 in. sq.	552	{ 150	{ .831	500	4.8	1.2		
			{ 150	{ .831					
67		581	{ 150	{ .900	515	5.2	1.2		
			{ 150	{ .864					
68		548	{ 150	{ .762	515	5.0	1.2		
			{ 150	{ .798					
	Mean Results ..	560	150	.831	510	5.0	1.2	131600	1149
69	Norway Spar, 6 ft. by 2 in. sq.	600	{ 200	{ .800	667	4.0	1.35		
			{ 200	{ .800					
70		600	{ 200	{ .760	617	4.0	1.25		
			{ 200	{ .740					
71		580	{ 200	{ .840	680	4.0	1.30		
			{ 200	{ .860					
	Mean Results ..	577	200	.800	655	4.0	1.30	182200	1474

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

105. *Additional Experiments made in the Royal Arsenal, by P. W. Barlow, Civil Engineer, on the Strength and Elasticity of various Woods of English and Foreign growth.*

No. of experiments.	Names of woods.	Specific gravity.	Weight in lbs. which produced 1 in. defect.	Breaking weight in lbs.	Value of E, from the formula $E = \frac{l^3 w}{32 a d^3 b}$.	Value of S, from the formula $S = \frac{l w}{4 a d}$.	REMARKS.	
1	Acacia, English growth	710	..	1195	144000	1867	} Specimens supplied by W. Withers, Esq.	
2	—, ditto	710	bore	1084	rope broke, the piece	little injured.		
3	Oak, fast grown	903	660	999	161100	1501		
4	—, slow grown	856	414	677	101100	1058		
5	—, fast grown	972	550	999	133000	1501		
6	—, slow grown	835	425	943	107400	1473		
7	—, superior quality ...	748	896	1447	210700	2261	} Very fine specimen; been in store two years.	
8	—, ditto	756	680	1304	166000	2037		
9	Tonquin Bean .	middle	1036	1114	338900	2050	} This timber was sent from Berbice by Captain Gipps, R. E.	
10		outside	1080	1332	2228	225700		3481
11	Locust	middle	972	1052	2110	250200		3303
12		outside	934	940	2294	229500		3568
13	Bullet Tree ...	middle	1029	1360	1724	332000		2696
14		outside	1029	1332	1668	325200		2606
15	Greenheart ...	middle	1015	1332	1892	325200		2956
16		outside	986	1388	1612	338900		2512
17	Cabacally	middle	907	952	1074	232400		2606
18		outside	892	940	1556	229500		2431
19	African Oak ..	middle	972	1168	1447	285400	2261	} From a very fine timber long in store.
20		outside	972	1168	1657	285100	2589	
21		middle	1015	1400	1643	314400	2567	
22	American Black Birch	outside	972	1097	1643	287000	2567	} A long time in store, very dry; the same tree.
23		middle	648	775	1279	189200	1967	
24		outside	633	775	1110	189200	1410	
25	Common Birch	middle	648	644	1027	157200	1604	} Dry, and of the same plank.
26		outside	669	831	1433	200000	2239	
27		middle	792	800	1164	195300	1820	
28	Ash	outside	630	884	1304	215900	2037	} Dry, and of the same tree.
29		middle	727	660	1304	161100	2007	
30	Elm	outside	777	660	1304	161100	2037	} Dry, and of the same deal.
31		middle	554	324	772	106400	1031	
32	Christiana Deal	outside	532	856	1052	209000	1644	} Dry, and of the same deal.
33		middle	698	772	940	188400	1480	
34	Memel Deal ..	outside	680	786	1108	191900	1731	} Dry, and of the same deal.
35		middle	590	856	1108	209000	1731	

Note.—In these experiments, the bearing distance was 50 inches, and the bars two inches square.

Experiments on the Strength of Bent Timber.

106. In naval architecture it is always necessary to make use of a great quantity of bent timber; which, as far as can be done, is selected out of natural grown pieces, as nearly as possible of the required form, and is commonly known in the dock-yards by the term *compass timber*, which was formerly contracted for at a higher rate than that of straight growth: but both compass and straight timber is now, I believe, sent in at the same price. The great call for the former, however, during the war, rendered it very scarce, and much time and labour were employed in examining the stacks, in order to select pieces proper for each required purpose; and as the pieces, when they could be obtained, generally exceeded the requisite dimensions, much was necessarily cut away, and a great difference was always found between the *first* and the *converted* contents: the pieces were also, frequently, very much grain-cut, which necessarily diminished their strength very considerably.

These inconveniences, and particularly the great difficulty in obtaining compass timber, led Mr. Hookey, at that time master boat-builder in Woolwich Dockyard, but now assistant builder, to extend a method which he had long practised, of bending boat timbers, to the bending of the largest ship timbers; and, having obtained permission to have

a machine constructed for the purpose, it was found to answer every possible expectation that could be formed of it; the largest timbers, viz. pieces 18 inches square, being brought to any required curve in about fifteen minutes after being placed upon the machine: a description of which, in its original state, (but it has since received some improvements,) may be seen in vol. xxxii. of the 'Transactions of the Society of Arts.'

The method of preparing the timber is as follows: a fine saw-cut is made from one end, or both, according to the form into which the timber is to be bent; the length of it being also different, according to the length of the piece and the degree of curvature: but commonly, in a curve, the height of which is about $\frac{1}{8}$ th or $\frac{1}{6}$ th of the whole length, the saw-cut from each end is about $\frac{1}{3}$ rd of the length. The piece is then boiled for some hours, depending upon its lateral dimensions, and placed upon the machine, when the screws, &c., being applied, the required curvature is obtained, as above stated, in about twelve or fifteen minutes; after which it is screw-bolted, and is then ready for use. The reader, by referring to figs. 11 and 12, Plate III., will readily understand the above description; these figs. representing the fragments of two pieces bent for the following experiments. It is only necessary to observe, that the keys *k*, *k*, and *K*, are no part of the original plan, but were suggested during our experiments.

The advantages attending this method of bending timber for the purposes of ship building, are, 1st, That it dispenses with the use of compass timber, should it again become very scarce; and, therefore, no impediment would arise to the service, if the necessary quantity of timber of this kind could not be in any way procured. 2dly, It saves a deal of the time and labour necessary for unstacking and restacking piles of timber, to procure pieces of requisite compass; any piece of the proper length and squarage being at once available with the application of the machine. 3dly, It saves a great quantity of timber, which is necessarily cut to waste in bringing compass timber to its required dimensions; the conversion, in some cases, taking away a considerable part of the original contents; while, in bending timber, the original and converted contents are nearly the same. But, notwithstanding these recommendations in its favour, there appears to be a prejudice, well or ill founded, against the adoption of it, and some objections have been offered to the practice: the first of which is, that boiling the timber, and the strain impressed upon it, have a tendency to weaken the pieces, and, consequently, the ship into which such timbers are introduced: and, secondly, that the bolts are not sufficient to keep the two parts in a proper degree of contact, so as to prevent the introduction of damp and moisture. The latter point must be left to the decision of the practical builder; but with regard to the strength,

this may be otherwise determined, and I therefore solicited permission of the Navy Board to be allowed to make experiments on bent pieces of natural growth, grain-cut, and others, bent on the principle of Mr. Hookey, and the results of these experiments will be seen in the following Table: from which it will appear, that, taking the medium between the natural grown pieces and those which are partly so and partly grain-cut, no defect in point of strength will be found on the side of those bent upon the above plan. I also wished to try what effect boiling and steaming timber had upon the ultimate strength without bending; the account of which is given in my third Report. From which it appears, that, although there is an obvious falling off in the strength of those pieces boiled for a long time, the defect is very small while the boiling or steaming is not continued beyond the proportion of an hour to an inch in thickness, which is the usual practice in the dockyard.

(COPY OF A REPORT TRANSMITTED TO THE HONOURABLE
THE PRINCIPAL OFFICERS AND COMMISSIONERS OF HIS
MAJESTY'S NAVY; CONTAINING)

107. *Experiments on the Strength of Bent Oak Scantlings :
1st, Of Natural Growth ; 2dly, Grain-cut ; and 3dly, On
those bent according to the Plan of Mr. Hookey. The
latter with a Saw-cut, and without it. Also the former
of these with and without keys.*

Note.—The pieces were each 6 feet long and 2 inches square, but they were
broken on props 5 feet apart.

No. of experiments.	Nature of the pieces.	Arch up or down.	First curve in inches.	Specific gravity.	Breaking weight.	Last deflection below the props.	Strength computed from the formula $S = \frac{l W \sec^2 \Delta}{4 a d^2}$
1	Natural growth.	up	8	804	680	— 2	1312
2	Do.	up	8	820	764	— 8	1504
3	Do.	down	8	822	768	10	1600
4	Do.	down	8	874	762	13	1647
5	Grain-cut.	up	7½	980	505	— 3	1161
6	Do.	up	8½	830	568	— 2	1122
7	Do.	down	7½	938	546	10	1137
8	Do.	down	8½	840	550	10	1146
9	Bent whole.	up	7½	798	667	— 1	1314
10	Do.	down	7½	810	617	13	1353
11	{ Saw kerf, but no	up	8½	886	517	+ 2	—
12	{ keys.	down	8½	856	517	15	—
13	{ Saw kerf, with	up	8½	754	712	+ 2	1407
14	{ square keys.	down	8½	732	662	14	1470
15	{ Saw kerf, with	up	6	873	717	+ 5	1447
16	{ cylindrical keys.	down	6	873	762	12½	1657

Note 1.—The last deflection, having the sign *plus* + prefixed, indicates that the pieces arched so many inches the contrary way before breaking ; and those marked *minus* —, wanted the number of inches following, of coming down to the level of the props.

Note 2.—The pieces laid with the arch up were necessarily supported by the outside of the props ; these, therefore, must be considered as being broke at 5 feet 3 inches, which was the distance from the outside of one prop to that of the other ; and this is the case even where the pieces bent the contrary way ; for, notwithstanding the middle of the piece came below the props, the half-lengths were still sufficiently curved to throw the principal bearing on the outside.

In each of the figs. 11 and 12, Plate III., *ABCD* represents a fragment of the scantlings; *aa*, *bb*, *cc*, the screw bolts, and *mn* the saw-cut; which latter is 2 feet, or one-third the length of the piece. In fig. 12, *K* represents the form of the key, which was of oak, 1 inch long and $\frac{1}{2}$ an inch deep, let in $\frac{1}{4}$ of an inch into each part; and in fig. 11, *k* and *k* are copper bolts, of $\frac{1}{2}$ -inch diameter; which, therefore, also laid $\frac{1}{4}$ of an inch into each part; and in both figures the keys passed through the whole thickness of the scantling.

The idea of this mode of keying was suggested in our first experiments on pieces of this description; viz. Nos. 11 and 12, in which it was found that the screw bolts were not sufficient to prevent the part above and below *mn* from sliding upon one another. This defect may not have place when pieces of this kind are introduced into a ship, in consequence of the number of tree-nails with which the futtocks are pierced, which have necessarily a tendency to prevent that slipping of the parts noticed above. But, even in this case, I am convinced that considerable stiffness would be gained by keying the pieces after the manner of fig. 11, where it may be observed, that hard wood, as sound oak or *lignum vitæ*, would answer equally as well as copper bolts; and farther, that as the neutral axis in any section of fracture is generally at about $\frac{3}{8}$ ths of the depth, there would be no loss of strength in the piece, provided the key did not exceed $\frac{1}{4}$ th of the whole depth.

N.B. Mean strength $\left\{ \begin{array}{l} \text{Nos. 1, 2, 3, 4, of natural growth, 743} \\ \text{Nos. 13, 14, 15, 16, bent and keyed, 713} \end{array} \right\}$

Additional Experiments.

108. In order to form a comparison between the strength of a piece of timber bent upon Mr. Hookey's principle, and a straight piece in its natural state, two pieces were formed from the same scantling, having been only parted by the saw; the bent piece was brought to a curve of $9\frac{1}{2}$ inches, and keyed, as in fig. 11, Plate III.; the two pieces were then broken at the same distance, viz. 5 feet; their other dimensions being also the same as those above. The results of these experiments are as follow:

Straight piece, not } deflected $5\frac{1}{2}$ inches; broke with
boiled . . . } 667 lbs.

Bent to a curve of } deflected to $14\frac{1}{2}$ inches; broke
 $9\frac{1}{2}$ in., arch down } with 727 lbs.

By a comparison with all the above results, we obtain the following proportional breaking weights, viz.:

Natural growth	743 lbs.
Bent on Hookey's principle, and keyed . . .	730
Bent, without a saw-cut	632
Grain-cut	562
Bent on Hookey's plan, without keys . . .	517
Straight, and in natural state, deduced from the } results of the 2nd specimen of the first Report }	764

Note.—In comparing the first two of the above numbers with the last, it should be remembered, that although the former were broken with less weight, it does not indicate a less degree of strength; the same weight producing a greater strain upon a bent than upon a straight piece, proportional to the secant squared of the angle of deflection.

*To the Hon. the Principal Officers and Commissioners
of His Majesty's Navy.*

(COPY OF A REPORT TRANSMITTED TO THE HONOURABLE
THE PRINCIPAL OFFICERS AND COMMISSIONERS OF HIS
MAJESTY'S NAVY; CONTAINING)

109. *Experiments on the Strength of Oak Timber, in its natural state, compared with similar pieces boiled and steamed for different periods.*

Note.—The following pieces of oak were all cut from the same log, the mean specific gravity of which was 822.

No. of experiments.	Boiled or steamed.	No. of hours.	Length in feet.	Square in inches.	Deflection with 100 lbs.	Ultimate deflection.	Breaking weight, lbs.	Mean breaking weight, lbs.
1	Natural state	0	6	2	·425	6·0	722	} 669
2	Natural state	0	6	2	·500	6·5	617	
3	Steamed	5	6	2	·450	6·0	617	} 669
4	Steamed	5	6	2	·425	7·0	722	
5	Steamed	10	6	2	·430	6·0	662	} 614
6	Steamed	10	6	2	·475	5·0	567	
7	Boiled	2	6	2	·500	5·0	567	} 614
8	Boiled	2	6	2	·425	6·5	662	
9	Boiled	4	6	2	·462	7·5	662	} 614
10	Boiled	4	6	2	·525	4·0	567	
11	Boiled	6	6	2	·550	6·0	597	} 589
12	Boiled	6	6	2	·425	5·5	582	
13	Boiled	8	6	2	·475	5·5	647	} 639
14	Boiled	8	6	2	·500	7·0	632	
15	Boiled	10	6	2	·550	5·5	567	} 607
16	Boiled	10	6	2	·500	6·0	647	
Nos. 17 and 18, bent and keyed on Hookey's plan, part of the same log, and broke at the same length, viz. 6 feet; and the same squarage, viz. 2 inches.								
17	Boiled	3	1st curve 10 in.	Arch up.	Breaking wt. 632			
18	Boiled	3	1st curve 10 in.	Do. down	Breaking wt. 636			

*To the Hon. the Principal Officers and Commissioners
of His Majesty's Navy.*

There is not in the above experiments that degree of uniformity that we might have expected, considering the pieces were all cut from the same log. It should be observed, however, that the two experiments, 11 and 12, ought not to be considered as equally conclusive with the others, as they each broke at a knot about 6 inches from the centre of the beam.

Rejecting these, therefore, there appears, generally, to be a slight loss in strength from boiling and steaming; but it is not very perceptible while that process is not continued beyond the time usually allowed in the dockyards.

In several experiments which I made on pieces boiled only for two or three hours, there was no apparent defect in strength; some of them even exceeding, and others falling a little short of, similar unboiled pieces: but as they were not all from the same timber, they would not, probably, be thought conclusive if they were detailed; on which account they are omitted.

On Trussed Girders.

110. We shall now conclude this course of experiments with the four following, on girders, trussed and plain; the two former, viz., No. 1 and No. 3, were very accurately made, and constructed on a scale of 2 inches to the foot, from the drawing given by Nicholson (Plate XXXIX., '*Carpenter's New Guide*'); the former being supposed to denote a 34-feet, and the other a 25-feet girder.

On the Deflection and Strength of Girders, trussed and plain.

No. of experiments.	Distance between the props.	Depth of the girder.	Breadth of the girder.	Weight in lbs.	Deflection in inches.	REMARKS.
1	ft. in. 5 8	ft. in. 0 2	ft. in. 0 1½	<div> 100 200 300 400 450 500 </div>	<div> .35 .67 1.05 1.47 1.75 2.25 </div>	Truss in 3 pieces; length of centre piece 1 ft. 6 in. Distance of extreme butments 4 ft. 10 in. 2 king bolts, 2 plate bolts, and 5 screw bolts.
2	5 8	0 2	0 1½	<div> 100 200 300 400 450 500 </div>	<div> .30 .60 .90 1.20 1.35 1.55 </div>	Without a truss.
3	4 2	0 2	0 1½	<div> 100 200 300 600 700 743 803 903 953 </div>	<div> .15 .30 .57 .87 1.20 1.30 1.45 1.50 broke </div>	Truss in 2 pieces; distance of extreme butments 3 ft. 4 in. 1 king bolt, 2 plate bolts, and 4 screw bolts.
4	4 2	0 2	0 1½	<div> 100 200 300 400 500 600 717 </div>	<div> .15 .27 .41 .57 .77 1.00 broke </div>	Without a truss.

Nos. 1 and 2 were not broken in the experiment; but it appears that the trussed beam was the weakest; or at least it gave the greatest deflections. The wood of No. 1 was certainly inferior to the untrussed beam, but still it was to have been expected that the trusses would have been more than equivalent to the difference in the former respect; but as it was not, the experiment seems to indicate that there is but little efficacy in a truss of that description.

The trusses of No. 3 came fairly into action with each other, and certainly added very considerably to the resistance of the girder.

On the Resistance to Pressure.

111. Besides the two kinds of strains, *i. e.* the tensile and transverse strains, to which timber is exposed in building and machinery, there is another of considerable importance to which we have only at present very slightly referred, and this is the strain that pillars, columns, &c., have to sustain when supporting weights in a vertical position; and it must be admitted that it is one less supported both by theory and experience than any other branch of the general subject of strength and resistance. It has indeed been found experimentally, according to Mr. Tredgold, in his ‘Treatise on Carpentry,’ “that when a piece of timber is compressed in the direction of its length, it yields to the force in a different manner according to the proportion between its length and area of its cross section;” and that in case of a cylinder whose length is less than seven or eight times its diameter, it is impossible to bend it by a force applied longitudinally, as the piece is destroyed by splitting before the bending takes place; but when the length exceeds this, the pillar will bend under a certain load and be ultimately destroyed by a similar kind of action to that which has place in the transverse strain.

112. *Crushing force.*—A few experiments have been made on the resistance which different woods offer to a crushing force when their length is incon-

siderable, principally by M. Rondelet, in his '*Art de Bâtir*,' and by George Rennie, Esq., in the '*Philosophical Transactions*' for 1818; but unfortunately the results differ very widely from each other.

According to M. Rondelet, it required a force of from 5000 to 6000 lbs. to crush a piece of oak of 1 inch base, and from 6000 to 7000 lbs. to crush a similar section of fir; whereas Mr. Rennie gives the following specific numbers, which are so much less than the former in the two cases which admit of comparison, as to throw considerable doubt on the subject.

Mr. Rennie's results are as stated below :

Base 1 inch square.—1 inch in height.

Elm, crushed with	1284 lbs.
American pine	1606
White deal	1928
English oak	3860
Do., length 4 inches, same base	5147
African oak, base 3 in., side length 81 in.					60480
Or 6720 per square inch.					

This seems to prove that the resistance increases in a much higher ratio than the area, but without further experiments it is impossible to derive any general rule.

113. *Resistance of columns to flexure.*—This is the most important question connected with the inquiry, but it is like the former, one on which few experiments have been made, and in which little has been

derived from theory, although it has engaged the attention of some of the most distinguished mathematicians of the last and present century. Experiments on this subject are, as we have said, very few indeed; those given by M. Girard in his '*Traité Analytique de la Résistance des Solides*,' are the only ones of any importance to which we can refer, and the results in these are by no means so uniform as might be desired.

The following is an abstract from M. Girard's first and second Tables. Table I. contains the results of his experiments on the vertical pressure of oak beams. The first column contains the number of the experiment; the second, the dimensions and specific gravity¹⁴ of the several pieces; the third and fourth, the height from the bottom to the point of greatest curvature; the former in the direction of the least thickness, and the latter in that of the greatest. The fifth and sixth exhibit the measure of the greatest deflection; the former in the direction of the least, and the latter in that of its greatest dimension: the seventh column shows the several weights under which those deflections were observed; the eighth, the time from the commencement; and the ninth contains remarks, &c.

We have only shown the effect of four different weights for each beam; but the author himself has

¹⁴ M. Girard gives only the weight of the pieces; but we have preferred changing the weights into the specific gravities, as furnishing a readier means of comparing one piece with another.

in some cases employed ten, twelve, or more different pressures, measuring the deflection, &c., for each; but as it was thought unnecessary to follow him through the whole, the results of his first two and last two charges in the first eighteen experiments have been given. Those columns also, which M. Girard has drawn from computation founded on his theory, are omitted.

Table II., which is an abstract from the author's second Table, contains the results of his experiments on the transverse deflection of such of the beams as were not broken in the experiments above referred to: they were supported at each end at different lengths, and in different positions, viz. first with their greatest thickness vertical, and then with their less.

The formulæ M. Girard employs to compute the weight under which a piece of timber ought to begin to bend when pressed vertically, from the deflection being given when charged with any weight horizontally, are as follow:

Let P represent half the weight when the piece is charged horizontally in the middle, and b the corresponding deflection; f half the length of the piece, and π the semicircumference of a circle to diameter 1.

$$\text{Then } \frac{P f^3}{3 b} = \text{absolute elasticity,}$$

$$\text{And } Q = \frac{\pi^2 P f}{12 b} = \text{the weight,}$$

under which the same piece will begin to bend when the pressure is vertical.

If, therefore, for the same depth and thickness, $\frac{P f^3}{3 b}$ were constant, the weight Q , under which a piece begins to bend, would be inversely as the square of the length: but M. Girard finds $\frac{P f^3}{3 b}$ nearly as the square of the length, or as f^2 ; and consequently Q varies, *cæteris paribus*, as f inversely.

The formula given by Dr. Young, in his 'Natural Philosophy,' differs from this. See Prob. vii. Art. 115.

Note.—In the following Table, where two heights and two versed sines are connected by a { with one weight, it shows that the piece bent in two places, in opposite directions.

TABLE I.—(CONTINUED.)

No. of experiments.	Dimensions and specific gravity of the pieces.	Height of the greatest curvature from the foot.		Versed sine of greatest curvature.		Weight in kilo-grammes.	Time in hours.	REMARKS.
		In the direction of the thickness.	In the direction of the breadth.	In the direction of the thickness.	In the direction of the breadth.			
10	Length 2.2631	Metres. 1.4613	Metres. 1.1366	Metres. .0079	Metres. .0062	11999	10.00	Recovered its first form.
	Breadth 0.1262	1.4613	0.9742	.0079	.0062	17320	12.91	
	Thickness 0.1015	1.4613	0.9742	.0113	.0062	17320	22.91	
	Sp. gr. 1036	1.2989	0.9742	.0135	.0068	20326	25.83	
11	Length 1.9484	0.9742	0.9742	.0045	.0079	17321	7.08	Recovered its first form.
	Breadth 0.1556	0.9742	0.9742	.0045	.0090	22940	10.00	
	Thickness 0.1330	0.9742	0.9742	.0051	.0101	33105	26.66	
	Sp. gr. 1102	39644	27.50	
12	Length 1.9484	0.9742	0.9742	.0079	..	22940	20.00	Recovered its first form.
	Breadth 0.1579	0.9742	0.9742	.0079	..	33123	25.00	
	Thickness 0.1306	0.6495	0.9742	.0068	.0146	39637	27.91	
	Sp. gr. 935	1.6237	0.9742	.0023	
13	Length 1.9484	0.6495	0.6495	.0045	.0056	17321	2.08	Recovered its first form.
	Breadth 0.1579	0.6495	0.6495	.0062	.0068	22939	3.33	
	Thickness 0.1015	0.6495	0.6495	.0068	.0108	39456	33.33	
	Sp. gr. 9870023	
14	Length 1.9484	1.4613	0.9742	.0045	.0040	11973	10.00	Broke under the last weight.
	Breadth 0.1601	1.4613	0.9742	.0045	.0045	17274	27.50	
	Thickness 0.1015	1.6237	1.2989	.0113	.0090	28509	40.41	
	Sp. gr. 1035	32996	50.41	
15	Length 1.9484	1.2989	0.6495	.0056	.0045	17294	10.00	Recovered its first form.
	Breadth 0.1330	0.9742	0.6495	.0051	.0045	22899	28.33	
	Thickness 0.1060	1.6237	1.4342	.0068	.0118	46952	86.66	
	Sp. gr. 1032	0.3247	..	.0011	
16	Length 1.9484	0.9742	0.9742	.0045	.0056	11998	18.33	Broke under the last weight.
	Breadth 0.1285	0.9742	0.9742	.0056	.0079	17317	20.00	
	Thickness 0.1082	0.6495	0.6742	.0045	.0135	37273	92.50	
	Sp. gr. 993	0.2435	..	.0011	
17	Length 2.2731	0.6495	0.9742	.0029	.0028	11998	10.00	Recovered its first form.
	Breadth 0.1579	..	0.6495	..	.0034	17320	20.00	
	Thickness 0.1082	1.6237	0.9742	.0056	.0045	33120	52.50	
	Sp. gr. 920	1.4613	0.9742	.0113	.0051	39630	57.50	
18	Length 2.5979	0.9742	1.2989	.0051	.0034	11999	10.00	Broke under the last weight.
	Breadth 0.1579	0.9742	1.2989	.0068	.0056	17321	20.00	
	Thickness 0.1353	
	Sp. gr. 916	1.6237	1.2989	.0146	.0079	37305	50.83	

TABLE II.

Girard's Experiments on the Deflection of Oak Beams, when supported at the ends, and loaded in the middle of their length.

Note.—These Beams are the same as those in the preceding Table.

No. of experiments.	Dimensions in metres, and specific gravity of the pieces.	Deflection in metres.	Weight in kilogrammes.	Absolute elasticity, computed from $\frac{P f^3}{3 \delta}$.	REMARKS.
1	Length 2·5978	·0180	1884	38250	These two experiments were performed on the same piece of wood, which was also the same as No. 1. first Table.
	Depth ·1579	·0238	2379	36524	
	Breadth ·1285	·0238	2932	37876	
	Sp. gr. 1038	·0373	3467	33954	
2	Length 2·2731	·0158	1884	29174	
	Depth ·1579	·0215	2395	27266	
	Breadth ·1285	·0248	2930	28909	
	Sp. gr. 1038	·0300	3470	28304	
3	Length 1·9484	·0045	1882	64461	These were all the same piece of wood, viz. No. 7. of the first Table.
	Depth ·1579	·0056	2393	65864	
	Breadth ·1285	·0113	4007	54634	
	Sp. gr. 973	·0153	4542	45744	
4	Length 1·6237	·0056	1877	29893	
	Depth ·1579	·0068	2388	31312	
	Breadth ·1285	·0119	4976	37288	
	Sp. gr. 973	·0141	5512	34844	
5	Length 1·6237	·0056	1876	29877	This piece was No. 13. of Table I.
	Depth ·1285	·0079	2388	26953	
	Breadth ·1579	·0119	3463	34313	
	Sp. gr. 973	·0158	4000	29973	
6	Length 1·9484	·0090	1874	32101	
	Depth ·1579	·0135	2385	27228	
	Breadth ·1015	·0271	3786	22669	
	Sp. gr. 987	·0316	4519	22038	
7	Length 1·9484	·0135	1874	21395	This piece was No. 15. of Table I.
	Depth ·1015	·0226	2383	16313	
	Breadth ·1579	·0271	2919	16500	
	Sp. gr. 987	·0474	3448	11212	
8	Length 1·9484	·0158	1871	18257	
	Depth ·1330	·0180	2380	20381	
	Breadth ·1060	·0282	2917	15942	
	Sp. gr. 1032				
9	Length 1·9484	·0232	1870	12437	
	Depth ·1060	·0893	2916	13267	
	Breadth ·1330				
	Sp. gr. 1032				

* P = half the charge, f half the length, and δ the deflection.

Solution of Practical Problems.

115. Having in the foregoing pages established the requisite data and formulæ for determining the dimensions of beams under every variety of transverse strain, it is proposed to give a few examples by way of illustration, in which I shall confine myself to the wood given in the preceding Table of Data; these having been carefully selected, and the experiments made with this particular object. The numbers for direct cohesion are drawn from Art. 14.

PROBLEM I.

To determine the Strength of Direct Cohesion of a piece of Timber of any given Dimensions.

Rule.—Multiply the area of the transverse section, in inches, by the cohesion per square inch, Art. 14, and the product is the strength required.

In practice, the weight the timber has to support should not exceed one-fourth of the strength as calculated by the rule.

Example I.—What weight will be required to tear asunder a piece of teak, 3 inches square?

In this case the tabular value is . . . 15000

The area of the section $3 \times 3 =$. . . 9

The weight required . . . 135000 lbs.

Example II.—The diameter of a rod of ash being

2 inches, and its specific gravity 700, what weight will be required to tear it asunder?

The tabular value is 17000

The area of the section $2 \times 2 \times .7854 =$. 3.1416

The product 53407.2 lbs.

Note.—If the weight be given and the area of section required, it is only necessary to divide the given weight by the tabular value of cohesion.

PROBLEM II.

To find the Strength of a Rectangular Beam of Timber, fixed at one end and loaded at the other.

Rule.—Multiply the value of S in the Table of Data, by the area, and the depth of the section in inches, and divide that product by the leverage in inches, and the quotient will be the weight required in lbs.

Note 1.—In case the beam is inclined, the leverage is the distance IL, or F'L', fig. 6, Plate III. When the beam is horizontal, the leverage is usually called the length.

Note 2.—In practice, the load ought not to be greater than one-fourth of the weight found by the rule; for permanent stretching or displacement of the fibres begins to take place as soon as the load exceeds about one-fourth of the breaking weight. This will be perceived by comparing the weights which the specimens bore without loss of elasticity,

with the weights that broke them, in the Table of Data.

Note 3.—If the load be distributed in any manner over the length of the beam, the horizontal distance between the point of support and a vertical line drawn through the centre of gravity of the load, must be taken for the leverage.

Example I.—A beam projecting 5 feet over the point of support, is 6 inches deep and 4 inches in breadth of Riga fir, and is intended to support a load at its extremity; it is required to determine the greatest load it would bear, and the load it may be exposed to without injury.

For Riga fir, $S=1108$, and the area being $6 \times 4 = 24$, the depth 6 inches, the leverage 5 feet = 60 inches, we have $\frac{1108 \times 24 \times 6}{60} = 2659.2$ lbs.

the greatest or breaking load; and $\frac{2659.2}{4} = 664.8$ lbs. for the load it would bear without injury.

Example II.—A cistern to contain 36 cubic feet, or one ton of water, is to be supported by two cantilevers: the projection of the cistern from the face of the wall being 4 feet, it is required to determine the size for the cantilevers.

Let the cantilevers be of larch, such as the 3rd specimen, then we find by the Table of Data, $S=1127$, and the depth 5 inches. The load on them will be 1 ton = 2240 lbs., and the weight will be uniformly distributed over the length; therefore, the distances of the centre of gravity from the wall

will be half the length, or 2 feet=24 inches, which is the leverage. This is the reverse of the preceding operation, on account of the weight being given.

$$\frac{2240 \times 4 \times 24}{1127 \times 5} = 38.2 \text{ inches nearly, for the area of}$$

both cantilevers, or $\frac{38.2}{2} = 19.1$ inches for the area of one of them; and if the section be rectangular, the depth being 5 inches, the breadth will be 3.82 inches for each cantilever.

PROBLEM III.

To determine the Strength of a Rectangular Beam of Timber when it is supported at the ends, and is loaded in the middle of its length.

Rule.—Multiply the value of S, in the Table of Data, by four times the depth in inches, and by the area of the section in inches, and divide the product by the distance between the supports, in inches, and the quotient will be the greatest weight the beam will bear in lbs.

Note 1.—If the beam be not horizontal, the distance between the supports must be the horizontal distance.

Note 2.—One-fourth of the weight found by the rule should be the greatest weight upon a beam in practice.

Note 3.—If the load be applied at any other point than the middle, it will be as the rectangle of the

segments, into which the point divides the distance between the supports, is to the square of half that distance; so is the weight found by the rule, to the weight the beam will sustain at the given point.

Note 4.—If the load be distributed in any manner whatever over the beam, the centre of gravity of the load must be considered its place, and its stress equal to the whole weight; unless part of such weight be sustained by the supporting points independently of the resistance of the beam.

Example I.—Required the weight a beam of Riga fir, one foot square, would sustain in the middle, its length being 20 feet?

In this case the tabular value of S is 1108, and the depth 12 inches, and the area 144 inches, the length 240 inches; consequently,

$$\frac{1108 \times 4 \times 12 \times 144}{240} = 32010 \text{ lbs.}$$

And the beam may be loaded in practice with $\frac{32010}{4} = 8002\frac{1}{2}$ lbs., without injury to its texture.

If the load were applied at 8 feet distance from the end, instead of being applied in the middle, then it would be 12 feet from the other end; and by Note 3, we have $8 \times 12 : 10 \times 10 :: 8002\frac{1}{2} : 8336$ lbs. nearly, for the weight the beam 12 inches square would support at 8 feet from the end; showing the advantage of applying the load as far from the middle as possible.

Example II.—To determine the size of a girder of Riga fir for a warehouse, where the distance between

the points of support is 18 feet=216 inches, and the greatest probable stress at the middle, including the weight of the floor itself, 20 tons.

The tabular number is

$$S = 1108, \text{ and } 20 \text{ tons} = 44800 \text{ lbs.}$$

Let us further suppose that the greatest depth of the timber intended for the purpose is 20 inches. By reversing the rule, we have

$$\frac{4 \times 44800 \times 216}{1108 \times 4 \times 20 \times 20} = 21.83 \text{ inches}$$

for the breadth of the girder, which would be obtained by bolting together two pieces, each 20 inches by 11 inches; or much better by putting the two pieces at the most convenient distance apart, that would admit of both resting on the sustaining piece.

If there be only 20 tons distributed uniformly over the surface of the floor, then a girder of 20 inches by 11 inches would be sufficient.

PROBLEM IV.

To determine the Dimensions of a Beam, capable of supporting a given Weight with a given degree of Deflection, when fixed at one end.

Rule.—Divide the weight in lbs. by the reduced tabular value of E ,¹⁵ multiplied by the breadth and deflection, both in inches; then the cube root of the quotient multiplied by the length in feet, will be the depth required in inches.¹⁶

Example I.—A beam of Riga fir is intended to bear a load of 665 lbs. at its extremity, its length being 5 feet, its breadth 4 inches, and the deflection not to exceed $\frac{1}{4}$ of an inch.

In this case the tabular value of E is 96; hence,

$$\frac{665}{96 \times 4 \times \frac{1}{4}} = 6.88$$
; the cube root of which is 1.902;
 hence, $5 \times 1.902 = 9.51$ inches, the depth required.

By reference to Example I. of Prob. II. it will be found that a beam of 6 inches depth would be suf-

¹⁵ The value of E in these rules is the tabular value divided by 1728, which renders it unnecessary to reduce the length in feet into inches.

For English oak, $E = 105$

For Riga fir, $E = 96$

¹⁶ This rule is applicable to the imperfect fixing which obtains in practice; but the more perfect the mode of fixing is, the nearer the deflection will be to half that determined by the rule, and ordinary cases will usually be a mean between these results.

ficient to bear the load ; but when, from the nature of the construction, only a limited degree of flexure can be allowed, this mode of calculation becomes necessary.

Note 1.—When the weight is uniformly distributed over the length of the beam, the deflection will be only $\frac{2}{3}$ ths of the deflection from the same weight applied at the extremity, and in the rule consider the weight reduced in this proportion.

Note 2.—If the beam be a cylinder, the deflection will be 1·7 times the deflection of a square beam, other circumstances being the same.

Note 3.—In the above examples the reduction of results to the differences depending on the specific gravity is not shown, neither is it applicable in practice ; but for theoretical comparison it is important, and may always be performed by stating, as the specific gravity of the tabular specimen is to the load supported in any example, so is the actual specific gravity of the specimen to the load it would support under similar circumstances.

PROBLEM V.

To find the Dimensions of a Beam, capable of sustaining a given Weight with a given degree of Deflection, when supported at both ends.

Rule.—Multiply the weight to be supported in lbs. by the cube of the length in feet. Divide this product by 32 times the reduced tabular value of E, (see Note 1, Prob. IV.,) multiplied into the given deflection in inches, and the quotient is the breadth multiplied by the cube of the depth in inches.

Note 1.—If the beam be intended to be square, then the breadth is equal to the depth, and the fourth root of the quotient is the depth required.

Note 2.—If the beam be a cylinder, multiply the quotient by 1.7, and then the fourth root will be the diameter of the cylinder.

Note 3.—When the load producing the depression is greater than one-fourth of the greatest stress the beam would bear, it is too great to be trusted in construction; but in timber this limit is seldom exceeded, on account of its flexibility.

Note 4.—If the load be uniformly distributed over the length, the deflection will be $\frac{5}{8}$ ths of the deflection from the same load collected in the middle. And in the rule, employ $\frac{5}{8}$ ths of the weight of the load instead of the whole load.

Example I.—The length of the fir shaft of a water-wheel being 20 feet, and the stress upon it

7 tons, it is required to determine its diameter so that its deflection may not exceed $\cdot 2$ of an inch.

The reduced tabular value of $E = 96$, or more exactly $32 E = 3075$, and 7 tons $= 15680$ lbs.; hence (by the Rule and Note 2) $\frac{1.7 \times 15680 \times 20^3}{3075 \times .2} = 346730$ nearly. The fourth root of this sum is 24.3 inches, the diameter required.

Shafts which are to be cut for inserting arms, &c., will require to be larger, in a degree equivalent to the quantity destroyed by cutting.

The flexure of shafts ought not to exceed $\frac{1}{100}$ of an inch for each foot in length, this being considered the limit; and it will be always desirable to make shafts as short as possible, to avoid bending.

Example II.—The greatest variable load on a floor being 120 lbs. per superficial foot, it is required to determine the depth of a square girder to support it, the area of the floor sustained by the girder being 160 feet, the length of the girder 20 feet, and the deflection not to exceed half an inch.

The reduced value of E for Riga fir is 96 or $32 E = 3075$, and the weight is $120 \times 160 = 19200$ lbs. uniformly distributed; hence (by Note 4) we have

$$\frac{\frac{1}{2} \times 19200 \times 20^3}{3075 \times \frac{1}{2}} = 62440.$$

The fourth root of this number is 15.8 inches, the depth required.

The deflection of $\frac{1}{40}$ th of an inch for each foot in length is not injurious to ceilings; indeed, the

usual allowance for settlement is about twice that quantity. Ceilings have been found to settle about four times as much without causing cracks, and have been raised back again without injury.

The variable load on a floor seldom can exceed half the quantity of 120 lbs. on a superficial foot, unless it be in public rooms; hence, the number may be taken from 60 to 120, according to circumstances.

The same rule applies to joists of different kinds for floors; the area of the floor supported by the joists being multiplied by from 60 to 120 lbs. per superficial foot, according to the use the room is designed for.

Example III.—To determine the size of a rafter for a roof to support the covering of slate, the distance between the supports being 6 feet, and the weight of a superficial foot, including the stress of the wind, being 56 lbs., and the deflection not to exceed $\frac{1}{40}$ th of an inch for each foot in length.

The tabular value gives

$$32 E = 3075, \text{ the weight} = 56 \times 6 = 336 \text{ lbs. ;}$$

hence (by Note 4),

$$\frac{5 \times 336 \times 40 \times 6^3}{8 \times 3075 \times 6} = 98.34.$$

If the breadth be made $2\frac{1}{2}$ inches, then

$$\frac{98.34}{2.5} = 39.3;$$

and the cube root of 39.3 is 3.4 inches, the depth required.

PROBLEM VI.

To determine the Dimensions of a Pillar or Column to bear a given Stress in the direction of its Axis, without sensible Curvature.

Rule.—Multiply the weight to be supported in lbs. by the square of the length of the pillar in feet, and divide the product by 80 times the tabular value of E, (Art. 104,) reduced as in Prob. IV., the quotient will be equal to the breadth multiplied by the cube of the least thickness; therefore, either the breadth or thickness will require to be fixed upon, before the other can be found.¹⁷

Note 1.—If the pillar be square, its side will be the fourth root of the quotient.

¹⁷ The rule is derived as follows :—The force f , which a column will bear without sensible flexure is

$$f = .8225 \frac{d^2 m}{l^2}; \text{ and } m = \frac{l^3 W}{4 d^2 \delta}$$

(see Dr. Young's Nat. Phil. ii. pp. 47, 48); hence, when l is in feet, we have

$$f = \frac{78.96 l W}{\delta}. \quad \text{But we have } W = \frac{32 E a d^3 \delta}{l^3};$$

$$\text{consequently, } f = \frac{78.96 E a d}{32 l^2}.$$

In the rule the number 80 is used for 78.96. If the above expression be divided by 1.7, it becomes a rule for a cylinder,

$$\text{or } \frac{1.4508 E d^4}{32 l^2} = f, \text{ or } \frac{1.5 E d^4}{32 l^2} = f, \text{ for simplicity.}$$

Note 2.—If the column be a cylinder, multiply the tabular value of E by 48 instead of 80. The fourth root of the quotient in the rule will be the diameter of the cylinder.

Example I.—What should be the least thickness of a pillar of oak to bear a ton without sensible flexure, its breadth being 3 inches, and its length 5 feet ?

The reduced tabular value of E for oak is 105, and 1 ton = 2240 lbs.; hence

$$\frac{2240 \times 5^2}{80 \times 105 \times 3} = 2.222.$$

The cube root of 2.222 is 1.31 nearly, which is the side as required.

Example II.—Required the side of a square post of Riga fir to support 10 tons, the pressure being in the direction of the axis, and the height of the post 12 feet.

The reduced tabular value of E is 96; hence

$$\frac{22400 \times 12^2}{80 \times 96} = 419.6 \text{ nearly;}$$

the fourth root of which is 4.53 inches, the side of the post as required.

The dimensions given by this rule are obviously too small to be used in practice. The rule only shows the extreme load that can be supported by a pillar under the theoretical condition that the pressure exactly coincides with the axis of the pillar; but this pressure will overpower the resistance of the pillar if it has the smallest deviation from the axis.

(See Dr. Young's Nat. Phil. ii. p. 47.) It is the more necessary to point out this circumstance, because it is the same in Girard's Rules, quoted in p. 167; and Poisson's Equation ('*Traité de Mécanique*,' Art. 160, tome i.). For the case where the force is applied at a distance from the axis, Poisson has left the solution incomplete. Dr. Young has given a solution of this case in his work above quoted; but it is not quite so convenient for application as one which may be obtained by assuming certain data, that are difficult to obtain in a simple form by calculation.

In the former editions of this work, other problems and questions were given connected with this subject; but the data are so uncertain, that it has been thought better to omit them,—no rule being preferable to one which may be erroneous.

ON THE TRANSVERSE STRENGTH OF BRICK, STONE, CEMENT, ETC.

116. THERE are but few cases in which it is important to know the transverse strength of the above materials, and we have but scanty information on the subject. The following includes nearly all I have seen.

Cohesive Power of Stone.

The first experiments, I know of, relative to the cohesion of stone, are those of M. Gauthey, a German engineer; who found, from the results of several trials, that a piece of stone, of what he denominated soft *givry*, 1 foot square and 1 foot long, required a weight of 5000 lbs. to break it across, one end being fixed in a rock, and the weight hung on at the other; and that hard *givry* required, under similar circumstances, 5600 lbs. to produce fracture. Taking our dimensions, therefore, in feet, we have

$$\text{Soft givry, } \frac{lW}{ad^2} = 5000.$$

$$\text{Hard ditto, } \frac{lW}{ad^2} = 5600.$$

Or taking, as we have done in timber, the dimensions in inches,

Soft givry, $S = \frac{lW}{a d^2} = 35.$

Hard ditto, $S = \frac{lW}{a d^2} = 39.$

I am not acquainted with the nature of this stone ; but its power is very inferior to three specimens of stone tested by George Rennie, Esq., at the London Docks. These specimens, which I saw, were certainly very fine ; but the difference between the strength of them, and the above, is very extraordinary, particularly the Welsh slate.

Experiments made by Mr. G. Rennie, upon the following Stones, generally used for paving.

The dimensions were, length 12 inches ; breadth 2½ inches ; depth 1 inch.
The stones were laid flat on two bearings, 10 inches apart, and the weights suspended from the middle of the stones.

Kinds of stones.	Weight it bore.			Weight of stone.	Value of $S = \frac{lW}{4 b d^3}.$
	cwt.	qr.	lbs.	lbs. oz.	
Green Moor Yorkshire Blue Stone	2	3	27	2 12	335
Ditto Ditto White Do.	3	0	23	2 12	359
Caithness—Scotland	7	2	17	3 0	857
Valentia—Ireland	7	3	3	3 2	871
Welsh	17	0	12	3 2	1961

On the Cohesive Power of Brick.

117. In order to ascertain the cohesion of brick, three common bricks were procured, which had been exposed to the weather for two years at least; and three of the same kind, of recent make; and three of the best stock. These were supported between two props, 8 inches apart, and then loaded in the middle till they broke. The least thickness of the bricks was $2\frac{1}{2}$ inches, and the greatest 4 inches; and they were placed with their less dimension vertical. The following are the results of these experiments:

Common old brick.	Common new brick.	Best stock.
1. 384 lbs.	1. 411 lbs.	1. 434 lbs.
2. 298	2. 411	2. 479
3. 347	3. 387	3. 420
<hr/>	<hr/>	<hr/>
Mean 3(1029	3)1209	3)1333
<hr/>	<hr/>	<hr/>
343 lbs.	403 lbs.	444 lbs.

Hence, taking the dimensions in feet:

$$\text{Common old brick } \frac{l W}{4 a d^2} = 3939$$

$$\text{Do. of recent make } \frac{l W}{4 a d^2} = 4631$$

$$\text{Best stock } \frac{l W}{4 a d^2} = 5115.$$

Strength of different Cements.

118. I am indebted for the following experiments, on the strength of different cements, to M. J. Brunel, Esq., who made them in reference to the construction of the tunnel under the Thames.

Experiment 1.—Against a brick wall a brick was attached by cement, its broadest surface to the wall, and with its length vertical to this brick, another was added; to this a third; and so on till thirteen bricks were thus cemented to each other: to the thirteenth brick another was added endwise; and, lastly, a fifteenth brick to the end of this, in the same position as the first thirteen. The cement supported this length of column without any appearance of breaking. Two bricks were then laid on the farthest extremity; and, lastly, four others in front of these: in laying on the last brick the column or arm broke at the wall.

Experiment 2.—In this experiment twelve bricks were cemented to each other exactly as above; and then nine bricks more were laid on, viz. by placing one over each of the last seven; and, lastly, two at the farthest extremity. The arm was left in this state without breaking.

These experiments were made with Parker and White's cement, which was perfectly dry in both cases before the additional bricks were added.

Experiment 3.—Eleven bricks were attached in the same manner, and several weeks after, twenty-one bricks were piled upon the farthest extremity. Adding the last brick caused the arm to break off at the wall.

Experiment 4.—Eleven bricks were attached to the wall edgewise; in this state the arm supported four bricks, and then broke at the wall.

These two experiments were made with Messrs. Turner and Montague's cement.

Experiment 5.—A column was built 6 feet high and 14 inches square, and when dry was laid lengthwise on two props, 5 feet 6 inches asunder; in this position a weight of 896 lbs. was laid over the centre, which it supported without breaking. It continued to bear this a considerable time.

Experiment 6.—Exactly the same experiment was tried on a column, using half cement and half sand; this bore the same weight for half an hour, and then broke.

These experiments were made with Mr. Shepherd's cement.

It may be proper to add, that in every case of fracture the brick itself gave way before the cement.

CRUSHING FORCE.

119. *Experiments on the resisting Power of various Building Materials, Stone, Brick, &c., to a Crushing Force.*

No. of experiments.	MATERIALS.	Specific gravity.	Crushing weight.
1	Portland stone, 1 inch cube	1284
2	Ditto 2 inches long	805
3	Statuary marble, 1 inch	3216
4	Craigleith do. do.	8688
5	Chalk, cube of 1½ inch	1127
6	Brick, pale red, do.	2085	1265
7	Roe stone, Gloucestershire, do.	1449
8	Red brick do.	2168	1817
9	Ditto, Hammersmith paviors', do.	2254
10	Burnt do. do.	3243
11	Fire brick do.	3864
12	Derby grit do.	2316	7070
13	Ditto, another specimen, do.	2428	9776
14	Killaly white freestone, do.	2423	10264
15	Portland do.	2428	10284
16	Craigleith white freestone, do.	2452	12346
17	Yorkshire paving, with the strata, do.	2507	12856
18	Ditto do. against strata, do.	12856
19	White statuary marble do.	2760	13632
20	Bramley Fall sandstone do.	2506	13632
21	Ditto against strata, do.	13632
22	Cornish granite do.	2662	14302
23	Dundee sandstone do.	2530	14918
24	Portland, a 2-inch cube	2423	14918
25	Craigleith, with the strata, 1½ inch cube.	2452	15560
26	Devonshire red marble do.	16712
27	Compact limestone do.	2584	17354
28	Granite, Peterhead do.	18636
29	Black compact limestone do.	2598	19924
30	Purbeck do.	2599	20610
31	Black Brabant marble do.	2697	20742
32	Freestone, very hard do.	2528	21254
33	White Italian marble do.	2726	21783
34	Granite, Aberdeen, blue kind do.	2625	24556

See Experiment by G. Rennie, Esq., Phil. Trans. 1818.

120. *On the Force necessary to overturn Walls and Columns.*

1. A column of soft givry (assuming the specific gravity 2000) is erected on a base 2 feet square, and its height is 20 feet. Required the force, acting perpendicular to its end, necessary to overturn it.

It is obvious here that the force necessary to produce the fracture will consist of two parts, viz. 1st, that which is necessary to produce an equilibrium with the weight of the wall, independent of the cohesion; and, 2nd, of a part sufficient to overcome the cohesion, independent of the equilibrium. The latter will vary with the area of the base of fracture and the point of application of the force; and the former with the weight of the column and the situation of its centre of gravity.

Generally, if W denote the weight of the wall, l the distance of the point of application of a direct force from the fulcrum about which the wall is to turn, and r the distance of the centre of gravity from the same, both in feet; then by the property of the lever $F = \frac{Wr}{l}$, the force necessary to produce an equilibrium.

And from the theory of the strength of materials

$$\frac{F l}{a d^2} = C, \text{ a constant quantity,}$$

where a is the breadth, and d the depth of the section of fracture in feet; whence $F' = \frac{a d^2 C}{l}$, the

force requisite to produce the fracture: therefore,
 $F + F' = \frac{Wr}{l} + \frac{ad^2C}{l}$, the whole force required.

In the present case,

$W = 2000$ oz., or 125 lbs., and $125 \times 2^2 \times 20 = 10000$, $r = 1$, $l = 20$,
 $a = 2$, $d = 2$, and let $C = 500$;

whence,

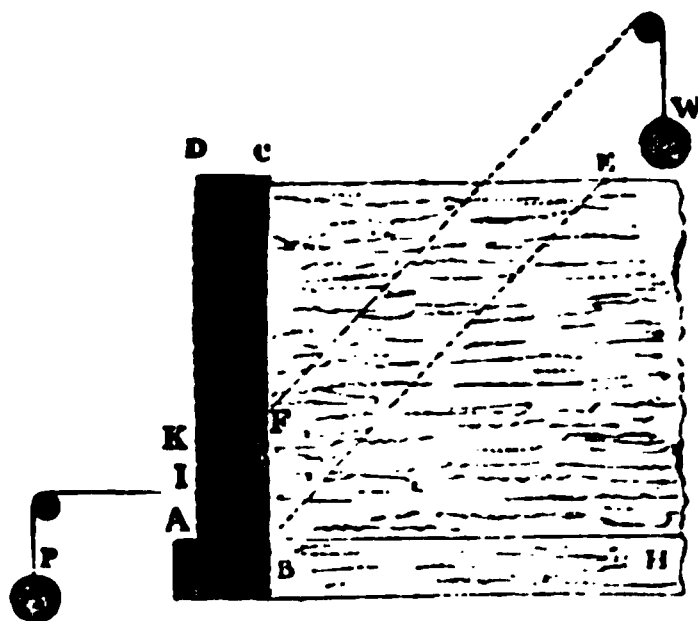
$$F + F' = \frac{10000}{20} + \frac{8 \times 500}{20} = 500 + 200 = 700 \text{ lbs.},$$

the force sought.

On the Pressure of Banks and the Dimensions of Revetments.

121. Having established above (at least approximately) certain data relative to the resistance and cohesion of walls and columns, it remains now to ascertain the pressure of earth against revetments, in order thence to determine their requisite dimensions, that an equilibrium may be established between those two forces.

For this purpose, let CBHE (in the annexed figure)



denote a bank of earth, the natural slope of which is

EB. Let the weight of the part CBE, one foot thick, $= W$, and make $BE = l$, $CB = h$, $CE = b$. From the theory of the inclined plane,

$$\text{as } l : h :: W : \frac{h}{l} W = W',$$

the weight which, attached to the centre of gravity of the sliding solid, would preserve it in equilibrio on the plane EB, supposing no friction between the two surfaces. The weight W' will, therefore, under this supposition, denote the quantity, FI the direction, and I the effective point of application of the force of the bank against the wall ABCD. And now, to find the horizontal force at I: since the triangles KFI and BEC are similar, we have by the resolution of forces

$$l : b :: W' : \frac{b W'}{l} = \frac{b h W}{l^2},$$

for the horizontal effect at I: also, since KA, from the nature of the centre of gravity $= \frac{1}{3}$ of DA, or $\frac{1}{3} h$;

$$\text{and } KI = \frac{h x}{b}, \text{ and } AI = \frac{1}{3} h - \frac{h x}{b},$$

(x being taken to denote the breadth of the wall at bottom,) the whole effect of the above pressure to turn the wall as a lever about a fulcrum at A, will be expressed by

$$\left(\frac{1}{3} h - \frac{h x}{b} \right) \frac{b h W}{l^2}, \text{ or } \left(\frac{1}{3} h - \frac{h x}{b} \right) \frac{b^2 h^2 s}{2 l^2},$$

s denoting the specific gravity of the earth.

Now, to find the dimensions of the revetment requisite to keep this force in equilibrio, let h' denote

the given height of the wall; S its specific gravity, or the weight of one cubic foot; x , as above, the thickness of the wall at the bottom; y the distance of the perpendicular, let fall from its centre of gravity upon its base, from the outward edge of the wall at bottom, viz. the point about which the wall turns; and a the area of its transverse vertical section; then, since we are only considering 1 foot in length, the same quantity, a , will also denote the solid content of the wall opposed to the bank; and, consequently, $a S$ will be its weight.

Therefore, by the preceding proposition,

$$F = y a S,$$

the resistance which the wall opposes in consequence of its weight, and

$$F' = C x^2,$$

the resistance from cohesion, C being a constant quantity, $\frac{1}{10}$ th of which we may take $= 500$, as in the preceding article; whence

$$y a S + C x^2$$

will be the whole resistance opposed to the bank; and, consequently, in case of an equilibrium, or of an equality between the force of pressure of the bank and the resistance of the wall, we shall have

$$y a S + C x^2 = \frac{b^2 h^2 s}{6 l^3} - \frac{b^2 h^2 s x}{2 b l^3};$$

a general formula, from which x , the breadth of the wall, in all cases may be determined.

If the wall be rectangular, then $y = \frac{1}{2}x$, and $a = h'x$, and the above becomes

$$\frac{1}{2}h'Sx^2 + Cx^2 = \frac{b^2h^2s}{6l^2} - \frac{b^2h^2sx}{2bl^2};$$

$$\text{or, } x^2 + \frac{bh^2sx}{Sh'l^2 + 2Cl^2} = \frac{b^2h^2s}{6Cl^2 + 3h'l^2S}.$$

If the wall be triangular, then $y = \frac{2}{3}x$, and $a = \frac{1}{2}h'x$, and the above becomes

$$\frac{1}{3}h'Sx^2 + Cx^2 = \frac{b^2h^2s}{6l^2} - \frac{b^2h^2sx}{2bl^2};$$

$$\text{or, } x^2 + \frac{bh^2sx}{\frac{1}{3}h'l^2S + 2Cl^2} = \frac{b^2h^2s}{2h'Sl^2 + 6Cl^2}.$$

Example I.—As an example, let the natural slope of a given soil, when unsupported, be 45° , and its specific gravity 2000, or the weight of a cubic foot, 125 lbs.; and let it be required to determine the breadth of a rectangular wall of soft givry necessary to support it: the wall and bank being both 12 feet high; and the specific gravity of the wall 2500, or 156 lbs. to the cubic foot.

Here $h' = 12$, $h = 12$, $b = 12$, $l = 12\sqrt{2}$, $S = 156$, $s = 125$, and $C = 500$.

Whence,

$$x^2 + 3.794x = 15.176;$$

$$\text{or, } x = -1.897 \pm \sqrt{(1.897^2 + 15.176)} = 2.435 \text{ feet.}$$

Example II.—Let all the data remain the same, to find the breadth at bottom of a triangular wall, that will keep the same bank in equilibrio.

Here, putting our second formula into numbers, we have

$$x^2 + 5.148x = 20.594; \text{ or,}$$

$$x = -2.574 \pm \sqrt{(2.574^2 + 20.594)} = 2.643 \text{ feet.}$$

This is but little different from the former, as ought obviously to be the case, because a great part of the resistance is due to the cohesion of the bottom section, that arising from the weight being comparatively small: it is singular, therefore, that the former datum has never (I believe) been introduced into the solution of the problem. Prony, who has attempted an elaborate solution of this proposition, has no reference to the wall's cohesion. It will be observed, also, that in the above investigation we have not considered the friction of the two surfaces: this is, of course, very considerable, and will reduce the thickness of the wall to a quantity less than the above. Experiments are, therefore, necessary to establish this point; in the mean time it may be observed, that as it is always desirable that the resistance of the wall should be more than equal to the pressure it has to sustain, it will be safer to omit it entirely than to introduce it without very correct data, drawn from the results of experiments carried on upon a large scale.

Example III.—Supposing the wall to be built of the best stock brick, which weighs 100lbs. to the cubic foot, and that a cubic foot of the earth weighs 96lbs.; also, that the bank is 12 feet high, and the natural slope of the soil is 30° : what must be the thickness of the rectangular wall that will just prevent the bank from slipping?

Example IV.—With the same data, required the thickness of the wall at bottom, supposing it in the

form of a triangular wedge, as in the second example above.

Example V.—To find the thickness of an upright rectangular wall necessary to support a body of water, the depth being 10 feet, and the wall 12 feet high, the specific gravity of water being 1000, and the best stock brick 2000.

Example VI.—Required the thickness of the wall at bottom, supposing the data the same as in the preceding example, but the wall to be in the form of a triangle, as in examples 2 and 4.

Note.—The pressure in the last two examples is to be estimated on the principles of the pressure of fluids.

122. *Remark.*—The above can only be considered as a very imperfect sketch of the theory of Revetments, at least as relates to its practical application, for want of the proper experimental data; being merely given, in connexion with our general theory of the strength of materials, for the sake of introducing considerations relative to the cohesion of walls, &c., which have been commonly omitted: and the consequence has been, that, according to all theories, (and there have been several,) the computed thickness of the wall has very far exceeded what was ever considered to be practically necessary.

To render the theory complete, with respect to its practical application, it is necessary to institute a course of experiments upon a large scale; first, upon

the strength of common cement and mortar; and, secondly, upon the force with which different soils tend to slide down, when erected into the form of banks. A well-conducted set of experiments of this kind would blend into one what many writers have divided into several distinct data. Thus some authors have considered first, what they call the natural slope of different soils, by which they mean the slope that the surface will assume when thrown loosely in a heap; very different, as they suppose, from the slope that a bank will assume that has been supported, but of which that support has been removed or overthrown. This, therefore, leads to the consideration of the friction and cohesion of soils, and what is denominated the slope of maximum thrust: but however well this may answer the purpose of making a display of analytical transformations, I cannot think it is at all calculated to obtain any useful practical results. I should conceive that a set of experiments, made upon the absolute thrust of different soils, which would include or blend all these data in one general result, would be much more useful, as furnishing less causes of error, and rendering the dependent computations much more simple and intelligible to those who are commonly interested in such deductions.

We may further observe, that the method of resolving the force of the bank at the point I, instead of the point F, which former is obviously the effective point as regards the lever by which the wall

turns, shows, that while the continuation of the slope falls within the base of the wall, the soil which forms it will add to the stability of the revetment; which is conformable to the experiments of Major-General Pasley. See vol. iii. of that author's 'Course of Military Instruction.'

ON THE STRENGTH OF CAST IRON.

Direct Cohesion.

123. CAST iron is but seldom employed to act as a tie, or to resist by its direct cohesive power, for which purpose it is not considered well calculated; not perhaps because it has not sufficient strength, but because its strength is not certain, and that it accommodates itself less to any cross strain than malleable iron. A bar of malleable iron will admit of considerable torsion without any great diminution of its direct strength, but in cast iron this is not the case, and any twist brought on a bar with a direct strain is pretty sure to produce fracture long before the whole of its direct strength is called into action. For this reason but few experiments have been made, and these give results certainly far within the proper limits. It will be seen in the next section, on the transverse power of cast iron, that, even admitting this material to be insuperably hard, its direct cohesive power must be at least ten tons per inch, and as it is certainly not insuperably hard or incompressible, it must necessarily exceed 10 tons; whereas the only three experiments we have, give a

mean of 8·14 tons, or about 18,000 lbs. per square inch, viz. :—

Experiment 1.—By Captain Brown, on a bar
1½ inch square, which was broken with 11·35 tons, tons.
or per square inch 7·26

Experiment 2.—By George Rennie, Esq., on
a bar
¾ inch square, cast horizontally, which was broken
with 1193 lbs.,
or per square inch 8·52

Experiment 3.—By the same on another bar
¾ inch square, cast vertically 1218 lbs.,
or per square inch 8·66

3) 24·44

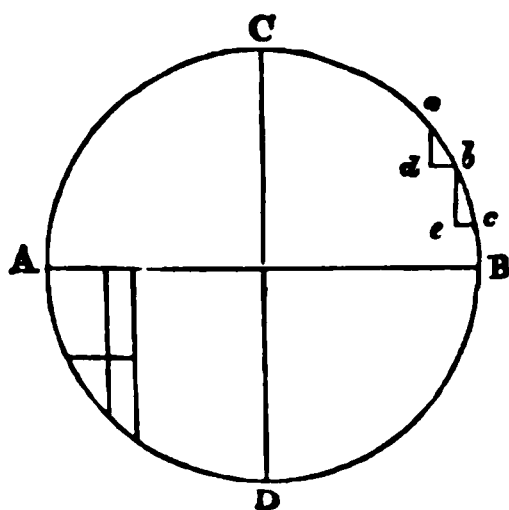
Mean . . . 8·14

On the Strength of Hydrostatic Presses.

124. It has been remarked that cast iron is seldom employed to resist a direct strain, but there are some cases in which this is unavoidable, and amongst others, in hydrostatic presses and water pipes; for the tendency of the internal pressure is here obviously to rend open the cylinder longitudinally, and its power of resistance is only the direct cohesion of the particles of metal in its longitudinal section. It would at first sight appear that the strength of a cylinder exposed to an internal pressure must be proportional to its thickness, but practically this is not

the case, it being found necessary to increase the thickness in a much higher proportion than in that of the strain. My attention was called to this apparent mystery some years back by Mr. Kier, who was engaged in the manufacture of hydrostatic presses, and it led me to the following investigation of the subject, which was presented to the Institution of Civil Engineers, and has been recently published in the first volume of their 'Transactions.'

Let ab , bc , be any small elementary part of the circumference, which may be taken as right lines, and let the pressure on each of them be called p , which, being proportional to them, may be represented by the elements themselves, ab , bc , these being perpendicular to the direction



in which the pressure acts. Resolve these pressures or forces into two rectangular forces, ad , db , and be , ec , of which, ad and be will represent forces acting perpendicular to their direction or parallel to AB , and db and ec forces parallel to DC . Confining ourselves at present to the former, if we conceive the semi-circumference DBC to be divided into its component elements, it is obvious that the sum of all the forces acting parallel to AB , will be equal to the sum of all the perpendiculars, ad , be , or to the whole diameter DC . That is, the sum of all the forces acting parallel to AB , will be to the sum of all the forces

or pressure on the semi-circumference DBC , as the diameter to the semi-circumference. But the pressure on the semi-circumference is equal to the number of inches in the same, multiplied by the pressure per square inch; consequently the force or pressure exerted parallel to AB , will be equal to the inches in the diameter, multiplied by the pressure per square inch, the ring being here supposed, for the purpose of simplification, only an inch deep. But to resist this pressure, we have the two thicknesses of the ring at D and C ; therefore the direct strains on the circumference at any one point, as D , will be equal to the pressure of the fluid per square inch, multiplied by the number of inches in the radius.

We should come to the same result more simply, but perhaps not so satisfactorily, by conceiving a section passing through the diameter DC ; then it follows that the pressure on this section, which is directly resisted at D and C , is equal to the number of square inches in the section, multiplied by the pressure per square inch. Therefore the strain on D or C singly, is equal to the pressure per square inch multiplied by the inches in the radius; the same as above.

Having thus found the strain at D and C , it would appear at first, as is stated above, only to be necessary to ascertain the thickness of metal necessary to resist this strain when applied directly to its length; this, however, is by no means the case, for if we imagine, as we must do, that the iron, in consequence

of the internal pressure, suffers a certain degree of extension, we shall find that the external circumference participates much less in this extension than the interior; and as the resistance is proportional to the extension divided by the length, according to the law *ut tensio sic vis*, it follows, that the external circumference, and every successive circular lamina, from the interior to the exterior surface, offers a less and less resistance to the interior strain: the law of which decrease of resistance it is our present object to investigate.

In the first place, it is obvious that whatever extension the cylinder or ring may undergo, there will be still in it the same quantity of metal; or, which is the same, the area of the circular ring, formed by a section through it, will remain the same, which area is proportional to the difference of the squares of the two diameters.

Let D be the interior diameter before the pressure is exerted, and $D + d$ its diameter when extended by the pressure. Let also D' be the external diameter before, and $D' + d'$ the diameter after the pressure is exerted; then from what is stated above it follows, that we shall have

$$\begin{aligned} D'^2 - D^2 &= (D' + d')^2 - (D + d)^2 \\ \text{or, } 2 D' d' + d'^2 &= 2 D d + d^2 \\ \text{or, } 2 D' + d' : 2 D + d &:: d : d'; \end{aligned}$$

or, since d' and d are very small in comparison with D' and D , this analogy becomes $D' : D :: d : d'$. That is, the extension of the exterior surface is to

that of the interior as the interior diameter to the exterior.

But the resistance is as the extension divided by the length; therefore the resistance of the exterior surface is to that of the interior as $\frac{D}{D'} : \frac{D'}{D}$ or as $D^2 : D'^2$. That is, the resistance offered by each successive lamina, is inversely as the square of the diameter, or inversely as the square of its distance from the centre; by means of which law the actual resistance due to any thickness is readily ascertained.

Let r be the interior radius of any cylinder, p the pressure per square inch on the fluid, t the whole thickness of the metal, and x any variable distance from the interior surface. Let also $rp = s$ represent the strain exerted at the interior surface, according to the principles explained in the preceding part of this paper. Then by the law last illustrated we shall have

$$(r+x)^2 : r^2 :: s : \frac{r^2 s}{(r+x)^2}$$

for the strain at the distance x from the interior surface; and consequently $\int \frac{r^2 s dx}{(r+x)^2} + \text{Cor.} =$ the sum of all the strains, or the sum of all the resistance. This becomes, when

$$x=t, R=r^2 s \left(\frac{1}{r} - \frac{1}{r+t} \right) = s \frac{rt}{r+t}.$$

That is, the sum of all the variable resistances due to the whole thickness t , is equal to the resistance

that would be due to the thickness $\frac{r t}{r+t}$ acting uniformly with a resistance s , or $r p$.

Application of this Rule for computing the proper Thickness of Metal in a Cyindric Hydraulic Press of given Power and Dimensions.

125. Let r be the radius of the proposed cylinder, p the pressure per square inch on the fluid, and x the required thickness: let also c represent the cohesive strength of a square inch rod of the metal.

Then from what has preceded it appears, that the whole strain due to the interior pressure will be expressed by $p r$, and that the greatest resistance to which the cylinder can be safely opposed is $c \times \frac{r x}{r+x}$: hence when the strain and resistance are in equilibrio, we shall have

$$r p = \frac{r x}{r+x} \times c \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{or } p r + p x = c x;$$

$$\text{whence } x = \frac{p r}{c-p} \text{ (the thickness) sought.}$$

Hence the following rule in words for computing the thickness of metal in all cases; viz., multiply the pressure per square inch by the radius of the cylinder, and divide the product by the difference between the cohesive strength of a square inch rod of the metal and the pressure per square inch, and the quotient will be the thickness required.

At present we have only considered the circumferential strain: to find the longitudinal strain, we have to multiply the area of the piston by the pressure per inch; while the resistance in this direction will be equal to the cohesive power of the metal multiplied by the area of the transverse section of the cylinder; so that when these are equal to each other, we shall have

$$3.1416 r^2 p = 3.1416 (2 r x + x^2) c,$$

which gives $x = r \left\{ \sqrt{\left(\frac{p}{c} + 1\right)} - 1 \right\}$. . . (2)

And it is obvious, that whichever of these two values of x , viz. (1) or (2), is the greatest, is the one which must be adopted. It will appear, however, that in all practical cases the former is the greater; for it is only when p exceeds c that the latter value of x can be ever equal to the former. Let us, for example, find the relative values of p and c , when these values of x are equal to each other, by making

$$\frac{r p}{c - p} = r \left\{ \sqrt{\left(\frac{p}{c} + 1\right)} - 1 \right\};$$

this gives $\frac{p^2}{(c - p)^2} + \frac{2 p}{c - p} = \frac{p}{c},$
or $p^2 c + 2 p c (c - p) = p (c - p)^2,$
or $p^2 - p c = c^2,$
whence $p = c \left(\frac{1}{2} \pm \frac{1}{2} \sqrt{5} \right).$

That is, these two values of x can only be equal to each other when p exceeds c in the ratio of $(\frac{1}{2} \pm \frac{1}{2} \sqrt{5}) : 1$; which is an impracticable pressure; for it is obvious from the first value of x , that no

thickness will be sufficient to resist an internal pressure which exceeds (per square inch) the cohesive power of a square inch rod of the metal; a result which at first sight appears to be paradoxical; but it will be observed that, with such a pressure, the interior surface will be fractured before the other parts of the metal are brought into action.

It will therefore be sufficient to attend wholly to the first expression; and here it may be observed, that x and r , with the same pressure and cohesive power, being always in the same ratio, we might reduce the rule for finding the thickness of metal to the following tabulated form, in which it would only be necessary to multiply the number standing against any pressure by the internal radius of the cylinder or piston for the thickness required: and this I have done in the Paper alluded to; but as it is certain, we have not at present a knowledge of the direct cohesive power of the metal, I have cancelled them in the present article. Mr. Tredgold, in his 'Treatise on Cast Iron,' by deductions from the transverse strength, makes the direct cohesive power about 20 tons; but this depends upon the assumption, that the neutral axis is in the centre of the area of fracture, for which there is no good reason. It will be seen, in a subsequent part of this Treatise, that in malleable iron, in rectangular bars, the neutral axis divides the area in the ratio of 1 to 4; and there is every reason to consider it to be much higher in cast iron. We can therefore, perhaps, not assume

it at more than 12 tons per inch ; and as not more than one-half of this ought to be practically depended upon, we cannot take c , in the preceding formula, at more than 6 tons. Supposing the press, therefore, to be intended to act with a power of 3 tons per inch, with a radius of 5 inches, we should have

$$x = \frac{3 \times 5}{6 - 3} = 5 \text{ inches for the thickness sought.}$$

In the same way, the thickness of a gun-metal cylinder, or leaden pipe, may be found, taking in each for the practical strength, one-third of the mean strength of the metal, as given in the following Tables.

On the Strength of Direct Cohesion of various Metals.

126. As but few applications of other metals than iron are called for under circumstances of great importance, the number of experiments upon them are very limited, and these are generally upon their direct strength. We shall content ourselves, therefore, with giving the following Table of Results, from experiments made in Woolwich Dockyard ; and others by George Rennie, Esq., Phil. Trans., 1818.

TABLES OF THE DIRECT COHESIVE POWERS OF VARIOUS METALS.

TABLE I.

The following Experiments were made by order of the Admiralty, with the Testing Machine in Woolwich Dock-yard, described in a subsequent Article. On King's Copper, Greenfell's Copper, and on the Patent Yellow Metal. By Mr. John Kingston.

KING'S COPPER.

Diameter of bolt.	Quantity stretched in four feet.		Breaking weight in tons.	Reduced to square inch.
	with tons	inch.		
1½	15 ..	·062	22 tons.	51189
1½	15 ..	·100	22	51189
1½	15 ..	·125	21½	50607
1	12 ..	·125	16½	48578
1	12 ..	·137	17	50050
1	12 ..	·125	17½	50786
0¾	9 ..	·125	12½	51286
0¾	9 ..	·125	12½	49135
0¾	9 ..	·085	13½	51062
0¾	6 ..	·125	9	47104
0¾	6 ..	·137	8½	45797
0¾	6 ..	·137	9	47104
			Mean ..	49499=22·1 tons.

GREENFELL'S COPPER.

1½	15 ..	·137	19½ tons.	45372
1½	15 ..	·125	19½	45372
1½	15 ..	·125	18	41881
1	12 ..	·125	15½	46369
1	12 ..	·150	15½	45633
1	12 ..	·150	14½	43425
0¾	9 ..	·100	13	50098
0¾	9 ..	·112	13	50098
0¾	9 ..	·087	13½	52989
0¾ ⅞	6 ..	·100	9½	47727
0¾ ⅞	6 ..	·125	9	44150
0¾ ⅞	6 ..	·125	8½	42832
			Mean ..	46329=20·7 tons.

TABLE—(CONTINUED).

PATENT YELLOW METAL.

Diameter of bolt.	Quantity stretched in four feet.		Breaking weight in tons.	Reduced to square inch.
	with tons	inch.		
$1\frac{1}{4}$	15 ..	·150	$23\frac{1}{4}$ tons.	51750
$1\frac{1}{2}$	15 ..	·230	23	50640
$1\frac{3}{4}$	15	19	41840
1	12 ..	·250	$16\frac{1}{2}$	50628
1	12 ..	·750	$18\frac{1}{2}$	55329
1	12 ..	·500	$20\frac{1}{2}$	59617
$0\frac{1}{2}$	9 ..	2·00	13	50098
$0\frac{1}{2}$	Defective		8	30830
$0\frac{1}{2}$	9 ..	2·00	$12\frac{1}{2}$	48172
$0\frac{3}{4}$	6 ..	1·70	$9\frac{1}{2}$	49720
$0\frac{3}{4}$	6 ..	3·00	■	41870
$0\frac{3}{4}$	6 ..	2·00	$9\frac{1}{2}$	49720
			Mean ..	49185 = 21·9 tons.

TABLE II.

Experiments on the Strength of Direct Cohesion of various Metals. By George Rennie, Esq. (from Phil. Trans. 1818).

No.	Metals.	Reduced to inch square.	
		lbs.	tons.
1	$\frac{1}{4}$ -inch cast iron bar, horizontal cast ... 1168	1193	8 51
2	Ditto vertical cast..... 1218		
3	Ditto, cast steel, previously tilted	8391	59·93
4	Ditto, blistered steel, reduced per hammer ..	8322	59·43
5	Ditto, sheer ditto, ditto	7977	56·97
6	Ditto, Swedish iron, ditto	4504	32·15
7	Ditto, English ditto, ditto	3492	24·93
8	Ditto, hard gun-metal	2273	16·23
9	Ditto, wrought copper, reduced per hammer	2112	15·08
10	Ditto, cast copper	1192	8·51
11	Ditto, fine yellow brass	1123	8·01
12	Ditto, cast tin	296	2·11
13	Ditto, cast lead	114	0·81

Experiments on the Resistance of $\frac{1}{4}$ -inch Iron Bars to a wrenching force.

127. The following experiments were made by George Rennie, Esq., and were published by him in the Phil. Trans., Part I., for 1818. The apparatus consisted of a wrought iron lever, 2 feet long, having an arched head of about 60° , and 4 feet diameter, of which the lever represented the radius: the centre round which it moved had a square hole, made to receive the end of the bar to be twisted. The lever was balanced, and a scale hung on the arched head; the other end of the bar being fixed in a square hole, in a piece of iron, and that again in a vice. The under-mentioned weights represent the quantity of weight put into the scale.

EXPERIMENTS

ON TWISTS CLOSE TO THE BEARING, CAST HORIZONTAL.

No.		lbs.	oz.
1.	$\frac{1}{4}$ -inch bars, twisted as under, with	10	14 in the scale.
2.	$\frac{1}{4}$ ditto, bad casting	8	4
3.	$\frac{1}{4}$ ditto	10	11
Average . . .		9	15

CAST VERTICAL.

4.	$\frac{1}{4}$	10	8
5.	$\frac{1}{4}$	10	13
6.	$\frac{1}{4}$	10	11
Average . . .		10	10

214 RESISTING POWERS OF VARIOUS METALS.

ON TWISTS OF DIFFERENT LENGTHS, HORIZONTAL CAST.

No.		Weight in scale.	
		lbs.	oz.
7.	$\frac{1}{4}$ by $\frac{1}{2}$ long	7	3
8.	$\frac{1}{4}$ by $\frac{3}{4}$ ditto	8	1
9.	$\frac{1}{4}$ by 1 inch ditto	8	8

VERTICAL.

10.	$\frac{1}{4}$ by $\frac{1}{2}$ ditto	10	1
11.	$\frac{1}{4}$ by $\frac{3}{4}$ ditto	8	9
12.	$\frac{1}{4}$ by 1 inch ditto	8	5

CAST HORIZONTAL, TWISTS AT 6 INCHES FROM THE BEARING.

13.	$\frac{1}{4}$ by 6 inches long	10	9
14.	$\frac{1}{4}$ by ditto ditto	9	4
15.	$\frac{1}{4}$ by ditto ditto	9	7

TWISTS OF $\frac{1}{2}$ -INCH SQUARE BARS, CAST HORIZONTALLY.

		qrs.	lbs.	oz.	
16.	$\frac{1}{2}$ close to the bearing .	3	9	12	end of the bar hard.
17.	$\frac{1}{2}$ ditto	2	18	0	middle of the bar.
18.	$\frac{1}{2}$ at 10 in. from bearing, } lever in the middle . . . }	1	24	0	

On Twists of different Materials.

128. These experiments were made close to the bearing, and the weights were accumulated in the scale until the substances were wrenched asunder :

No.		lbs.	oz.
19.	Cast steel	19	9
20.	Sheer steel	17	1.
21.	Blister steel	16	11
22.	English iron	10	2
23.	Swedish iron	9	8
24.	Hard gun-metal	5	0
25.	Fine yellow brass	4	11
26.	Copper	4	5
27.	Tin	1	7
28.	Lead	1	0

It will of course be understood, that these experiments give only the relative resistance to torsion, and not the actual resistance. On this subject the reader should consult Tredgold's 'Practical Essay on the Strength of Cast Iron.'

216 RESISTING POWERS OF VARIOUS METALS.

129. *Experiments by George Rennie, Esq., on Resistance of Cast Iron to a crushing force; from Phil. Trans. for 1818.*

Size of the prism.		Specific gravity.	Crushing weight.	Mean from each set.	REMARKS.
Side of base.	Height.				
inch.	inch.		lbs.	lbs.	
↓	↓	7033	1454	} 1440	{ These specimens were from one block.
Do.	Do.	Do.	1416		
Do.	Do.	Do.	1449		
↓	↓	6977	1922	} 2116	Iron from a block.
Do.	Do.	Do.	2310		
Do.	↓	Do.	2363	} 1758	{ These specimens were from the same block.
Do.	↓	Do.	2005		
Do.	↓	Do.	1407		
Do.	↓	Do.	1743		
Do.	↓	Do.	1594		
Do.	↓	Do.	1439		
↓	↓	6977	10561	} 9773	{ These specimens were from the same block as above.
Do.	Do.	Do.	9596		
Do.	Do.	Do.	9917		
Do.	Do.	Do.	9020		
↓	↓	7113	10432	} 10114	{ These specimens were from horizontal castings.
Do.	Do.	Do.	10720		
Do.	Do.	Do.	10605		
Do.	Do.	Do.	8699		
↓	↓	7074	12665	} 11136	These specimens were vertical castings.
Do.	Do.	Do.	10950		
Do.	Do.	Do.	11088		
Do.	Do.	Do.	9814		
Do.	Do.	Do.	11006		
↓	↓	} 7113	9455	} 9414	Horizontal casting.
Do.	Do.		9374		
↓	↓	} 7074	9938	} 9982	Vertical casting.
Do.	Do.		10027		
↓	↓	7113	9006		{ Horizontal castings.
Do.	↓	Do.	8845		
Do.	↓	Do.	8362		
Do.	↓	Do.	6430		
Do.	↓	Do.	6321		
↓	↓	7074	9328		{ Vertical castings.
Do.	↓	Do.	8385		
Do.	↓	Do.	7896		
Do.	↓	Do.	7018		
Do.	↓	Do.	6430		

130. *Similar Experiments on different Metals.*

Size of the prism.		Specific gravity.	Crushing weight.	Mean from each set.	REMARKS.
Side of base.	Height.				
inch.	inch.		lbs.		
$\frac{1}{4}$	$\frac{1}{4}$	Cast copper.	7318	{ Crumbled by the pressure.
Do.	Do.	Brass.	10304	{ Fine yellow brass reduced $\frac{1}{8}$ th with 3213 lbs.; $\frac{1}{4}$ with 10304 lbs.
Do.	Do.	{ Wrought copper. }	6440	{ Reduced $\frac{1}{8}$ th with 3427 lbs.; $\frac{1}{4}$ with 6440 lbs.
Do.	Do.	Cast tin.	966	{ Reduced $\frac{1}{8}$ th with 552 lbs.; $\frac{1}{4}$ with 966 lbs.
Do.	Do.	Cast lead.	483	Reduced $\frac{1}{4}$ with 483 lbs.

In these experiments, after the metals had been compressed to a certain extent, the resistance is stated to have been enormous.

On the Transverse Strength of Cast Iron.

131. The form in which cast iron is most frequently employed is to resist a transverse strain, as in rafters, girders, &c., &c., and numerous experiments have been made to determine the requisite data for computing the proper dimensions in these cases. Amongst the earliest experiments of this kind were those of Mr. Banks, in his ‘Treatise on the Power of Machines.’ These were made by resting the ends of square inch bars on supports at

3 feet distance, and then loading them with weights at their centre till fracture took place: the results were as below :

No. of experiment.	Distance of supports.	Depth.	Breadth.	Breaking weight.	Mean.
1	36 inches	1	1	756	756
2	36	.. 1	1	756	
3	30	.. 1	1	1008	840
				reduced to 36 inches.	
4	36	.. 1	1	963	972
5	36	.. 1	1	958	
6	36	.. 1	1	994	
7	36	.. 1	1		
average of three other experiments . .					730
8	36	.. 1	1	864	869
9	36	.. 1	1	874	
10	36	.. 1	1		897
				by Mr. George Rennie	
					6)5064
					Mean. 844

Now we have seen (Arts. 28 and 45), that in cases of transverse strains, the supports and load being applied as above,

$\frac{1}{3} d d' t = \frac{1}{4} l W,$

where d' is the depth of the area of tension and t the ultimate resistance per square inch; consequently,

$d' t = \frac{3}{4} l W = 22788;$

and as d' cannot exceed 1, t cannot be less than

22,788 lbs., or less than 10 tons, as has been stated (Art. 123). But there can be no doubt that cast iron is not, any more than other bodies, incompressible; d' , therefore, is less than 1, and the strength of direct cohesion more than 10 tons; unfortunately, however, the exact amount is not known. Mr. Tredgold makes it 20 tons, but this is quite inconsistent with some subsequent and well-conducted experiments by Mr. Hodgkinson, of Manchester.¹ The latter gentleman thinks it probable that in the commencement of the strain the neutral axis may divide the area in equal parts, as Mr. Tredgold assumes, but that it changes its position as the strain increases, as I have found to be the case in malleable iron in which the axis descends, as greater weights are applied.

It has been observed (Art. 40) that the position of the neutral axis is not of much importance in the case of timber, bars or beams of this material being generally rectangular where strength is required; and the strength of one being known, that of others may be computed without reference to this datum: but it is very different in cast iron, because in this, bars may be cast of various forms, and the strength of these cannot be computed without knowing the position of the axis in question. To compensate for this want of information, however, Mr. Hodgkinson has supplied us with numerous results on bars of

¹ See 'Manchester Memoirs,' vol. v.

different forms, which will be given in the sequel. From the preceding mean result we obtain for our value of S in cast iron rectangular bars,

$$S = \frac{lW}{4ad^2} = 7596, \text{ or } 7600 \text{ nearly.}$$

Mr. Tredgold's Experiments.

132. In these the depth of the bar was $\cdot65$ of an inch, and the breadth 1.3 inch. They were securely fixed at one end, the load being applied at the other, the leverage being in each case 2 feet.

No. of experiment.	Kind of iron.	Length.	Breadth.	Depth.	Breaking weight.	Value of S , $S = \frac{lW}{4ad^2}$
1	Old Park	inches. 24	inch. 1.3	inch. $\cdot65$	lbs. 184	8040
2	Adelphi	24	1.3	$\cdot65$	173	7560
3	Alfreton	24	1.3	$\cdot65$	168	7341
4	Scrap iron	24	1.3	$\cdot65$	174	7638
					Mean	7645

These values of S agree very nearly with that obtained from the preceding mean.

We may, therefore, with confidence state the constant (S) for rectangular cast iron bars to be

$$S = 7620.$$

On the Deflection of Cast Iron when submitted to a Transverse Strain.

133. On this subject Mr. Tredgold ² has furnished us with the four following results; the bars were as those given above, two of each kind having been cast for the purpose of the experiment.

EXPERIMENT 1.

OLD PARK IRON.

Two specimens run from this kind of pig iron, each 3 feet in length; smooth, clean, and regular castings. The section of the bars rectangular, depth 0·65 inch, breadth 1·3 inch; the supports 2·9 feet or 35 inches apart, the load suspended in the middle.

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·1 inch.	Bent 0·1 inch.
120	0·2	0·203
162	0·265	0·275
182	0·305 small set.	0·31 { set barely perceptible.
190	0·32 set ·005	0·33 set ·005.

² Treatise on the Strength of Cast Iron.

The iron was slightly malleable in a cold state; yielded easily to the file. The fracture dark grey, with little metallic lustre, fine grained and compact.

We may consider 162 lbs. as the greatest load it would bear without impairing its elastic force, and 0·27 as the mean between the flexures produced by this weight, or $\delta=0\cdot27$.

$$\text{Whence } E = \frac{l^3 W}{32 a d^3 \delta} = 2251800.$$

EXPERIMENT 2.

ADELPHI IRON.

The specimens of this iron were clean, good castings, of the same dimensions as the above; that is, depth 0·65, breadth 1·3 inch, distance of supports 35 inches.

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·1 inch.	Bent 0·1 inch.
120	0·2	0·205
162	0·26 no set.	0·27 no set.
182	0·30 set ·0075	0·305 set ·005

Taking the mean deflection with 162 at ·265, we find

$$E = \frac{l^3 W}{32 a d^3 \delta} = 2294200.$$

EXPERIMENT 3.

ALFRETON IRON.

Same dimensions and distance of supports as in the preceding, viz.

$$d = \cdot 65, a = 1 \cdot 3, l = 35.$$

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·1 inch.	Bent 0·1 inch.
120	0·2	0·195
162	0·27 no set.	0·28 no set.
183	0·31 small set.	0·325 small set.

Taking ·275 as the mean deflection with 162 lbs., we find

$$E = \frac{l^3 W}{32 a d^3 \delta} = 2210800.$$

EXPERIMENT 4.

SCRAP IRON.

These bars were run from old iron; they were uneven on the surface. Dimensions as before.

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·9 inch.	Bent 0·09 inch.
120	0·18	0·18
162	0·25 no set.	0·255 no set.
180	0·28 no set.	0·285 do.
190	0·30 small set.	0·30 { set not certain.
210	0·34 set ·005.	0·34 set ·004.

On these experiments Mr. Tredgold observes, that these bars showed no signs of a permanent set with 180 lbs.; but to whatever cause this greater range of elastic power may be owing, it would certainly be unsafe to calculate upon it. The iron was very hard to the file, and very brittle fragments flying off when hammered on the edge, instead of indenting as the preceding specimens.

Taking ·2525 as the mean deflection with 162 lbs., we have

$$E = \frac{l^3 W}{32 a d^3 \delta} = 2407800.$$

Excluding this as an unusual specimen, we have as a mean from the other three experiments,

$$E = 2254000$$

for the mean elastic power of cast iron to the nearest fourth figure; the other places are supplied by ciphers for the sake of simplification, their real value being unimportant.

Comparison of the Strength, Stiffness, &c., of Cast Iron, with good English Oak.

134. By the Table of Data, (Art. 104,) it appears, that the value of S, for the best specimen of English oak, is 1672; and from the preceding experiment for cast iron, $S = 7645$, that is, strength of

oak : cast iron :: 1 : 4.5 nearly.

Stiffness oak : cast iron :: 1 : 13 nearly.

Sp. grav. oak : cast iron :: 1 : 8 nearly.

If we consider that 170 lbs. in these experiments is just within the elastic power, we find

$$S = \frac{lW}{4ad^3} = 2705,$$

which is a little more than one-third of the greatest value of S, viz. 7645. Cast iron may, therefore, be considered to have its elasticity destroyed with about one-third the weight that will produce fracture; it ought, therefore, not to be loaded in permanent constructions to more than this amount.

Of the Section of Greatest Strength.

135. If cast iron beams, rafters, &c., were employed generally of a rectangular form like timber, the above data would be all that would be required; but as this metal may be cast of any form at pleasure, it becomes an object of great importance to know which is the strongest form of section with a given load and under different kinds of strains. If the position of the neutral axis in cast iron were known, and if it were found to preserve a constant law under all circumstances, these sections might be computed; but at present this datum has not been determined, nor the direct strength of cohesion: we must be content, therefore, with the results of actual experiments on particular forms, of which a great variety have been made by Mr. Hodgkinson, of Manchester, and published by him in the 'Manchester Memoirs' (vol. v.), and from which the following abstract is made.

136. *Experiments on the Transverse Strength of Cast Iron of various Sections. By Eaton Hodgkinson, Esq.*

EXPERIMENT 1.

Beam with equal rib at top and bottom.

Distance between the supports 4 feet 6 inches,
depth of beam $5\frac{1}{8}$ inches.

Area of top rib $= 1.75 \times .42 = .735$ in.

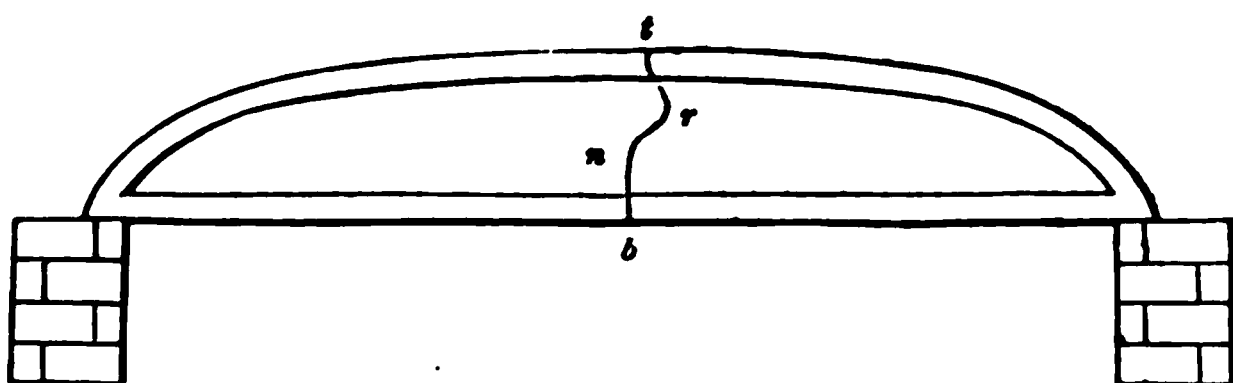
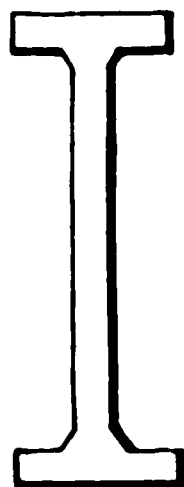
Do. bottom rib $= 1.77 \times .39 = .690$

Thickness of vertical part }
between the ribs . . . } $= .29$

Area of cross section . . . $= 2.82$

Weight of casting $36\frac{1}{4}$ lbs.

Breaking weight 6678 lbs.



The form of fracture is represented by the line $tbnr$, where $tr = .6$ inch and $bn = 2.5$ inches, the figure being a side view of the beam.

EXPERIMENT 2.

Beam with areas of section top and bottom rib as 1 : 2.

Distance between the supports 4 feet 6 inches,
depth of beam $5\frac{1}{8}$ inches.

Area of top rib $1.74 \times .26 = .45$ in.

Do. of bottom rib $1.78 \times .55 = .98$

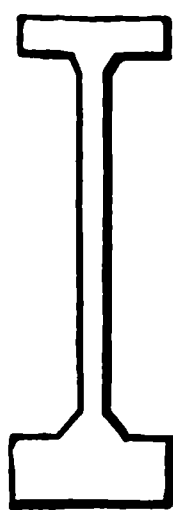
Thickness of vertical part . $= .30$

Area of cross section . . . $= 2.87$

Weight of casting 39 lbs.

Breaking weight 7368 lbs.

Form of fracture nearly as in experiment 1.



EXPERIMENT 3.

Beam with top and bottom rib as 1 : 4.

Distance between the supports 4 feet 6 inches,
depth of beam $5\frac{1}{8}$ inches.

Area of top rib $1\cdot07 \times \cdot30 = \cdot32$ in.

Do. of bottom rib $2\cdot1 \times \cdot57 = 1\cdot2$

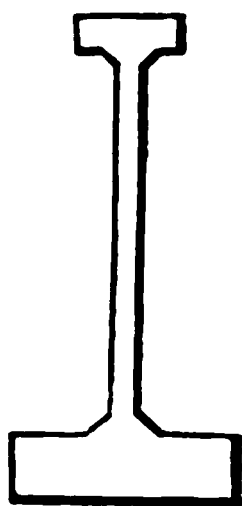
Thickness of the vertical part = $\cdot32$

Area of cross section . . . = $3\cdot02$

Weight of casting 40 lbs.

Breaking weight 8270 lbs.

Fracture as in experiment 1; $tr = \cdot6$.



EXPERIMENT 4.

Beam cast in common form; Messrs. Fairbairn and Lillie's model.

Distance between supports and depth of beam
as before.

Thickness at A = $\cdot32$

B = $\cdot44$

C = $\cdot45$

F E = $2\cdot27$

D E = $\cdot52$

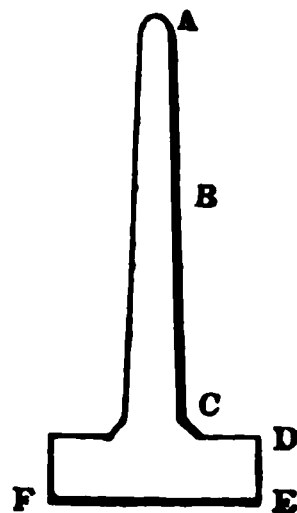
Area of section = $3\cdot2$ in.

Weight of casting = $40\frac{1}{2}$ lbs.

Deflection with 5758 lbs. $\cdot25$ in.

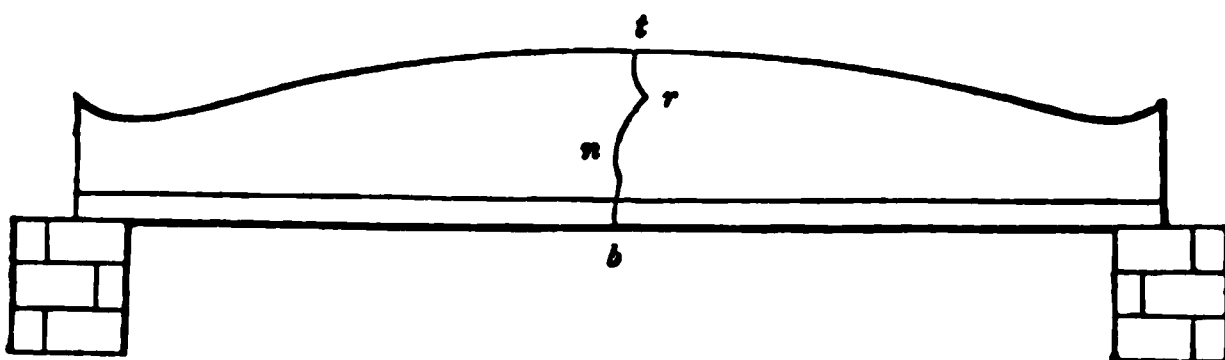
7138 $\cdot37$

Breaking weight = 8270 lbs.



The beam twisted a little before breaking; this, however, was not usually the case in the other beams in the same model.

Form of fracture as in figure; $tr = .75$.



All the preceding experiments were made on beams cast on their side from iron of which the following is a description :

$\frac{1}{3}$ of Blaina, No. 2, } Welsh.
 $\frac{1}{3}$ of Blaina, No. 3, }
 $\frac{1}{3}$ of W. I. S., No. 3, Shropshire.

This mixture is a strong iron, and therefore well suited for beams.

EXPERIMENT 7.³

This was on a beam from the same model as that in experiment 4 ; it was cast erect, but upside down, as usual, and therefore ought not to be compared with the preceding ones.

Distance between supports as before.

Thickness at A = .30

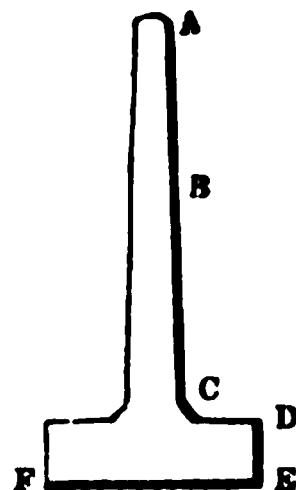
B = .37

C = .425

F E = 2.28

D E = .53

Area of the above section = 2.28 in.



³ Experiments 5 and 6 are omitted, being defective.

Weight of beam = 38 lbs.

Deflection with 6679 lbs. .37 inches.

9495 .50

9297 .62

Breaking weight = 9503 lbs.

It twisted in a serpentine manner before it broke. The form of fracture was nearly as in experiment 4; but here $tr = 1.0$, and $bn = 2.5$.

Remark.—In the future experiments, all the beams, except otherwise mentioned, were cast erect, but upside down, as there is an accession of strength from that cause. Those in experiments 8, 9, 11, 12, and 21, were elliptical, and were indeed from the model of the first three experiments, its top and bottom ribs being further changed.

EXPERIMENT 8.

Beam from the same model as that of experiment 3, the top rib in the casting being to the bottom as 1 to $3\frac{1}{2}$ nearly.

Distance between supports as before.

Area of top rib = $1.05 \times .32 = 0.34$ in.

of bottom rib = $2.15 \times .56 = 1.20$

Thickness of vertical part = .33

Area of cross section = 3.08 inches.

Weight of casting $39\frac{1}{2}$ lbs.

Breaking weight 8263 lbs. = 73 cwt. 89 lbs.

It broke very near to the middle.

The form of fracture was nearly as in the figure to experiment 1, but here $bn = 2.5$ and $tr = .55$.

EXPERIMENT 9.

In this the model of the above had 1 inch in breadth added to its bottom rib.

Ratio of the ribs 1 to $4\frac{1}{2}$ nearly.

Distance between supports as before.

Area of top rib $= 1.05 \times .34 = 0.357$

Do. of bottom rib $= 3.08 \times .51 = 1.570$

Thickness of vertical part $= .305$

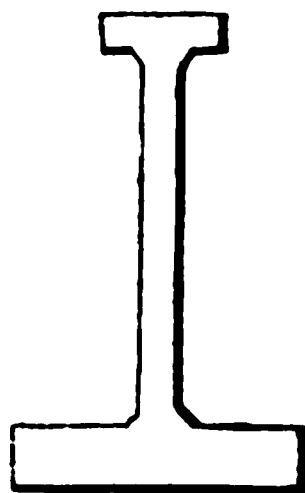
Area of section $= 3.37$ inches.

Weight of beam $= 44\frac{3}{4}$ lbs.

Breaking weight $= 10727$ lbs. $= 95$ cwt. 87 lbs.

It broke by tension 4 inches from the middle, but slanting towards it; and there seemed to be a small flaw in the bottom rib, at the place of fracture.

Here $t r = .6$ inch.



EXPERIMENT 10.

Common beam, cast upside down, in the usual manner. This, like the rest, was from the same model as that in experiment 4.

Distance between supports as before.

Thickness at A $= .29$

B $= .425$

C $= .46$

F E $= 2.3$

D E $= .53$

Area of section = 3.16 inches.

Weight of beam = $40\frac{1}{2}$ lbs.

Breaking weight = 8823 lbs.

It broke $1\frac{1}{2}$ inch from the middle. The form of fracture was nearly as in experiment 4.

Here $bn = 2.25$ and $tr = .8$.

EXPERIMENT 11.

Beam from model of experiment 9, only its top and bottom rib altered as above.

Ratio of rib 1 to 4 nearly.

Distance between supports and depth as before.

Area of top rib = $1.6 \times .315 = 0.5$ in.

Do. bottom rib = $4.16 \times .53 = 2.2$.

Thickness of vertical part = .38.

Area of section = 4.50 inches.

Weight of beam = 57 lbs.

Deflection with 11186 lbs = .40 in.

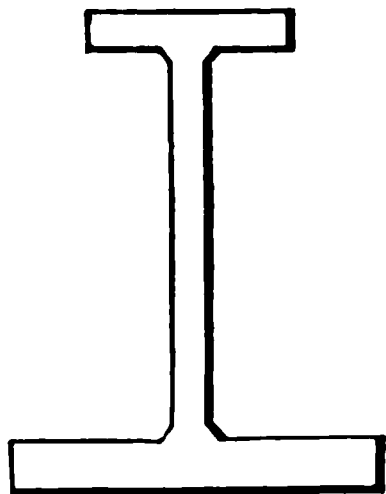
12698 .45

13706 .52

Breaking weight = 14462 lbs.

It broke by tension 1 inch from the middle.

$bn = 2.5$ inches.



EXPERIMENT 12.

The model of this beam differed from that of the last, in having a broader bottom flange.

Ratio of rib, 1 to $5\frac{1}{2}$ nearly.

Distance of support as before.

Area of top rib $= 1.56 \times .315 = 0.49$

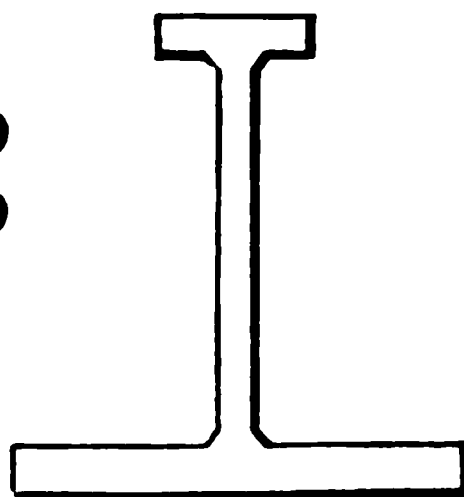
Do. bottom rib $= 5.17 \times .56 = 2.89$

Thickness of vertical part $= .34$ in.

Area of section $= 5$ inches.

Weight of beam $= 67\frac{1}{4}$ lbs.

Breaking weight 16730 lbs.



EXPERIMENT 13.

Distance between supports as before.

Thickness at A $= .29$

B $= .425$

C $= .53$

D E $= .565$

F E $= 2.34$

Area of section $= 3.32$ inches.

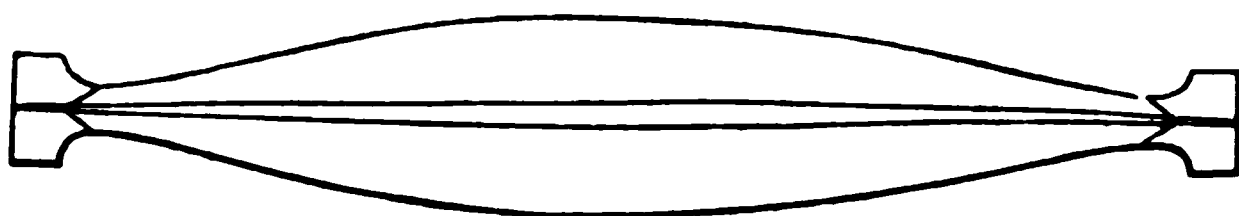
Weight of beam $= 41$ lbs.

It broke at $1\frac{1}{2}$ inch from the middle with 8942 lbs.

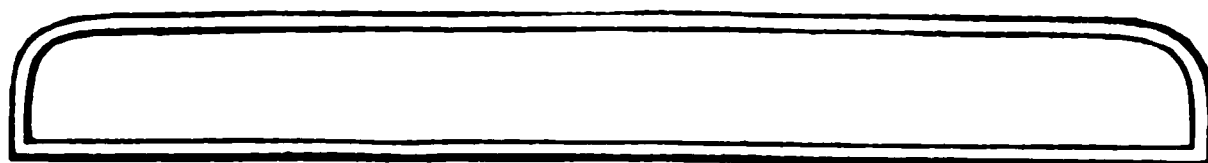
Form of Beam altered.

The beams in all the future experiments were of equal height through their whole length, and had their top and bottom ribs uniform in thickness, but tapering towards the ends, the bottom rib being parabolic. They are represented in the next page by the vertical plan and elevation, where the sections of their middle are as in the following experiments: and the sections, from their middle towards the ends, as in experiments 11, 9, 3.

PLAN.



ELEVATION.



This form was adopted to save metal, by reducing the bottom rib, which was likely to become very large.

EXPERIMENT 14.

Distance between supports 4 feet 6 inches, and depth of beam $5\frac{1}{8}$ inches, as before.

Area of top rib $= 2.3 \times .315 = .72$

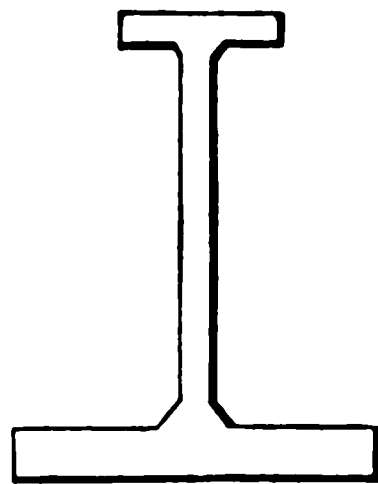
Do. bottom rib $= 4.06 \times .57 = 2.314$

Thickness of vertical part $= .33$

Area of section $= 4628$ inches.

Breaking weight $= 15024$ lbs.

It broke by tension very near to the middle.



EXPERIMENT 15.

In this experiment the breadth of the bottom rib only was increased as before.

Distance between supports and depth as before.

Area of top rib $= 2.35 \times .29 = .68$

Do. of bottom rib $= 5.43 \times .537 = 2.916$

Thickness of vertical part $= .35$

Area of section = 5.292 inches.
 Breaking weight 16905 lbs.
 It broke by tension.

EXPERIMENT 16.

Beam from the same model, but with further increased bottom rib.

Distance between supports and depth as before.
 Area of bottom rib = $6.8 \times .502 = 3.413$ inches.
 Breaking weight = 14336 lbs. nearly.

EXPERIMENT 17.

Beam of the *common form*, from the same model as the preceding ones, (see fig. to experiment 4.)

Distance between supports as before.

Weight of casting $39\frac{1}{2}$ lbs.

Weight.	Deflection.
6218	.28 inches.
7138	.33

Breaking weight = 7598 lbs.

EXPERIMENT 18.

Beam from the same model as that in experiment 16.

Distance of supports as before.

Top rib = $2.3 \times .28 = .64$

Bottom rib = $6.61 \times .54 = 3.57$

Thickness of vertical part = $.34$

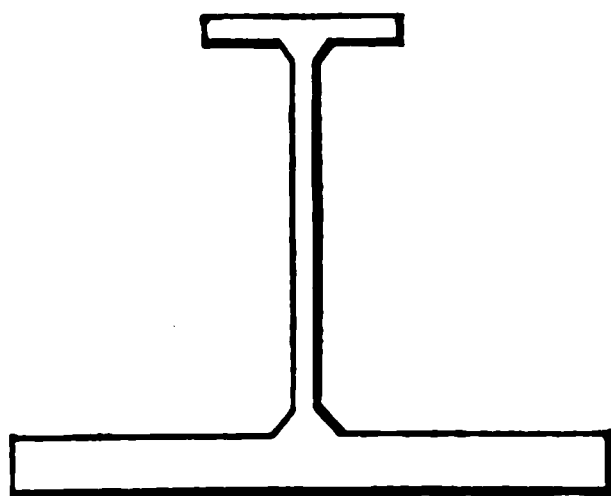
Area of section = 5.86 inches.

Weight of casting $68\frac{1}{2}$ lbs.

Breaking weight 19441 lbs.

This beam broke very nearly in the middle by tension, as before.

EXPERIMENT 19.



Distance of supports 4 feet 6 inches, and depth of beam $5\frac{1}{8}$ inches, as before.

Area of top rib $= 2.33 \times .31 = .72$

of bottom rib $= 6.67 \times .66 = 4.4$

Thickness of vertical part $= .266$

Area of section $= 6.4$, or $6\frac{2}{5}$ inches.

Weight of beam $= 71$ lbs.

This beam broke in the middle by compression with 26084 lbs., a wedge separating from its upper side.

EXPERIMENT 20.

Beam from the same model as that in the last experiment.

Distance between supports as before.

Area of top rib $= 2.3 \times .28 = .64$

of bottom rib $= 6.63 \times .65 = 4.31$

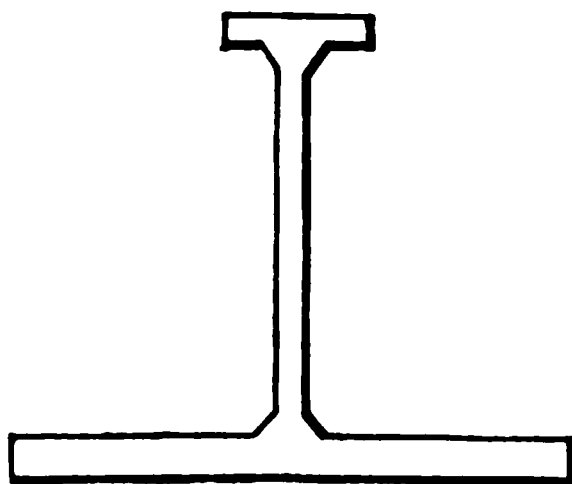
Thickness of vertical part = $\cdot 335$

Area of section $6\cdot 5$, or $6\frac{1}{2}$ inches.

Weight of beam = $74\frac{3}{4}$ lbs.

It broke in the middle of the beam by tension, with 23249 lbs. nearly.

EXPERIMENT 21.



This was an *elliptical* beam from the same model as that in experiment 12, and those preceding it, the bottom rib being further increased, and being like as in them of equal breadth through the whole length of 5 feet.

Distance between supports as before.

Area of top rib = $1\cdot 54 \times \cdot 32 = \cdot 493$

of bottom rib = $6\cdot 50 \times \cdot 51 = 3\cdot 315$

Thickness of vertical part = $\cdot 34$

Ratio of ribs $6\frac{1}{2}$ to 1.

Area of section = $5\cdot 41$ inches.

It broke very near the middle by tension, with 21009 lbs. nearly.

EXPERIMENT 22.

This beam was of the common form, from the

same model as before, for comparison with the three preceding ones.

Distance between supports as before.

Thickness at A = $\cdot 30$

B = $\cdot 42$

C = $\cdot 45$

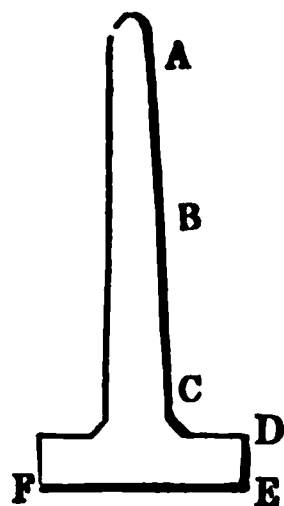
D E = $\cdot 51$

F E = $2\cdot 28$

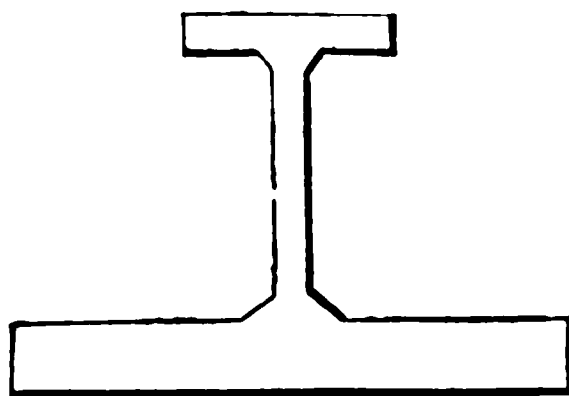
Area of section = $3\cdot 17$ inches.

Weight of beam = 40 lbs.

This beam bore 8965 lbs., and broke in the middle with considerably less than 9327 lbs.



EXPERIMENT 23.



Distance between supports 7 feet.

Depth of beam $4\cdot 1$ inches.

Area of top rib = $2\cdot 25 \times \cdot 33 = \cdot 74$

of bottom rib = $6\cdot 00 \times \cdot 74 = 4\cdot 44$

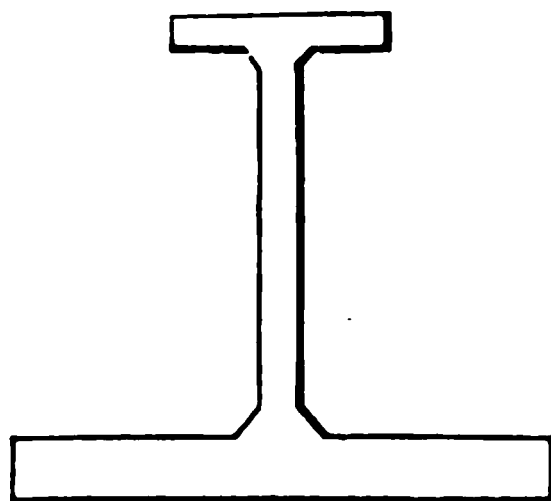
Thickness of vertical part = $\cdot 40$

Area of section = $6\cdot 54$

Weight of casting = 114 lbs.

Breaking weight, 6 tons, 103 lbs.

EXPERIMENT 24.



Distance between supports 7 feet.

Depth of beam 5·2 inches.

Area of top rib = $2\cdot25 \times \cdot35 = \cdot79$

of bottom rib = $6\cdot00 \times \cdot77 = 4\cdot62$

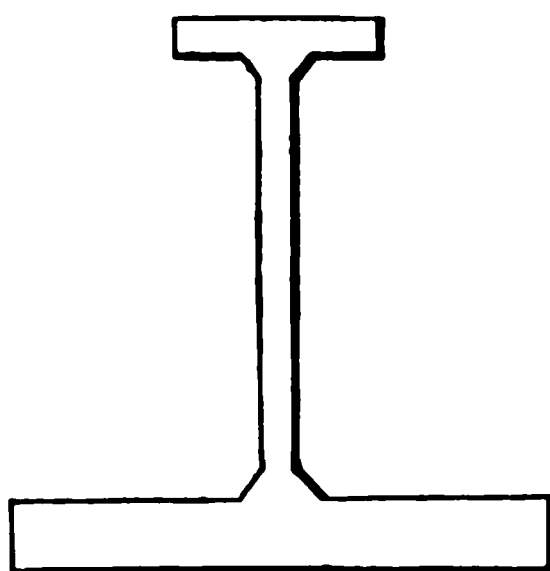
Thickness of vertical part = $\cdot34$

Area of section = 6·94 inches.

Weight of casting = 128 lbs.

Breaking weight, 6 tons, 15 cwt. 9 lbs.

EXPERIMENT 25.



Distance between supports 7 feet.

Depth of beam 6·0 inches.

Area of top rib = $2.2 \times .33 = .73$

of bottom rib = $5.95 \times .75 = 4.46$

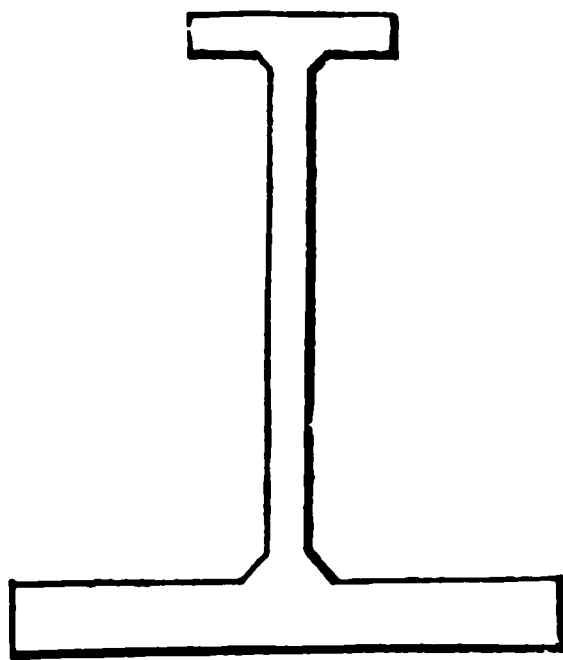
Thickness of vertical part = .355

Area of section = 7.08 inches.

Weight of casting = $127\frac{1}{2}$ lbs.

It broke by tension in the middle with this last weight, 15129 lbs., after standing a minute.

EXPERIMENT 26.



Distance between supports 7 feet.

Depth of beam 6.93 inches.

Area of top rib = $2.25 \times .34 = .765$

of bottom rib = $6.05 \times .75 = 4.537$

Thickness of vertical part = .38

Area of section = 7.67 inches.

Weight of casting = 146 lbs.

Breaking weight, 9 tons, 18 cwt.

EXPERIMENT 27.

Distance between supports 7 feet.

Depth of beam 6.98 inches.

Area of top rib $= 2.2 \times .36 = .79$
 of bottom rib $= 7.0 \times .69 = 4.83$

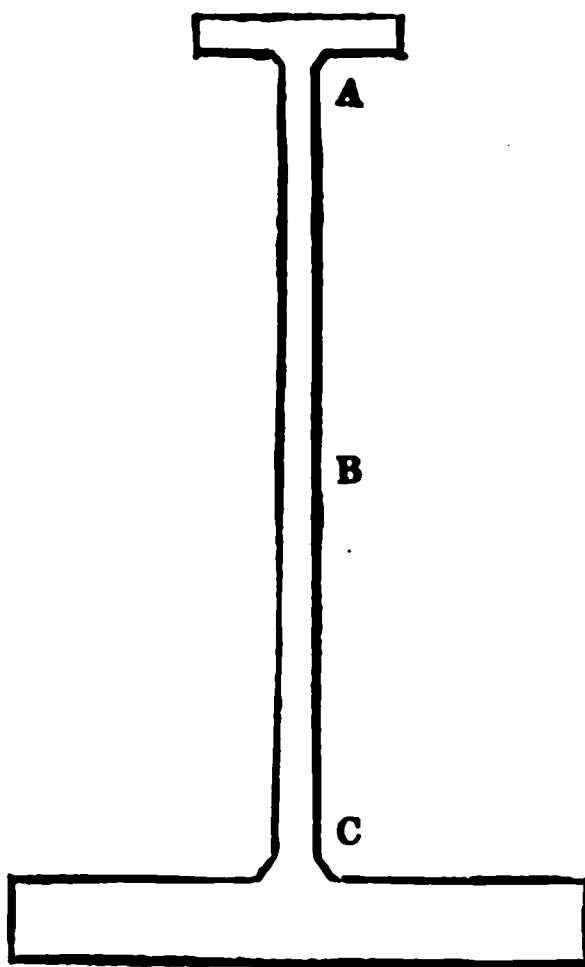
Thickness at A $= .27$

B $= .33$

C $= .60$

Breaking weight 11056 lbs.

EXPERIMENT 30.



Distance between supports 9 feet.

Depth of beam $10\frac{1}{4}$ inches.

Weight of beam 227 lbs.

Area of top rib $= 2.1 \times .27 = .57$

of bottom rib $= 6.14 \times .77 = 4.72$

Thickness at A $= .20$

B $= .25$

C $= .35$

Breaking weight 28672 lbs.

EXPERIMENT 31.

Distance between supports 4 feet 6 inches.

Depth of beam 5·1 inches.

Weight of beam 88 lbs.

Area of top rib = $2·15 \times ·24 = ·52$

of bottom rib = $7·60 \times ·72 = 5·472$

Thickness at A = ·27

B = ·44

C = ·48

Area of section = 7·90 inches.

Breaking weight 12 tons 11½ cwt.

EXPERIMENT 32.

Distance between supports 9 feet.

Depth of beam 5½ inches.

Weight of beam 192 lbs.

Area of top rib = $2·25 \times ·3 = ·67$ in.

of bottom rib = $7·7 \times ·76 = 5·85$

Thickness at A = ·36

B = ·42

C = ·60

Breaking weight 15196 lbs.

EXPERIMENT 33.

Distance between supports 9 feet.

Depth of beam 10¼ inches.

Weight of beam 244 lbs.

Area of top rib = $2·2 \times ·33 = ·73$ in.

of bottom rib = $7·6 \times ·75 = 5·70$

Thickness at A = $\cdot 15$

B = $\cdot 38$

C = $\cdot 35$

Breaking weight 32200 lbs.

EXPERIMENT 34.

Beam of common form, from the same model as before, and cast on its side for comparison.

Distance between supports 4 feet 6 inches.

Depth of beam in its middle $5\frac{1}{8}$ inches.

Weight of beam $36\frac{1}{2}$ lbs.

Thickness at A = $\cdot 27$

B = $\cdot 40$

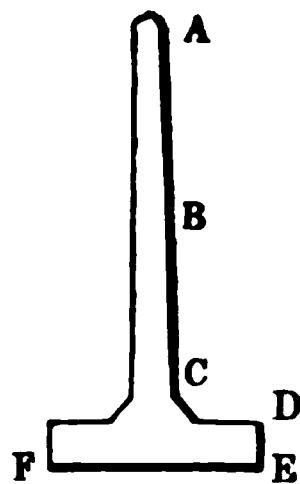
C = $\cdot 44$

F E = 2.27

D E = $\cdot 46$

Area of section = 2.921 inches.

Breaking weight 8792 lbs.



EXPERIMENT 35.

A beam of the *common form*, and from the same model, and iron, cast erect, as usual.

Distance between supports 4 feet 6 inches.

Depth of beam in its middle $5\frac{1}{8}$ inches.

Weight of beam 37 lbs.

Thickness at A = $\cdot 27$

B = $\cdot 355$

C = $\cdot 43$

F E = 2.26

D E = $\cdot 47$

Breaking weight 9044 lbs.

To the above, we may add the following experiments by George Rennie, Esq.

EXPERIMENTS.

137. *On the Transverse Strength of Cast Iron Bars of various Figures.*

No.	Description of bar.	Weight of bar.	Distance of bearings.	Breaking weight.
		lbs. oz.	ft. in.	lbs.
1	Bar of 1 inch square	10 6	3 0	897
2	{ Ditto of 1 inch ditto	9 8	2 8	1086
3	{ Half the above bar		1 4	2320
4	{ Bar of 1 inch square through the diagonal	2 8	2 8	851
5	{ Half the above bar .. :		1 4	1587
6	{ Bar of 2 in. deep, by $\frac{1}{2}$ inch thick ..	9 5	2 8	2185
7	{ Half the above bar		1 4	4508
8	{ Bar 3 inches deep, by $\frac{1}{2}$ inch thick	9 15	2 8	3588
9	{ Half the above bar		1 4	6854
10	Bar 4 inches, by $\frac{1}{2}$ inch thick	9 7	2 8	3979
	Equilateral triangles, with the angle up and down :			
11	{ Edge or angle up	9 11	2 8	1437
12	{ angle down	9 7	2 8	840
13	{ Half the first bar		1 4	3059
14	{ Half the second bar		1 4	1656
	A feather-edged, or \perp bar was cast, whose dimensions were			
15	{ 2 inches deep, by 2 wide, edge up ..	10 0	2 8	3105
16	{ Half of ditto			

N. B. All the above bars contained the same area, though differently distributed as to their forms.

138. *Experiments made on Bars of 4 inches deep by $\frac{1}{4}$ inch thick, by giving it different forms, the bearing at 2 feet 8 inches, as before.*

	lbs.
17 Bar formed into a semi-ellipse weighed	7 lbs. 4000
18 Do. parabolic on its lower edge	3860
19 Do. of 4 inches deep, by $\frac{1}{4}$ inch thick	3979

Experiments on the Transverse Strain of Bars, one end made fast, the weight being suspended at the other at 2 feet 8 inches from the bearing.

	lbs.
20 An inch square bar bore	280
21 A bar 2 inches deep, by $\frac{1}{4}$ inch thick	539
22 An inch bar, the ends made fast	1173
23 The paradoxical conclusion of Emerson was tried, which states, by cutting off a portion of an equilateral triangle (see page 114 of Emerson's 'Mechanics'), the bar is stronger than before, that is, a part stronger than the whole. The ends were loose, at 2 feet 8 inches apart, as before. The edge from which the part was intercepted was lowermost; the weight was applied on the base above; it broke with 1129 lbs., whereas in the other case it bore only	840

We have given the above experiment as it is reported by Mr. Rennie; but it is at variance, as well as experiments 11, 12, 13, 14, with all the similar experiments on wood, reported in pages 135, 136, and 137.

139. *Experiments on the Transverse Strength of Steel. By M. Duleau.*⁴

Description of specimens.	Distance between the supports.	Breadth.	Depth.	Deflection with a wt. of 10 kilo- grammes.
	Mètres.	Millimètres.	Millimètres.	Millimètres.
Cast steel, English, marked HUNTSMAN; perfectly re- gular, untempered, but brittle	} 0·98	5·9	13·3	8·4
German steel, (of cementation,) marked FORTSMAN, and three deer heads, used for razors; dimensions irregular				
Same kind of steel	1·845	25·7	21·6	2·8
Same piece, on edge	1·845	21·6	25·7	2·2
Same kind of steel	1·845	21·9	28·5	1·8
Do. do.	1·35	54·8	25·5	0·55
Same piece, on edge	1·35	25·5	54·8	0·27
Same kind of steel	1·35	26·6	52·0	0·3

⁴ Essai théorique et experimental sur la Résistance du Fer forgé.

ON THE STRENGTH OF MALLEABLE IRON.

140. It is only since the commencement of the present century that malleable iron has been employed in situations which rendered it desirable to know with certainty its strength under different circumstances. With the exception of anchors and chains, malleable iron was seldom employed to resist by itself very great strains, its general application having been to connect and tie together different parts of a structure under circumstances which rendered it difficult, and not essentially necessary, to know with accuracy its ultimate force of resistance: all that is requisite in such cases is, that the iron shall exceed the strength of the other parts, and as the quantity thus employed in any case was inconsiderable, it was of little importance if more iron than was really necessary was used, and which, therefore, was commonly done, and its actual strength disregarded. But since the time alluded to, malleable iron has been introduced for several important purposes, in which it is employed by itself to resist enormous strains, as in the case of ships' cables, suspension bridges, and railway bars; it is, therefore, of the greatest importance that we should be able, from a knowledge of its actual

strength, to proportion the several parts, so that while we ensure perfect safety on the one hand, we may not on the other unnecessarily employ more of the material than is requisite, for all the weight thus introduced beyond what safety requires is always unnecessary, and frequently injurious.

The first application of malleable iron, which rendered this knowledge indispensable, was the invention of iron cables, by Captain Brown, and he accordingly was the first person who constructed a machine capable of making experiments on a sufficiently large scale to be depended upon: this was made to work by wheel-work and a well-balanced system of levers, but subsequent experimenters have generally employed the hydrostatic press, a machine admirably suited to such a purpose; commonly, however, in these the force was estimated by the pressure on a small valve, which was very defective on to accounts, *i. e.* 1st., that the friction of the leathers, which is very considerable with large strains, was not included; and 2ndly, the proportion between the valve and piston was too great. Such machines, therefore, commonly overrated the strain, and the motion of the balance weight was too small to be sufficiently perceptible.

To avoid these two evils, the Admiralty have had an excellent machine of this kind constructed in Woolwich Dockyard, for testing their iron cables, in which the strain is brought on by hydrostatic pressure, but its amount is estimated by a system

of levers balanced on knife edges, which act quite independently of the strain there is upon the machine, and exhibit sensibly, a change of pressure of $\frac{1}{8}$ th of a ton, even when the total strain amounts to 100 tons. It is also furnished with a valve according to the plan above alluded to, but this serves only to show the great defect of such apparatus; for while the lever scale is, as above stated, sensible to $\frac{1}{8}$ th of a ton, the other will scarcely move with a change of 2 tons, its indications being less and less sensible as the strains become more and more great.

Proving Machine in the Dockyard, Woolwich.

141. This machine was constructed by Messrs. Bramah, of Pimlico, and is no doubt one of the most perfect of the kind which has been executed; and as all the following experiments on railway bars, and many of the others on the tensile force of iron and other metals, were made with this press, the following description of it will not be unacceptable to the reader. It consists of two cast iron sides, cast in lengths of $9\frac{1}{2}$ feet each, with proper flanches for abutting against each other and for fixing the whole to sleepers resting on a secure stone foundation. The whole length of the frame is $104\frac{1}{2}$ feet, equal to $\frac{1}{8}$ th the length of a cable for a first-rate; so that the cables are tested in that number of detached lengths, which are afterwards united by shackle bolts.—

The press is securely bolted down at one end of the frame, and the cylinder is open at both ends. The solid piston is $5\frac{1}{4}$ inches in diameter in front and $10\frac{1}{2}$ inches behind, so that the surface of pressure is the difference of the two, viz.

$$\left(\frac{21}{2}\right)^2 - \frac{21}{4}\right)^2 \times .7854 = 65\frac{1}{2} \text{ inches.}$$

The system of levers hung on knife edges are attached to the other end of the frame, and the cable is attached by bolt links to this and to the end of the piston rod. The levers being properly balanced, and the cable attached to a short arm rising above the axis, this draws the other arm downwards, and at a distance equal to twelve times the short arm, is a descending pin and ball: this acts in a cup placed on the upper part of the arm of the second lever, and this again acts on a third. The first two levers are under the floor, and pass ultimately into an adjacent room, where a scale carrying weights is conveniently placed, and the whole combination is such that every pound in the scale is the measure of a ton strain; and as we have stated, the whole acts with such precision that $\frac{1}{8}$ th of a pound, more or less, in the scale very sensibly affects the balance. At the same place is situated a scale, acted upon by the water pressure from the charge pipe of the press; and the valve in this pipe is of such dimensions that, together with the lever by which it acts, the

power is again such that a pound should balance a ton ; but the friction is here so great that it requires several pounds to make a sensible change in the apparent balance, and for this reason this scale is never used. The forcing pumps are in another adjacent room, and are worked by handles, after the manner of a fire-engine. At first, six pistons are acting, and the operation proceeds quickly ; but as the pressure and strains increase, the barrels are successively shut off, till at length the whole power of the men is employed on one pair of pumps only, and on this the action is continued till the proof strain is brought on the cable. A communication is then opened between the cistern and cylinder, and every thing is again restored to equilibrium.

The foregoing general description will be better understood by a reference to our Plates VI. and VII. ;¹ the former exhibiting the pumps, and the latter the press frame, levers, &c. Fig. 1, Plate VI., is a representation of the pumps in elevation, of which an end view is given in fig. 2. There are two pumps working in each of the frames, A, B, C, fig. 1, and

¹ The drawings here referred to were made from a very accurate and excellently executed model of this machine, constructed by an Egyptian youth, Mahomet Al Moonga, sent to this country by Mahomet Ali Pasha, for instructions as an engineer : he was placed under the tuition of Mr. John Kingston, assistant-engineer in His Majesty's Dockyard, Woolwich, under whose able instructions, he not only constructed this machine, but also other models and drawings, highly creditable to his industry and talents.

the latter is also exhibited in fig. 2. Of these, the pair in the frame A are the largest bore, viz. $1\frac{1}{4}$ inch ; those in B are 1 inch, and in C $\frac{3}{4}$ inch. The manner of acting on each of the pumps will be sufficiently understood by the drawing fig. 2, and the process of working the whole by referring to fig. 1, in which DD, DD are rails for the persons employed to pump with, as in the fire-engine. The holes for receiving these rails are also shown at DD, fig. 2. EE is an iron cistern containing the water, into which of course the several pump barrels, or pipes from them, descend, and are there supplied with valves opening upwards to prevent the return of the water. The piston or plunger is solid, and by its action the water is forced along the pipe or passage *aa*, *bb*, *cc*, and ultimately to the descending pipe *d*, which passes under the floor, rising again under the centre of the press or cylinder, entering it as shown at *d*, fig. 3, Plate VII.

This description seems all that is required to explain the operation of the pumps when they are all in action ; but when great strains are called for, it is necessary to shut off, first, one pair of pumps at A, and then the other pair B, so that the whole force of the men is then employed on the pair C only. The means of effecting this is very simple and ingenious. *ss* are two standards which serve as supports for the ends of the axles *gg*, each carrying a bevelled wheel *ww*, and these again work the horizontal bevelled wheels with which they are in connection ; and thus the vertical axle, carrying also

the wheel *h*, is made to revolve by turning the handle *m*, and this again turns the wheel *h'* and its axle, but in an opposite direction. The lower part of each of the axles is a screw working in a nut, so that the axle rises and falls by turning the handle *m*, which is indeed the whole object of this part of the apparatus; the end of these axles being terminated conically, and each applies to a like formed hole in the water channel. When all the pumps are at work, the wheel *h'* is turned till its plug stops a hole entering the channel *a*, and the water is forced forward; the plug on the axle *h* being at this time raised, which opens a passage towards *b b*, and the same from *b b* to *c c*.

Suppose, now, it were required to shut off the pumps *A*, the handle *m* is turned, the axle *h'* is raised, and the other *h* depressed, till its plug shuts off the channel *a a* from *b b*; at the same time, the plug on the axle *h'* being raised, a communication is opened to a lateral pipe communicating with the cistern, so that the pump *A*, although it continues in action as before, only raises the water from, and returns it to the cistern; and precisely the same applies to the pumps *B*.

The pair of pumps *C* have not this apparatus, and can only be opened to the cistern by means of a similar apparatus working horizontally by means of the handle *n n*, which, communicating with a plug which opens the main channel *c, b, a*, either to the pipe *d* to communicate the pressure to the press,

or with the waste pipe *e*, thereby permitting the water to return to the cistern. This, therefore, is done at the end of each experiment, and the water remains to be propelled as before when required.

We come now to a description of the press and its mode of operation. It has been stated that its whole length consists of eleven frames, of which only one is shown in the drawing at A B, A, B, figs. 1 and 2, Plate VII., the former being a plan and the latter an elevation. A part of the last frame is, however, shown in both figures, as is also that which supports the press: a cable is also represented in each figure as being tested. We may however observe, that in order to show the connecting pieces, the floor or platform of the bed (fig. 1) is removed; it consists of strong oak planks resting on the cross pieces seen in fig. 1. These cross pieces are not employed in the frame carrying the cylinder, the flanches of which, *f, f, ff*, answer this purpose: fig. 3 is a section of the press, with the solid piston passing through, having the smaller end screwed into the larger, leaving the difference of the two areas for the action of the water, as already explained; each end passes through collars of leather, to prevent leakage, and of course the action of the piston is rendered perfectly uniform and in the same line: the frame A B, adjacent to that carrying the press, is supplied with two planed iron slides *dd*, on which moves a cross piece *aa*, supporting the end of the piston, the form of which

beyond the frame will be sufficiently seen by the drawing. To this one end of the cable is attached by a shackle bolt, and in like manner to the other end of the frame in connection with the system of levers at k , the operation of which is still to be explained. The cable, prior to the experiment, rests on the platform of the bed, as above stated ; $g h$ is a heavy bent lever, as represented in fig. 2, turning on a knife edge seen at g ; to this lever above, as at k , are two cheek pieces, held in their places by a strong bolt, and to the other end of these cheek pieces the last link of the cable is secured in the same manner ; $l m$ is a transverse lever under the floor of the room, turning on a knife edge at l , and passing beyond m into an adjacent room, where it is connected at n (fig 4) with the hanging rod $o n$; this is again connected at o with the lever $o p$, having its fulcrum at q , the top of the fixed standard $q r$, which is securely fixed to the floor of the room $r s$; $t t$ is a stone platform for supporting the scale and weights w . The first lever $g h$ acts on the lever $l m$ as follows : $h v$ is a descending bolt, furnished at its end with a ball, and w is a socket on a piece rising from the lever $l m$, and which, of course, presses the lever $l m$ downwards. It is now obvious, that when a strain is brought on the cable it pulls the bent end k of the lever $k g h$ forward ; this depresses the end h , and the ball v ; this presses on the socket w of the lever $l m$; at n this pulls on the hanging rod $o n$, and this on the lever $o p$, which

raises the scale till such weight is introduced as balances the strain.

The arms of the several levers are so proportioned that a pound at w' shall balance a ton at k , but as this is difficult to be exactly brought out where all the parts are so large, there is an adjusting screw p which moves the point of support of the scale to and fro till this exact proportion is attained; and a detached apparatus which belongs to the machine, enables the engineer in charge to ascertain at any time the accuracy of its action against actual weights up to four tons. The machine has lately been tested in this way, and its action found to be perfectly satisfactory, the scale exhibiting very perceptibly a change of strain of $\frac{1}{8}$ th of a ton. It may be observed, that the axle of the large lever g turns in holes somewhat elliptical, and its end is borne up so as to carry the knife edge g to its proper position by a heavy weight x at the end of a long lever xy , which turns on an axle at z , attached to the side of the frame. This lever is seen dotted in fig. 5, which is the side of the frame fig. 2, removed to show the large lever gh more distinctly; precisely a similar lever acts on the other side frame against the axle g , to bear up its other extremity.

It only remains to say a few words more in reference to the press. The experiment being performed, the ram will be run out towards E, and will require to be pushed in towards A, prior to another trial. This is effected by means of the

rack $q\ r$, attached or detached at pleasure from the ram or piston at E ; it is acted upon by the pinion on the axle $Z\ Z$, which is turned by hand by means of the wheel $W\ W$; but when the experiment commences, this pinion is thrown out of gear with the rack by pushing the axle a little endwise, so as to clear the teeth of each from the other.

Let us now describe one experiment: 1st, The length of cable to be tested is laid upon the bed of the press. The ram or piston of the press is run forwards as far as its shoulder towards A , by means of the wheel $W\ W$ and rack $r\ q$. The handles $m\ m$ (fig. 1, Plate VI.) are turned so as to open entirely the water channel to the press; a screw plug is also opened on the top of the cylinder to allow the air to escape, and when water shows itself at this aperture by the working of the pumps, this is screwed in. Every thing is now ready; the pumping commences, and all six pumps being in action, the piston retreats fast, till the cable begins to strain; the process is then slower, and after a time, when the strain is considerable, the pumps A are shut off, and afterwards, if necessary, the pumps B ; the operation then continues on the pumps C only, till the proper strain is obtained, which is ascertained by a person at the scale, who continues to add pound after pound in the scale till the pounds' weight are equal to the tons' strain required. As soon as the scale rises with this weight, he pulls a handle which rings a bell in the pump-room, and

the operation ceases; the wheel *nn* (fig. 1, Plate VI.) is then turned so as to open the compressed water to the waste or return pipe *e*, through which the water returns to the cistern ready for performing the next experiment.

The experiments on bars, bolts, &c., described in the following pages, were made in the same manner, by employing two short lengths of cables and making the trials by means of attachments to their two ends towards the centre of the bed or platform.

142. *Table showing the different kinds of best Bower Cables at present employed in the British Navy, with the corresponding Iron Cables, and the Proof Strain for each.*

Rates of Ships.	Best bower hempen cables, 100 fathoms.				Number of threads in each.	Breaking strain by experiment.	Diameter and weight of the bolt of the iron cable substituted for the preceding.	Strain for the proof.
	Sizes, circum.	Weight.						
	in.	cwt.	qr.	lbs.		tons. cwt. qr.		tons.
First-rate, large	25	114	2	7	3240	..	2½ inches. 218 cwt.	} 81
middle	24	105	2	17	2988	..		
small .	23	96	2	27	2736	114 0 0		
Second-rate . .	23	96	2	27	2736			
Third, large . .	23	96	2	27	2736			
small .	22	89	0	12	2520	89 0 0	2 inches. 186 cwt. 2 qrs.	} 72
Fourth, 60 guns .	21	80	0	22	2268			
58 do. .	19	66	0	21	1872			
50 do. .	18½	62	1	14	1764	..	1½ inch. 170 cwt. 2 qrs.	} 63
Fifth, 48 do. .	18	58	2	6	1656	63 0 0		
46 do. }	17½	56	0	1	1584	..	1¼ inch. 145 cwt. 3 qrs.	} 55
42 do. }								
Sixth, 28 do. .	14½	38	0	21	1080	40 0 0	1½ inch. 87 cwt. 2 qrs.	} 34
Ship, sloop . .	13½	33	0	10	936	..	1¼ inch.	
Brig, large . .	13½	33	0	10	936	..	74 cwt. 3 qrs.	} 28
Ditto, small . .	11	21	2	15	612	..	1½ inch. 61 cwt. 1 qr.	

From the above Table the immense advantage of iron cables will be distinctly seen, and particularly when we consider that a hempen cable, on a rocky bottom, is destroyed in a few months, while the other will sustain no perceptible injury.

EXPERIMENTS ON CHAINS AND CABLES. 261

143. *Actual experimental Strength of Chain, made of various Descriptions of re-manufactured Foreign and English Iron, performed 2nd September, 1816, at Captain Brown's Manufactory.*

	tons. cwt.
1½ inch Old sable, 1½ inch square bars, cut into pieces 2 feet long, piled and rolled into bolts of 1½ inch	73 10
1½ inch Old sable, ditto, ditto	80 0
1½ inch Gurcoft new sable, ditto, ditto	71 0
1½ inch Keiolsken, Archangel, inch square bars, cut into lengths of two feet, piled and rolled into bolts	71 0
1½ inch Old bolts, found promiscuously, piled and fagotted by hand-hammers at my works	71 10
1½ inch full . . English bars, piled and rolled	86 0
1½ inch bare . . Ditto do	80 0

Further Experiments, made 13th September, 1816.

	tons. cwt.
1½ inch Old Dutch bolts, fagotted by hand-ham- mers at my works	71 0
1½ inch No. 1, ⅝ square, (Welsh iron,) hammered into blooms, and rolled into bolts, at the King and Queen works	78 10
1½ inch No. 2, ¾ inch square, (Welsh,) manufac- tured as above	73 5
1½ inch No. 4, Welsh iron, fagotted by hand- hammers at my works	88 10
1½ inch No. 6, ⅝ in. square ditto, rolled, but not hammered, at the King and Queen works	76 0
1½ inch King and Queen scrap iron	80 5

The links of these chains were of an oval-like form, 6 inches in the clear.

S. BROWN.

The mean of these experiments gives 76 tons for the strength of a double bolt of $1\frac{1}{2}$ inch diameter, *in the cable form*, which corresponds to about $21\frac{1}{2}$ tons per square inch. Now by the same machine, the mean strength of wrought iron, per square inch, is 25 tons (see the following experiments); therefore, the strength of iron in the cable form is to that of the simple bolt in about the ratio of 43 to 50. But in these cables the links were without stays: when these are introduced, as in Brunton's patent cable, the strength is very nearly equal to that of the iron in the simple bar form; so that the stay may be said to increase the strength by about one-sixth part; at the same time, however, it must be considered that the weight is also increased, although perhaps in a somewhat less ratio.

Experiments on Direct Cohesion of Malleable Iron.

144. The next important application of malleable iron was in the construction of suspension bridges, also the invention of Capt. Brown. Subsequently, viz. in 1814, it was proposed by the late distinguished engineer, Thomas Telford, Esq., to suspend a bridge of this kind over the River Mersey, at Run-corn, of 1000 feet span. In an undertaking of this magnitude, it became essentially necessary to know very exactly what strength could be depended upon in the material to be employed; and Mr. Telford

accordingly undertook an extensive series of experiments, both on the strength of malleable iron bolts, and on iron wire, with which he obligingly supplied me for the first edition of my ‘Essay on the Strength of Materials.’ These are given below, in the form in which they were recorded at the time of making them, at Messrs. Brunton’s iron cable manufactory. The other experiments were in like manner supplied to me by Capt. Brown. It is only necessary to observe, that Messrs. Brunton’s machine, being an hydrostatic press, registering by means of a valve, has a tendency to overrate its power, while Capt. Brown’s, perhaps, slightly underrates its power; but his results certainly agree best with subsequent experiments made by myself on the machine in his Majesty’s Dockyard, Woolwich.

145. *Experiments on the direct Strength of Cohesion of Malleable Iron, made at Messrs. Brunton and Co.’s Patent Chain Cable Manufactory, with an Hydrostatic Machine, or Bramah Press, constructed by Mr. Fuller. By Thomas Telford, Esq.*

BAR No. 1.

Cylindrical Bar of South Wales Iron, manufactured by S. Homfrey, Esq.

April 5th, 1814.	{	Length of bar when put in . . .	2 feet 2 $\frac{1}{2}$ inches.
		Ditto when taken out . . .	2 6 $\frac{7}{8}$
		Diameter when put in . . .	0 1 $\frac{3}{8}$
		Ditto when taken out . . .	0 1 $\frac{1}{8}$
Torn asunder by 43 tons 11 cwt.			

BAR No. 2.

*Cylindrical Bar of South Wales Iron, manufactured by
S. Homfrey, Esq.*

April 15th, 1814.	{	Length of bar when put in . . .	2 feet 3½ inches.
		Ditto when taken out . . .	2 6½
		Diameter when put in . . .	0 1½
		Ditto when taken out . . .	0 1¼
		Torn asunder by 52 tons 15 cwt. 1 qr. 10 lbs.	
		Time, 34 minutes.	

BAR No. 3.

Square Bar of Staffordshire Iron.

May 17th, 1814.	{	Length of bar when put in . . .	1 foot 5½ inches.
		Ditto when taken out	1 11¼
		Side of square when put in	0 0¾
		Ditto when taken out	0 0 ⁶ / ₁₆
		Began to stretch with 12 tons ; broke with 15 tons 5 cwt. 3 qrs. 4 lbs. Time, 9½ minutes.	

BAR No. 4.

Square Bar of Staffordshire Iron.

May 17th, 1814.	{	Length of bar when put in . . .	1 foot 7 $\frac{1}{4}$ inches.
		Ditto when taken out . . .	1 9 $\frac{1}{4}$
		Side of square when put in . . .	0 1 $\frac{1}{12}$
		Ditto when taken out . . .	0 0 $\frac{5}{8}$
		Began stretching with 32 tons; broke with 32 tons 6 cwt. 4 lbs. Time, 16 minutes.	

BAR No. 5.

Square Bar of Welsh Iron, 1 inch square.

May 5th, 1817.	{	With 18 tons stretched . . .	0 $\frac{1}{4}$ inches.
		Ditto 21 tons ditto . . .	0 $\frac{1}{4}$
		Ditto 23 tons ditto . . .	0 $\frac{3}{4}$
		Ditto 25 tons ditto . . .	1
		Ditto 27 tons ditto . . .	2 $\frac{1}{4}$
		Ditto 29 tons ditto . . .	2 $\frac{3}{8}$ { Broke with this weight.

BAR No. 6.

Bar of Swedish Iron, 1 inch square.

May 5th, 1817.	{	Began to stretch with 17 tons.
		Stretched ² with . . . 20 tons, $\frac{1}{16}$ th inch.
		Ditto with 27 tons, $\frac{3}{8}$ ths.
		Ditto with 29 tons. Broke at a flaw.

BAR No. 7.

Bar of fagotted Iron, from scrap Iron. By Mr. Howard, of Rotherhithe. 1 inch square.³

May 5th, 1817.	{	Began to stretch with 16 tons.
		Stretched with . . . 20 tons, $0\frac{3}{8}$ inch.
		Ditto with 25 tons, $0\frac{1}{4}$
		Ditto with 28 tons, $2\frac{3}{8}$
		Ditto with 29 tons. { Broke with this weight.

BAR No. 8.

Bar of common Staffordshire Iron, 1 inch square.

May 5th, 1817.	{	Began to stretch with 19 tons.
		Stretched with . . . 24 tons, $0\frac{1}{2}$ inch.
		Ditto with 28 tons, $0\frac{5}{8}$
		Ditto with 29 tons, $0\frac{6}{8}$
		Ditto with 30 tons, 1
		Ditto with 31 tons. { Broke with this weight.

² The stretchings were measured on 12 inches in the middle of the bar.

³ A similar bar began to stretch with 18 tons, and broke with the same weight as above ; viz. 29 tons.

BAR No. 9.

*Cylindrical Bar of common Iron, 2 inches diameter.*⁴

May 21st, 1817.	{	With 45 tons.	{	Began to stretch; about $\frac{1}{16}$ th of an inch on 12 inches in the middle. The machine being relieved, the bar shortened $\frac{1}{16}$ th of an inch.
		With 50 do.		Stretched $\cdot 125$ inch; relieved and shortened as before.
		With 55 do.		Do. $\cdot 25$; do. do.
		With 60 do.		Do. $\cdot 26$
		With 70 do.		Do. $\cdot 375$ inch; recover ^d very little when the machine was relieved.
		With 75 do.		Do. $\cdot 544$; do. do.
		With $80\frac{1}{16}$ do.		Do. $\cdot 75$; reduced in diameter to $1\frac{1}{16}$ th inch.
		With 85 do.		Do. $\cdot 86$; no perceptible change.
		With 90 do.		Do. $1\cdot 00$; do. do.
		With 95 do.		Do. $1\cdot 35$; reduced in diameter to $1\frac{1}{8}$ th inch.
		With 100 do.	{	Do. $2\cdot 2$; do. do. to $1\frac{1}{2}$ nearly.

With the last weight the bar gave evident signs of fracture; and, in a few minutes, gradually gave way.

⁴ The whole length of the above bar was 2 feet; and it stretched in its whole length $2\frac{7}{8}$ inches; of which $2\frac{1}{2}$ inches were in 12 inches in the middle part. The whole time of making this experiment was 3 hours; and it was performed with the utmost care.

The machine was frequently relieved; and, when re-applied, constantly brought up the weight to what it was before, but never exceeded it; which is evidence of its accuracy.

Note.—It is a curious fact, and deserving the attention of philosophers, that frequently, at the moment of rupture, the bar acquires such a degree of heat in the fractured part, as scarcely to enable a person to hold it grasped in his hand without a painful sensation of burning.

Reduction of the above to 1 inch square.

	tons.	cwt.	
No. 1, reduced to 1 inch square, gives	29	6	Welsh.
No. 2,	29	16	Ditto.
No. 3,	27	3	Staffordshire.
No. 4,	27	10	Ditto.
No. 5,	29	0	Welsh.
No. 6,	29	0	Swedish.
No. 7,	29	0	Fagotted.
No. 8,	31	0	Staffordshire.
No. 9,	31	16	
	9)	263 11	
Mean strength of an inch square bar	29	5 $\frac{2}{3}$	

146. *Experiments on Iron Bars and Cables, made at the Patent Iron Cable Manufactory of Captain S. Brown, Mill Wall, Poplar, on a Machine which acts on the Principle of the Weigh-Bridges. From a Report presented to the Author by the above Gentleman.*

(COPY.)

Mill Wall, Poplar, 28th May, 1817.

Experiments on different Descriptions of Iron.

BAR No. 1.

A bar of Swedish iron, 3 feet 6 inches long, $1\frac{5}{16}$ inch square, required a strain of 40 tons 19 cwt. to tear it asunder in a straight line. It stretched during the operation $\frac{3}{16}$ ths of an inch. No perceptible alteration in the general appearance of the bar, except at the place of rupture, where it was reduced to $1\frac{1}{16}$ th of an inch.

The particles remarkably small and close, of a whitish grey colour ; not the least heated in the operation.

BAR No. 2.

Another piece, 3 feet 6 inches long, same bar, required a strain of 39 tons 15 cwt. to tear it asunder in a straight line. It stretched $\frac{1}{4}$ th of an inch, the bar being torn into cracks in various places. It reduced to $1\frac{1}{16}$ th of an inch at the place of rupture. The particles remarkably close and small, as before, intermixed with a few fibrous specks.

Colour, whitish grey ; not heated at the time of rupture.

BAR No. 3.

A Swedish bar, 3 feet 6 inches long, (different mark,) $1\frac{3}{16}$ ths inch square, required 33 tons 10 cwt. to tear it asunder in a straight line. This bar was exceedingly soft and ductile, having stretched 3 inches in the operation ; and reduced at the place of rupture to $\frac{7}{8}$ ths of an inch. It broke extremely fibrous, exhibiting

no particles. The complexion silvery ; very much heated at the place of rupture.

BAR No. 4.

A bolt of Russia old sable, marked C C N, 3 feet 6 inches long, $1\frac{5}{8}$ ths inch diameter, required a strain of 36 tons 2 cwt. to tear it asunder in a straight line. This iron, very soft and ductile, stretched $2\frac{1}{2}$ inches, and reduced at the place of rupture to 1 inch in diameter. This iron appeared at the place of rupture in the form of a scarf, as if it had been cut with a pair of shears : the surface so smooth, that there was no appearance of fibres or particles : its fibrous quality was, however, sufficiently indicated by the whole appearance of the bolt.

BAR No. 5.

A bar of Welsh iron, denominated No. 3 ; 3 feet 6 inches long, $1\frac{1}{2}$ inch square, required a strain of 38 tons 1 cwt. to tear it asunder. This iron possessed considerable ductility, but reduced in diameter more gradually than in the two preceding experiments. It stretched 2 inches ; and was reduced at the place of rupture to $1\frac{1}{8}$ th inch. The complexion of this iron, when looking directly down upon the place of rupture, was a dingy blue, and when held horizontally to the light and viewed obliquely, bright and fibrous, though not so white or silvery as the foreign iron. Very much heated at the place of rupture.

BAR No. 6.

A bar of common Welsh iron, 3 feet 6 inches long, $1\frac{1}{8}$ th inch square. It required a strain of 31 tons. This bar had little ductility, and suffered no general derangement in the operation. It broke directly across the bar ; and measured at the place of rupture $1\frac{1}{8}$ th inch. The particles of this iron were fine, and exceedingly condensed, resembling steel ; and

there appeared nothing of a fibrous nature in it : indeed, its complexion and texture seemed to be at variance with the general rules for judging of the quality of iron. Its measure of strength, however, was most accurately ascertained.

BAR No. 7.

A highly interesting one. A bolt of Welsh iron denominated No. 3; 12 feet 6 inches long, 2 inches in diameter, required a strain of 82 tons 15 cwt. to tear it asunder. When subject to a strain of 68 tons, it stretched 3 inches, and was reduced to $1\frac{1}{8}$ ths inch in diameter. When the strain was increased to 74 tons 15 cwt., it had stretched 6 inches, and was reduced $\frac{1}{8}$ th of an inch gradually in the diameter. With 82 tons it stretched 14 inches. With 82 tons 15 cwt., the bolt broke about 5 feet from the end, the levers being exactly balanced. It had stretched during the whole process $18\frac{1}{2}$ inches; and measured at the place of rupture $1\frac{1}{8}$ inch in diameter.

SAMUEL BROWN.

BAR No. 8.

A bolt of Welsh iron, $1\frac{3}{8}$ inch diameter, 5 feet in length, was torn asunder by a force of $43\frac{1}{2}$ tons.

With 28 tons its diameter was reduced to 1·4 inches.

With 35 tons 1·35 inches.

With 40 tons 1·30 inches.

With 43 tons the bolt broke, having lengthened during the experiment 7 inches. Considerable heat about the section of fracture.

This is the only one of the above experiments at which I was present.

Reducing the above to inch square.

	Tons.
No. 1 Swedish iron 1 square inch,	23·77
No. 2 Ditto ditto	23·19
No. 3 Ditto ditto	23·75
No. 4 Russia ditto	26·55
No. 5 Welsh ditto	24·35
No. 6 Ditto ditto	24·90
No. 7 Ditto ditto	26·33
No. 11 Ditto ditto	26·34
Mean 25 tons.	
The mean of Mr. Telford's experiments is $29\frac{1}{4}$ tons.	
<hr/>	
Mean of the two 27 tons.	

147. *Experiments made on Malleable Iron Bolts on the Testing Machine in Woolwich Dockyard. By the Author.*

The machine on which these experiments were made having been already described, it only remains to explain the manner in which the bars were held in order to be submitted to the strains requisite to produce rupture. In the experiments described in the preceding pages, the ends of the bars were upset, as it is termed; that is, they had their ends hammered up into a conical lump, and were inserted into conical sockets made in two parts, which were placed over the ends and united by hoops. But there seems in this way a danger of injuring the texture of the iron; and it is, moreover, inapplicable in other metal without actual damage. The machine

which I employed was invented by Mr. John Kingston, assistant-engineer in the above establishment; it is very simple and effectual, as will be seen by the following description:

Mr. Kingston's Nippers for Testing Metal Bars.

These are represented in Plate VI., figs. 6, 7, 8, 9. Here *a, a, a, a*, figs. 6, 7, are heavy wrought iron sockets; *b, b*, strong double hooks of the same metal; *c, c, c, c*, links of the same metal, which connect the sockets *a, a*, &c., with the hooks *b, b*, the latter of which have eyes seen in fig. 7, by which they are connected with the cable proceeding from the ram of the press at one end, and with the levers at the other; and the ends of the sockets at *a, a*, have grooves to receive the links to prevent them from slipping. Figs. 8 and 9 are the nuts, on an enlarged scale; they are formed in two parts with a cylindrical hole in each, which being placed together have a coarse screw cut in them; fig. 8 shows their form on the side, which is slightly bevelled, as is the socket. *B B* is the bar; this is placed in the nuts as represented in figs. 6 and 7, and slightly driven with a hammer: the strain being now applied, the nuts are drawn into the sockets, which nip the bars more and more strongly as the strain increases, without any danger of slipping.

148. *Experiments on Malleable Iron Bars with the above Machine, in his Majesty's Dockyard, Woolwich. By the Author.*

BAR No. 1.—SOLLY'S PATENT IRON.

Round bar 1 inch in diameter, broke with a strain of 21 tons. It stretched before the fracture $10\frac{1}{2}$ inches in 8 feet in the middle ; its whole length was 10 feet 2 inches.

Strength per square inch 26·7 tons.

The bar broke at a part where it had been nicked with a chisel. It was therefore tried again, the marked part being inserted in the nippers, and the breaking weight was now 23 tons, or strength per square inch $29\frac{1}{4}$, and stretch $2\frac{1}{2}$ inches more.

BAR No. 2.—SOLLY'S PATENT IRON.

Square bar 1 inch in diameter, broke with $23\frac{1}{2}$ tons at the place where it had been nicked with a chisel to mark it. It stretched $13\frac{1}{2}$ inches in 8 feet. In consequence of this defect the broken parts were again tried, and one of these, after being broken, was again tried ; the following are the results :

	Tons.
Second trial, breaking weight	$26\frac{1}{2}$
Third do. do.	$26\frac{1}{2}$
Fourth do. do.	$25\frac{3}{4}$

The following are results of other experiments on iron of good medium quality :

	Tons.
1. Bar 1 inch square, breaking weight	24
2. Ditto ditto	$25\frac{1}{2}$
3. Round bar reduced to inch square	$25\frac{1}{2}$
4. Ditto ditto	26
Mean strength	$25\frac{1}{4}$

In the preceding experiments the mean of Messrs. Brunton's and Brown's experiments gives 27 tons, but from these experiments I consider that we ought not to assume the strength of good medium iron at more than 25 tons per square inch. It will be seen by subsequent experiments that the elasticity is destroyed with about 10 tons, and that iron ought not to be strained beyond its elastic power.

149. Experiments on the strength of Yorkshire Iron, by M. I. Brunel, Esq. These were made on bars reduced in the centre part (per hammer) to $\frac{3}{8}$ ths and $\frac{1}{2}$ ths, or $\frac{1}{2}$ inch square; but the results are all reduced to rods of 1 inch square.

Experiments of the Direct Cohesive Power of Hammered Iron. By M. I. Brunel, Esq.

Iron denoted <i>best</i> $\frac{3}{8}$ ths in the middle.			Iron denoted <i>best best</i> $\frac{3}{8}$ ths in the middle.			Iron denoted <i>best</i> $\frac{1}{2}$ in the middle.		
No.	Began to stretch.	Breaking weight.	No.	Began to stretch.	Breaking weight.	No.	Began to stretch.	Breaking weight.
	Tons per inch.	Tons per inch.		Tons per inch.	Tons per inch.			Tons.
1	21	29·8	1	28·16	35·12	1		27
2	24	32	2	27·4	36·4	2		31·12
3	18·15*	25*	3	24·16	32·16	3		31·62
4	22	34·19	4	27·16	33·10	4		32·25
5	20	34·6	5	22·15	31·14	5		32·75
6	20	28·2	6	25·18	31·15	6		30·00
7	23·2	28·2	7	22·3	31·9			
8	24	31·6	8	21·9	29·6			
9	26·9	32·11	9	23·9	31·7			
10	23·1	28·12	10	21·9	30·7			
Mean	22·2	30·4		24·4	32·3			30·8

* The Experiment No. 3 of the first series was obviously defective.

The mean strength of these bars considerably exceeds that drawn from the preceding Articles; a circumstance which may, it is presumed, be explained from the fact of their having been reduced per hammer.

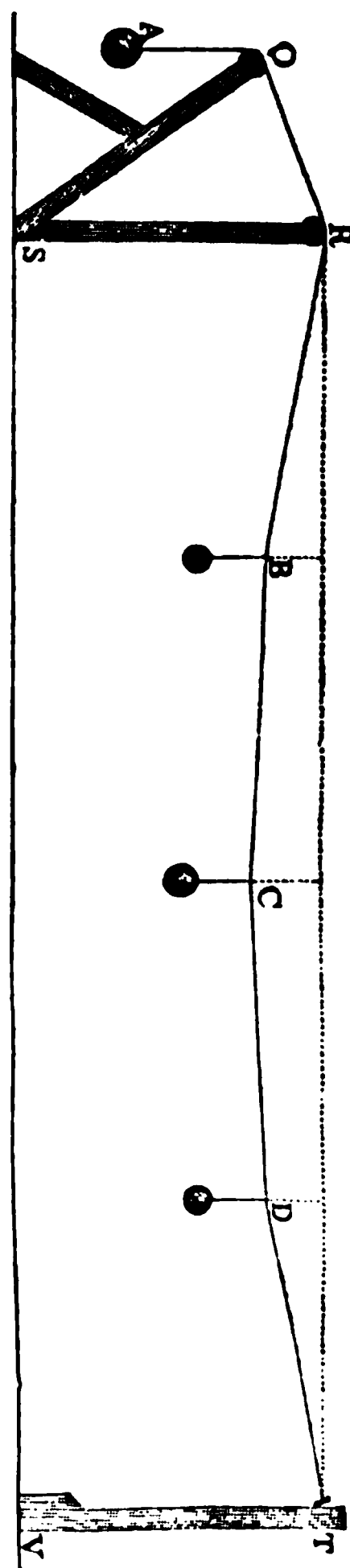
Experiments on the Strength of Iron Wire.

150. Amongst other propositions for suspension bridges, that of iron wire for the purpose has been included, and bridges of this kind have been executed; and as far as actual strength and facility of joining are concerned, it would appear to have a preference, but it is not thought applicable to the larger constructions of this kind. Mr. Telford, however, in the infancy of the practice, thought it desirable to try its strength under various circumstances, by submitting it to strains as nearly resembling those of the bridge itself as possible, with a statement of which he kindly furnished me in the form given in the following Tables.

In order to comprehend the tabulated results, it will be necessary to explain the apparatus with which the experiments were made: these are presented in the annexed figure.

Here RS, TV, represent the supporting pillars upon which the wire was extended; QS, another prop over which the wire passed; being placed at such an angle as made it coincide with the direction of the resultant of the vertical and horizontal tensions, in order to prevent any strains upon the other support, RS.

A, B, C, D, represent the places of the several weights with which the wire was loaded; C being in the centre of the length, and B and D at $\frac{1}{4}$ th of the length from each end; and the deflections from the horizontal line RT were measured at these points, as the different weights were applied.



EXPERIMENT No. 1.

Distance of the Props, 100 feet; Weight of 100 feet of Wire, 29½ ounces; Diameter, rather more than $\frac{6}{70}$ ths of an inch; and it broke when suspended vertically, at a medium of different trials, with 531 lbs.

Weight at A, in- cluding the Wire Q A.	Weight at B.	Weight at C.	Weight at D.	Deflec- tion at B.	Deflec- tion at C.	Deflec- tion at D.	REMARKS.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	ft. in.	ft. in.	ft. in.	
5 6½	0 0	0 0	0 0	..	4 10	..	{ Deflections at B and D not taken.
10 5	0 0	0 0	0 0	..	2 11½	..	
30 5½	0 0	0 0	0 0	..	0 10½	..	
do.	0 0	1 0½	0 0	..	1 8	..	
do.	0 0	2 0½	0 0	..	2 7	..	
do.	0 0	5 0½	0 0	..	4 11	..	{ The weight at C being taken off, the deflection be- came 11 in.
176 0	5 0	30 4	5 0	2 1	4 6½	2 1	
do.	9 0	30 4	5 0	2 5½	4 10½	2 2½	Raised weight A 1 in.
226 0	9 0	56 0	5 0	3 11	7 10½	3 7½	
286 0	9 0	56 0	5 0	2 8¾	5 11½	2 6½	
342 0	9 0	56 0	5 0	2 3½	5 0¾	2 1¾	
do.	9 0	66 0	5 0	2 5	5 4½	2 3½	
do.	9 0	72 0	5 0	2 7	5 9½	2 5½	
do.	9 0	77 0	5 0	2 7	5 10	2 5½	
do.	9 0	81 0	5 0	2 9¾	6 4¾	2 8	
do.	9 0	87 0	5 0	2 10½	6 6½	2 8½	
do.	15 0	71 0	15 0	2 11¾	6 3¾	2 11¾	
402 0	15 0	71 0	15 0	2 8½	5 8¾	2 8½	{ Broke after sustain- ing these weights for a short time.
402 0	30 0	56 0	30 0	

EXPERIMENT No. 2.

Distance of the Props, 31 feet 6 inches ; the same specimen of Wire as in Experiment No. 1, but had not been before used ; the two Ends of the Wire, in this Experiment, were fixed, after drawing it as tight as possible ; viz. to within less than $\frac{1}{8}$ th of an inch of a horizontal Line ; and the Weights applied only in the centre.

End at R and T fixed.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	REMARKS.
Fixed	0	lbs. 10 $\frac{1}{2}$	0	..	ft. 0 in. 2.83	..	
do.	0	20 $\frac{1}{2}$	0	..	0 5.5	..	
do.	0	30 $\frac{1}{2}$	0	..	0 7.75	..	
do.	0	40 $\frac{1}{2}$	0	..	0 10	..	
do.	0	50 $\frac{1}{2}$	0	..	1 0	..	
do.	0	60 $\frac{1}{2}$	0	..	1 1.75	..	
do.	0	70 $\frac{1}{2}$	0	..	1 3.5	..	
do.	0	80 $\frac{1}{2}$	0	..	1 5	..	
do.	0	90 $\frac{1}{2}$	0	..	1 6.5	..	
do.	0	100 $\frac{1}{2}$	0	..	1 8	..	
do.	0	110 $\frac{1}{2}$	0	..	1 9.75	..	
do.	0	120 $\frac{1}{2}$	0	..	1 10.75	..	
do.	0	130 $\frac{1}{2}$	0	

Just bore the last weight, and then broke.

EXPERIMENT No. 3.

Distance of Props, 100 feet ; Diameter, $\frac{1}{10}$ th of an inch ; Weight of 100 feet = 2 lbs. 9 oz. : bore vertically 736 lbs., but broke with 738 lbs.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	REMARKS.
lbs.	lbs.	lbs.	lbs.	ft. in.	ft. in.	ft. in.	
362	0	0	0	..	0 5	..	
362	30	15	30	2 2	2 11 $\frac{1}{2}$	2 1 $\frac{1}{2}$	
362	35	30	35	2 8	3 10 $\frac{3}{8}$	2 7 $\frac{1}{2}$	
362	40	35	40	2 11 $\frac{1}{10}$	4 3 $\frac{1}{2}$	2 10 $\frac{1}{2}$	
362	40	41	40	3 3	4 11	3 2 $\frac{1}{2}$	
468	56	41	56	3 4 $\frac{6}{10}$	4 9 $\frac{4}{10}$	3 4 $\frac{7}{10}$	
498	56	41	56	3 0 $\frac{4}{10}$	4 3 $\frac{6}{10}$	3 0 $\frac{6}{10}$	
558	61	41	61	3 1 $\frac{1}{2}$	4 4 $\frac{1}{2}$	3 1 $\frac{1}{2}$	
608	76	76	76	3 5 $\frac{2}{10}$	5 3 $\frac{2}{10}$	3 6 $\frac{1}{2}$	{ Fixed the wire at A.
Fixed	56	56	56	3 0	4 6 $\frac{7}{10}$	2 11 $\frac{1}{2}$	
do.	71	68	71	3 3 $\frac{2}{10}$	5 0	3 4	Refixed the wire.
do.	do.	do.	do.	3 4 $\frac{7}{10}$	5 1 $\frac{2}{10}$	3 4 $\frac{7}{10}$	
do.	77	74	77	3 6 $\frac{2}{10}$	5 4 $\frac{8}{10}$	3 6 $\frac{8}{10}$	Refixed the wire.
do.	77	74	77	3 3 $\frac{7}{10}$	4 11 $\frac{8}{10}$	3 3 $\frac{2}{10}$	

Bore this weight ; but in attempting to add 4 lbs. more to the weights at B and D, the wire broke.

EXPERIMENT No. 4.

The same Wire as in last Experiment. Distance of the Props, 31 feet 6 inches.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflec- tion at B.	Deflec- tion at C.	Deflec- tion at D.	REMARKS.
Fixed	lbs. 0	lbs. 0	lbs. 0	ft. in. ..	ft. in. 0 0½	ft. in. ..	Both ends fixed.
do.	40	41	40	0 7½	0 10½	0 7½	
do.	44	47	44	0 8½	1 0½	0 8½	
do.	50	47	50	0 9	1 0½	0 9	
do.	56	47	56	0 9¾	1 1½	0 9½	
do.	56	53	56	0 10½	1 2	0 9¾	
do.	61	53	61	0 10½	1 2½	0 10¼	
do.	61	59	61	0 10¾	1 3½	0 10¾	
do.	67	68	67	1 0	1 4½	0 11½	
do.	71	68	71	1 0	1 4½	1 0	
do.	71	76	71	1 0½	1 5½	1 0½	

With the last weights suspended a few minutes, the wire broke.

EXPERIMENT No. 5.

Distance of the Props, 100 feet ; Diameter, $\frac{6}{100}$ ths of an inch ; Weight of 100 feet, 16½ ounces. Vertically, the Wire bore 277 lbs. a few minutes, and then broke.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflec- tion at B.		Deflec- tion at C.		Deflec- tion at D.		REMARKS.
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
180	0	0	0	0	1 $\frac{1}{4}$	0	1 $\frac{1}{4}$	0	1 $\frac{1}{4}$	{ Took off the weight A, and tightened the wire.
180	6	5	6	1	0 $\frac{7}{8}$	1	5 $\frac{1}{2}$	0	11 $\frac{1}{2}$	
180	12	10	12	1	10 $\frac{1}{2}$	2	7 $\frac{3}{4}$	1	9 $\frac{1}{2}$	
210	16	14	16	2	3 $\frac{1}{2}$	3	2 $\frac{1}{2}$	2	2	
248	16	14	16	2	2 $\frac{1}{2}$	3	2 $\frac{1}{2}$	2	2 $\frac{1}{2}$	
Fixed	16	14	16	1	9 $\frac{1}{2}$	2	7 $\frac{1}{4}$	1	9 $\frac{1}{4}$	
ANOTHER PIECE OF THE SAME WIRE.										
Fixed	0	0	0	0	2 $\frac{3}{4}$	0	4	0	3 $\frac{1}{4}$	
do.	16	15	16	2	4	3	5	2	4 $\frac{1}{2}$	
do.	22	19	22	2	7 $\frac{1}{2}$	3	10	2	8 $\frac{1}{10}$	

In attempting to increase these weights to 25, 26, and 27 lbs., the wire broke at a defective place.

EXPERIMENT No. 6.

Same Wire as in the preceding Experiment. Distance of the Props, 31 feet 6 inches.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflec- tion at B.	Deflec- tion at C.	Deflec- tion at D.	REMARKS.
Fixed	lbs. 22	lbs. 30	lbs. 22	ft. in. 0 11½	ft. in. 1 6	ft. in. 0 10½	
do.	28	30	28	1 1½	1 6½	1 0½	
do.	30	30	30	1 1½	1 6½	1 1½	
do.	30	35	30	1 1½	1 7½	1 1½	

Broke in attempting to add 4 lbs. more at B and D.

EXPERIMENT No. 7.

Distance of the Props, 140 feet ; Diameter, $\frac{1}{21}$ of an inch ; Weight of 140 feet, 14 ounces. Broke, vertically, with 157 lbs.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflec- tion at B.	Deflection at C.	Deflec- tion at D.	REMARKS.
lbs.	lbs.	lbs.	lbs.	ft. in.	ft. in.	ft. in.	
120	0	0	0	0 1½	0 1½	0 1½	
120	6	5	6	2 8	3 5 ⁸ / ₁₀	2 7½	
120	12	10	12	4 8 ³ / ₁₀	6 4½	4 7 ⁷ / ₁₀	
120	15	20	15	7 1½	10 0	7 0½	
132	15	20	15	6 3½	8 9½	6 4½	
132	21	25	21	8 8½	11 11	8 7	
150	21	25	21	7 11½	10 10	7 0	
150	25	25	25	8 3	10 11	8 2	Broke.

EXPERIMENT No. 8.

Same Wire as in the last Experiment. Distance of the Props, 31 feet 6 inches.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.		Deflection at C.		Deflection at D.		REMARKS.
	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
Fixed	0	0	0	0	5½	0	5½	0	4½	
do.	6	5	6	1	1½	1	4½	1	1½	
do.	12	10	12	1	4½	1	8	1	3½	
do.	16	15	16	1	6½	1	10½	1	4½	
do.	20	20	20	1	7½	2	1	1	6½	

Broke in attempting to add 2 lbs. at B, 4 lbs. at C, and 2 lbs. at D.

EXPERIMENT No. 9.

The same Wire as last Experiment, and the Props the same distance ; viz. 31 feet 6 inches.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.		Deflection at C.		Deflection at D.		REMARKS.
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
120	20	30	20	2	6	3	3½	2	2½	
120	25	30	20	2	9½	3	7	2	5	
120	31	34	31	3	5 ⁴ / ₁₀	4	4½	2	11½	
120	34	34	34	3	6½	4	5½	3	1½	
120	34	42	34	3	9½	4	11½	3	2½	
120	34	50	34	4	0	5	3½	3	4	
150	34	50	34	3	3 ⁶ / ₁₀	4	4½	2	9 ⁸ / ₁₀	
150	34	55	34	3	6½	4	8½	3	0	
150	37	55	37	3	9 ⁴ / ₁₀	5	0	3	2½	
150	37	56	37	3	9½	5	0	3	2½	
156	37	56	37	3	9½	5	0	3	2½	
160	39	57	39	3	9 ² / ₁₀	5	0 ² / ₁₀	3	2 ² / ₁₀	Broke in attempting to add 6 lbs. more.

Note.—The above experiments were made at the Patent Iron Cable Manufactory of Messrs. Brunton & Co.

EXPERIMENT No. 10.

Distance of the Props, 900 feet; Diameter of Wire, $\frac{1}{10}$ th inch; Weight of 900 feet, 28 lbs. by the steel-yard; Weight of 100 feet, 3 lbs. 3½ oz. by the scales. Mean vertical Strength, from 9 Experiments, 630 lbs.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Distance of C from the ground.		REMARKS.
	lbs.	lbs.	lbs.	ft.	in.	
Fixed	0	0	0	15	6	{ On account of the length of the wire the curvature was measured from the ground; which latter was about 22 feet from the horizontal line, between the props or points of suspension.
do.	28	14	28	4	0½	
do.	28	17	28	3	4	
do.	28	19	28	3	0	
do.	28	20	28	2	10	
do.	28	21	28	2	5½	{ Removed the weights and re-tightened the wire.
do.	28	22	28	2	4	
do.	0	0	0	16	8	
do.	28	0	28	9	1	Broke the wire; not at a joint.
do.	28	14	28	4	8	
do.	28	17	28	..		

This experiment was made at Ellesmere; the points of suspension were, at one end a building, at the other a tree.

151. The nine experiments from which the mean vertical strength of 630 lbs. was deduced, are as follow :

1st broke with	lbs. 616
2nd	„	616
3rd	„	620
4th	„	652
5th	„	616
6th	„	637
7th	„	616
8th	„	646
9th	„	651
									<hr/> 9)5670
Mean of 9 Experiments									<hr/> 630 lbs.

The wire broke in these experiments at joints or unsound places ; it may therefore be considered the minimum of strength.

The mean of twelve other experiments, on wires of the same diameter, but of different specimens, was 634 lbs.

Strength per square inch, 36 tons.

The following Table shows the strength of the different specimens reduced to square inches :

Experiment	Diam.	Strength per square inch	Tons.
1 . . .	$\frac{6}{70}$		35·7
„ 2 . . .	$\frac{1}{8}$	„	42·0
„ 3, 4 . . .	$\frac{1}{10}$	„	42·9
„ 5, 6 . . .	$\frac{6}{100}$	„	38·1
„ 7, 8, 9 . . .	$\frac{1}{11}$	„	35·8
„ 10 . . .	$\frac{1}{10}$	„	36·1
			<hr/> Mean 38·4

Considerable discrepance will be observed between the strength of the wire in experiments 3, 4, and 10, which are of the same diameter. Perhaps a mean strength of 36 tons for a wire of less than, or not exceeding $\frac{1}{10}$ th inch diameter, is all that can be depended upon.

EXPERIMENTS.

152. *On the Momentum which Wires stretched, as in the preceding Experiments, will bear before breaking.*

Experiment 1. A piece of wire, which bore vertically 277 lbs., was stretched between two props, 140 feet distant from each other, till the versed sine, or deflection in the centre, was only $4\frac{3}{4}$ inches.

A 5 lb. weight was then tied to a cord, and the other end fastened to the middle of the wire; the length of the cord between the weight and the wire was 10 feet 6 inches. The weight being now lifted up to the level of the wire, it was let fall and struck the ground, but without injuring the wire.

Shortened the cord to 7 feet 7 inches, and proceeded as above: it did not strike the ground, nor did it injure the wire.

With the same length of cord, and a 10 lb. weight instead of the 5 lb., proceeding in the same manner: struck the ground but did not break the wire.

But the same weight hung by a string 6 feet 7 inches, let fall as above, broke the wire at a joint.

Note.—The distance of the middle of the wire from the ground was 13 feet 6 inches.

By the laws of falling bodies, we have for the

1st momentum	$(8 \times \sqrt{10.5}) \times 5 = 129$
2nd „	$(8 \times \sqrt{7.58}) \times 5 = 110$
3rd „	$(8 \times \sqrt{7.58}) \times 10 = 220$
4th „	$(8 \times \sqrt{6.58}) \times 10 = 204$

As the last momentum is less than the preceding, we may infer that the wire was damaged in the third trial.

Experiment 2. Distance of the props, 31 feet 6 inches. Diameter of the wire, $\frac{1}{10}$ th inch. Stretched to within $\frac{1}{8}$ th of an inch of a straight line.

A 10 lb. weight was tied to the middle of the wire by a cord 7 feet 9 inches long: it was lifted up to the level of the wire, as in the last experiment, and then let fall; but it did not break the wire.

A 15 lb. weight was tied, and let fall in the same manner, without breaking the wire.

A 20 lb. weight was then tried. It did not break the wire.

A 25 lb. weight being let fall from the same height, *broke the wire.*

Here our four momenta are,

1st momentum	$(8 \times \sqrt{7.75}) \times 10 = 222.6$
2nd „	$(1 \times \sqrt{7.75}) \times 15 = 333.9$
3rd „	$(8 \times \sqrt{7.75}) \times 20 = 445.2$
4th „	$(8 \times \sqrt{7.75}) \times 25 = 556.5$

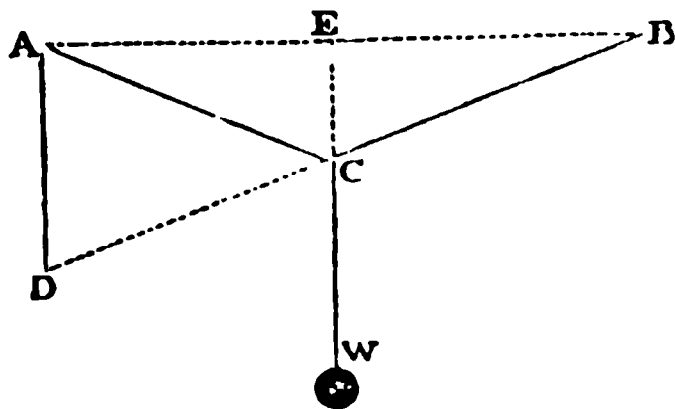
Comparing these momenta with the direct vertical strength, we have

1st vertical strength	. . .	277 lbs.	momentum	220
2nd ditto for wire of $\frac{1}{8}$ inch,		630 lbs.	ditto	556.5

that is, in the 1st experiment, the number expressing the momentum is less by $\frac{1}{5}$ th than the vertical strength; and in the 2nd by $\frac{1}{8}$ th: but it is probable that in the latter the wire would have been broken with a less weight than 25 lbs.

153. *Comparison of the preceding Experiments on extended Wires, with their Strengths computed theoretically.*

In experiment No. 2, page 278, it appears that a piece of wire, whose vertical strength was 531 lbs., being stretched on props 31.5 feet apart, and having a weight of 120.25 lbs. hung at its middle point, had that point deflected 1 foot $10\frac{3}{4}$ inches, and that it afterwards broke with the addition of 10 lbs. Let us endeavour to compute how much this 10 lbs. exceeded what was absolutely necessary to break the wire: or, which is the same, let there be given the distance of the props, the deflection, and the tension of the



wire, to find the weight which, suspended from its middle point, will produce the rupture.

Let A, B, in the preceding figure, represent the two fixed points; C E the deflection; A C B the wire: then it is obvious that the point C is kept in equilibrium by three forces; viz. A C, which denotes the tension of A C, or the equal tension of C B; and the unknown weight, W, *plus* half the weight of the wire, *w*. Now, when three forces, acting on a material point, preserve that point in equilibrium, each of the three forces is equal and directly opposed to the resultant of the other two. If, therefore, C B be produced to meet the vertical A D, D E will denote the resultant of the two forces, T, and $(W + w)$, representing by T the tension of A C: therefore, A D C will be the triangle of forces which keeps the point C in equilibrium; of which the side A D will denote the vertical force or weight, $W + w$; and by the nature of the construction $A D = 2 C E$; whence then we have as

$$A C : A D \text{ or } 2 C E :: T : W + w,$$

$$\text{or } W = \frac{2 C E \times T}{A C} - w.$$

Now, $C E = 1.8958$ feet, or $2 C E = 3.7916$.

Also, $A C = \sqrt{(A E^2 + E C^2)} = 15.86$.

And by the data of Experiment 1, $w = .29$ lbs.

$$\text{Whence } W = \frac{3.7916 \times 581}{15.86} - .29 = 126.65 \text{ lbs.}$$

This is about 4 lbs. less than the weight found by the experiment.

We may arrive at the same conclusion on principles a little different from the above, and somewhat more general; viz. since the weight W is kept in equilibrio by the tensions of AC and CB ; and since this weight, W *plus* w , the weight of the wire, is the only vertical force in the system, if we denote the tension of the wires AC and CB by T and T' , and the angles EAC , ECB , by a and a' , and resolve these two forces each into its component horizontal and vertical force; we must have the two former equal to each other, and the sum of the other two equal to the sum of the vertical weights, $W + w$; that is, we shall have

$$\begin{aligned} T \cos a &= T' \cos a' \\ T \sin a + T' \sin a' &= W + w; \end{aligned}$$

from which equations the two tensions, T and T' , may be determined, whatever may be the ratio of the two parts AC , CB ; but in our case, as these are equal, the first equation disappears, and the second becomes

$$2 T \sin a = W + w, \text{ or}$$

$$T = \frac{W + w}{2 \sin a}.$$

Or if T be given, and W required,

$$W = 2 T \sin a - w.$$

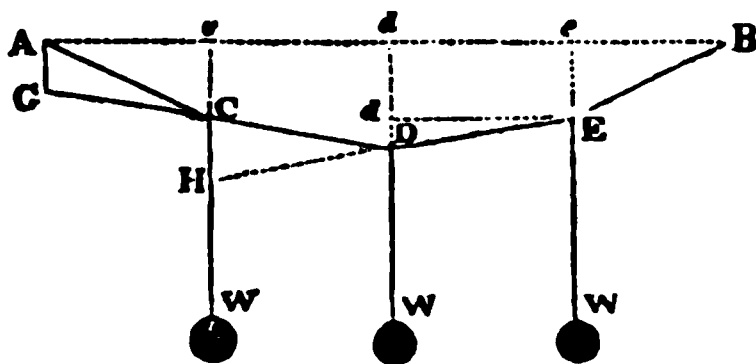
In the experiment above referred to,

$$T = 531, \sin a = .1195593, \text{ and } w = .29.$$

Whence

$$W = 2 \times 531 \times .1195593 - .29 = 126.65 \text{ lbs., as before.}$$

In a similar manner might be computed the tensions of the extreme points, when there are more than one weight, as in the third and subsequent experiments: but it will be, perhaps, more simple to begin here by computing the tensions of the two adjacent sides, CD and DE; which may be effected



precisely in the same manner as in the preceding case. For it is a principle in mechanics, that if a system of forces be in equilibrio, no alteration will take place in that state, by supposing any two or more of its points to become fixed: we may, therefore, suppose the points C and E fixed, and compute the tension of CD, or DE, exactly as above; viz. calling the angle DCE = α' , and the centre weight W' , and the tension t , we shall have

$$t \sin \alpha = \frac{1}{2} W' + \frac{1}{4} w,$$

$$t = \frac{W' + \frac{1}{2} w}{2 \sin \alpha'},$$

where w is the whole weight of the wire: then, having the tension t , the weight W , and the angle DCW, compute the value of the resultant of these two forces, which will obviously be the tension of AC; that is, if we denote this tension by T , we shall have

$$T = \sqrt{\frac{t^2 + W^2 + 2 \sin \alpha' t W.}{U}}$$

In experiment 3, $W' = 74$, $w = 2.5625$, and $\sin a = 0.6685$, whence

$$t = \frac{75.2812}{.1337} = 563 \text{ lbs.}$$

$$\text{And } T = \sqrt{563^2 + 77^2 + .1337 \times 563 \times 77} = 573.$$

This gives the tension too little: let us therefore compute the same from the 1st deflection; that is, by resolving T into two forces, the one horizontal, and the other vertical; and equating the latter with half the sum of the weights, *plus* half the weight of the wire; for as the whole system is retained in equilibrio by the two extreme tensions, the vertical component of each ought to be equal to half the entire vertical force, or half the whole weight. This consideration gives

$$T \sin a = \frac{1}{2} (W + W' + W'' + w),$$

where a denotes the angle $C A c$.

In the 3rd experiment,

$$a = 7^\circ 32' \text{ and } \sin a = .1311.$$

$$\text{Whence } T = \frac{230.56}{.2622} = 879.$$

If now we take the mean of our two results, we shall have

$$\frac{879 + 573}{2} = 726 \text{ lbs.}$$

Whereas the vertical strength, as determined from experiment, was 736 lbs.

The two different results given by the two methods, show that the system had assumed a form incon-

sistent with a perfect state of equilibrium, supposing the several lengths, or distances, Aa , cd , &c., to be equal: but it is obvious, that besides the probable unequal extensibility of the wire, that the point C , as the wire stretches, will approach towards A , and recede from the perpendicular; for D being exposed to equal actions on each side, will continue in the same vertical: this will obviously have a tendency to increase the angle a , and decrease the angle a' ; and, consequently, to increase the value of the tension computed according to the former method, and to diminish the same according to the latter, and therefore approximate them towards that medium result we have obtained above, which differs only 10 lbs. from what was found experimentally; viz. about 1 lb. out of 73 lbs.

In the 4th experiment,

$$a' = 2^\circ 51' \sin a' = .04893, \text{ and } \frac{1}{2} w = .39 \text{ lbs.}$$

$$t = \frac{76.39}{.09786} = 790 \text{ lbs.} \quad \text{And}$$

$$T = \sqrt{\{790^2 + 71^2 + .0944 \times 790 \times 71\}} = 797 \text{ lbs.}$$

According to the second principle, viz.

$$T \sin a = \frac{1}{2} (W + W' + W'' + w),$$

we have

$$W + W' + W'' + w = 218.79, \text{ and } \sin a = .12648;$$

$$\text{whence } \frac{218.79}{.25296} = 864 \text{ lbs.};$$

the mean of which is 831 lbs. instead of 736 lbs., which is in excess by about $\frac{1}{8}$ th part.

154. It will be observed, however, that these

methods are only approximative; but they are perhaps more intelligible to many readers than if we had entered upon the problem with all the generality that belongs to the doctrine of equilibrium of flexible bodies: but it may not be amiss to give a sketch of this general method, at least as applied to the action of vertical weights upon a perfectly flexible line.

Here we may suppose any number of weights $W, w, w', \&c., W'$; and a corresponding number of distances $L, l, l', l'', \&c., L'$, which may be equal or unequal: the tensions of these lines we may denote by

$$T, t, t', t'', \&c. T',$$

and their several angles, with reference to a horizontal axis Ax , passing through A , by

$$a, a, a', a'', \&c. a',$$

and their angles with reference to the other axis Ay ,

$$b, \beta, \beta', \beta'', \&c. b'.$$

Also, let n be the co-ordinate of the point B with reference to Ay , and m its co-ordinate as referred to $B y$.

Then if we resolve each of the tensions into its corresponding horizontal and vertical components, we shall have from the theory of equilibrium,

$$T \cos a + t \cos a + t' \cos a' + \&c. T' \cos a' = 0,$$

$$T \cos b + t \cos \beta + t' \cos \beta' + \&c. T' \cos b' = 0.$$

And by means of the co-ordinates,

$$L \cos a + l \cos a + l' \cos a' + \&c. L' \cos a' = n,$$

$$L \cos b + l \cos \beta + l' \cos \beta' + \&c. L' \cos b' = m,$$

and by the known property of cosines,

$$\begin{aligned}\cos^2 a + \cos^2 b &= 1, \\ \cos^2 a' + \cos^2 b' &= 1.\end{aligned}$$

From which six equations the six unknown quantities, viz. T , T' , $\cos a$, $\cos b$, $\cos a'$, $\cos b'$, may be determined; after having first computed t , t' , &c., and $\cos a$, $\cos a'$, &c., in functions of T , $\cos a$, and W , w , w' , &c., which may, in all cases, be effected on the general principle of the composition of forces; that is, taking t as the resultant of T and W , t' as the resultant of t and w , and so on.

The computations, however, if the number of weights be considerable, become extremely laborious, and difficult to execute: but if, as in the experiment, we limit the weights to three, and consider the two extreme ones equal to each other, and the points A and B as being situated in the same horizontal line; then, as the several tensions and angles from each extreme are equal, we may reduce the above equations to three; in which, however, we have still to compute $\cos a$ in functions of T , $\cos a$ and W ; on which account we prefer, in this case, retaining the six equations under the form,

$$\begin{aligned}T \cos a &= t \cos a, \\ T \cos b &= t \cos \beta + W, \\ l \cos a + l \cos a &= \frac{1}{2}n, \\ T \cos b &= \frac{1}{2}(W + w + W,) \\ \cos^2 a + \cos^2 b &= 1, \\ \cos^2 a + \cos^2 \beta &= 1.\end{aligned}$$

From which these several quantities may be determined, in functions of each other.

If we denote the less deflection c C by d , and the greater D d by $d+d'$, we shall have

$$\frac{d}{l} = \cos b, \text{ and } \frac{d}{l} = \cos \beta;$$

and substituting these in the first four equations, and denoting the entire weight of the system by π , we, shall have, after reduction,

$$T\sqrt{l^2-d^2}=t\sqrt{l^2-d'^2}$$

$$Td=td'+W$$

$$\sqrt{l^2-d}+\sqrt{l^2-d'^2}=\frac{1}{2}\pi$$

$$Td=\frac{1}{2}l\pi.$$

From which we may determine any one of these quantities in terms of the others: but it will be observed here, as in our partial solution, that if we suppose both deflections d and d' as known quantities, there will be a superfluity of data; viz. we shall have more equations than unknown quantities; and by assuming values for both these, we may give such as are inconsistent with the other data, and therefore also inconsistent with a state of perfect equilibrium: it is proper, therefore, in the solution of these equations, to include one of these quantities with the data, and one with the *quæsitæ* of the problem: in which case a rational solution will be obtained.

We shall not attempt the numerical solution of these equations; but the reader who is desirous of doing so will find no other difficulty than what belongs to the algebraical operations: we shall content ourselves with the approximative numbers as

above determined, considering it useless to expect a nearer approximation between theory (which is founded on a supposition of a perfect uniformity of matter, and the most accurate mode of action) and experiments, in which every kind of irregularity with regard to the composition of the material, and all the errors of fixing, observing, &c., are presented: indeed the agreement between the two deductions may be considered a confirmation of the correctness of the theory, and of the accuracy with which the experiments were performed; and on the basis of the two combined every confidence may be placed, as to computation, relative to works which from their magnitude bid defiance to any experiment, except that of their actual construction.

Calculation of the Strength of a Suspension Bridge, on the supposition of its forming a perfect Catenary Curve.

155. The foregoing experiments and computations, although they would probably have constituted the only data on which Mr. Telford would have proceeded in his proposed construction of the Runcorn bridge, yet they can only be considered as roughly approximative to the real case. And it must perhaps be admitted, that by assuming the bars to form a perfect catenary, we still only approximate. The approximation is however much more close in this case than in the former, and sufficiently so for all practical purposes.

The properties of the catenary are investigated in most treatises on mechanics ; we shall not, therefore, retrace steps which have been so often taken, but merely bring under one point of view these several relations ; referring such of our readers as may be desirous of actual investigations, to the several works in which they may be found, particularly to Poisson, ‘ *Traité de Mécanique*,’ whence the following have been selected :

Let l denote the length of the catenary ;

l' the distance of its points of suspension ;

c the angle between the tangent at the point of suspension, and the above horizontal line of distance ;

A the tension of the chain at the same point ;

T the tension at any other point ;

x any variable absciss ;

y the corresponding ordinate ;

s the corresponding arc ;

h the weight of an inch, or a foot, &c., of the chain x , y , s , &c., being taken in the same unit of measure.

This notation being established, the following are the principal properties of this curve ; viz.

$$1. \quad \frac{l'}{l} = \frac{\cos c}{\sin c} \text{hyp log } \frac{\cos c}{1 - \sin c}.$$

$$2. \quad \frac{A \sin c}{h} = \frac{1}{2} l, \text{ or } A = \frac{h l}{2 \sin c}.$$

$$3. \quad T = \sqrt{\{A^2 - 2 A h s \cdot \sin c + h^2 s^2\}}.$$

Which, at the lowest point, becomes

$$T = A \cos c.$$

$$4. \frac{1}{2} l = \frac{A \sin c}{h}, \text{ and } \frac{1}{2} l' = \frac{A \cos c}{h} \text{hyp log } \frac{\cos c}{1 - \sin c}.$$

$$5. x = \frac{A \cos c}{h} \text{hyp log } \left\{ \frac{A - hy \mp \sqrt{\{(A - hy)^2 - A^2 \cos^2 c\}}}{A (1 - \sin c)} \right\}.$$

$$6. y' = \frac{A (1 - \cos c)}{h}.$$

Where y' is the ordinate to the middle or lowest point of the curve :

$$7. s = \frac{A \sin c}{h} \pm \sqrt{\frac{\{(A - hy)^2 - A^2 \cos^2 c\}}{h}}.$$

These formulæ are not all necessary for the solution of our problem, but are given as embracing the principal properties of this curve.

156. Let us now suppose a bar of iron, which we must consider as flexible, to be fixed to two points of suspension, 1000 feet distant, the lowest point of the curve being $\frac{1}{20}$ th of the whole distance, or 50 feet; and let it be required to find the length of the bar and its action on the points of suspension, the weight h of one foot of it being given. In the present question, assuming the specific gravity of iron 7788, and the diameter of the bar $\sqrt{18}$ inches, we find $h = 48$ lbs.; also $l' = 1000$ feet; whence, by formula 6,

$$y' = \frac{A (1 - \cos c)}{h}, \text{ or}$$

$$A (1 - \cos c) = y' h = 48 = 50 = 2400.$$

And formula 4 gives

$$\frac{1}{2} l' = \frac{A \cos c}{h} \text{hyp log } \frac{\cos c}{1 - \sin c} = 500.$$

$$\frac{2400 \cos c}{48 (1 - \cos c)} \text{hyp log } \frac{\cos c}{1 - \sin c} = 500.$$

$$\text{Whence } \frac{\cos c}{10 (1 - \cos c)} \text{hyp log } \frac{\cos c}{1 - \sin c} = 1;$$

which, by approximation, gives angle $c = 11^\circ 15'$ nearly.

And hence, by formula 1, we find

$$l = 1008 \text{ feet} = \text{length of the catenary.}$$

Again, by formula 2,

$$A = \frac{h l}{2 \sin c} = \frac{1008 \times 48}{\cdot 39018} = 124005 \text{ lbs., or}$$

about 55 tons, the tension at the point of support.

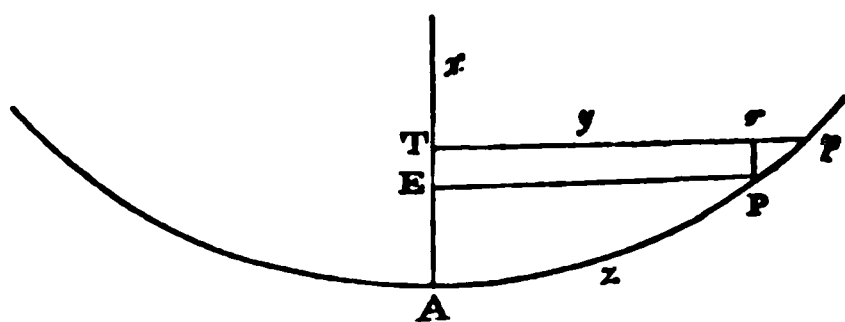
In a similar way, the tension and length being given, the depth of the curve may be computed.

It is not necessary, however, to have recourse to this mode of calculation, Mr. Davies Gilbert having, in an extremely ingenious paper, in the Phil. Trans. for 1826, supplied two Tables, by means of which, every circumstance connected with these kinds of calculation becomes merely a matter of tabular inspection, as explained in the following article.

Tables for Computing all the circumstances of Strain, Strength, &c., of Suspension Bridges. By Davies Gilbert, Esq., F.R.S. (Philosophical Transactions for 1826.)

157. It may be proper to premise that in our preceding experiments we have seen that the mean ultimate strength of malleable iron is 25 tons per square inch, which is equal in weight to a bar of the same dimensions whose length is 16,500 feet, which is sometimes called the modulus of the strength of iron, and is constant for bars of all dimensions. In like manner the strain or tension on a bar may be expressed by the number of feet in length of a bar of the same dimension.

In the following Table I., Mr. Gilbert uses the same method, but the unit instead of being a foot is $\frac{1}{100}$ th of the half distance of the points of support or of the ordinate of the semi-catenary; and the values of x , z , a and T , have also for their unit $\frac{1}{100}$ th



of the same length, x being the greatest depth of the curve, and z the length of the semi-catenary, a the modulus of tension at the lowest point of the curve, and T , the tension at the point of support. In Table II. the unit is $\frac{1}{100}$ th of the modulus of tension a in *feet* at the lowest point of the curve, and x, y, z and T , have also the same unit; x and z in this Table

being the length of each absciss with its corresponding arc, for every unit of y , reckoning from the lowest point of the curve.

Suppose, for example, the span of a proposed bridge were 800 feet, and its greatest deflection 50 feet. Here the unit or $\frac{1}{160}$ th of the half span is 4, and consequently the value of x in the Table is 12.5. Now, opposite 12.565, (which is the nearest tabular number,) we have the tension T at the point of support 412.56, and this, as the unit is 4 feet, is equal to 1650.24 feet of a bar of the same section, whence the tension in lbs. or tons becomes known. We find also a , the tension, at the lowest point 400, whence

$$400 \times 4 = 1600 \text{ the tension in feet.}$$

Since the tension is thus found to be 1600, the hundredth part of this is 16, which is therefore the unit of Table II., and the several values of x and z multiplied by 16, will be their corresponding values in feet for each unit of y . Thus the maximum and minimum tension of the bar, and the lengths of the several suspending bars, are determined, or rather perhaps their difference of length; for their absolute length will of course depend upon the depth of the platform below the lowest point of the curve. It has been seen that in the case here assumed the greatest tension is 1650 feet; whereas the ultimate strength is 16,500 feet; the bar is therefore stretched with only $\frac{1}{10}$ th of strain that would destroy it; and supposing the weight of the suspending bars, roadway, &c., to be $\frac{2}{3}$ rds of the weight of the bar, the strain would still be only 2750, or $\frac{1}{6}$ th of the full

power of the iron, that is, about $4\frac{1}{2}$ tons per inch, whereas we have seen that iron will bear 9 or 10 tons per inch without destroying its elasticity or power of restoration.

In the Menai Bridge the span is about 580 feet, and the unit, therefore, of our first Table 2·90; the greatest deflection is 43 feet, therefore $\frac{4\cdot3}{2\cdot9} = 14\cdot8 = x$ nearly, whence $T = 354\cdot8$, which, multiplied by 2·9, gives for the modulus of maximum tension 1028·9 feet from the weight of the bars alone. This weight, according to Mr. Provis's statement, is 394 tons, and the whole weight, including platform, &c., 643 tons: hence $394 : 643 :: 1028\cdot9 : 1680$, the whole strain.

The strength therefore here is nearly 10 times greater than the strain, independently of a passing load. Again, the tabular value of $a = 340$, and $340 \times 29 = 986$: then $394 : 643 :: 986 : 1610$, value of a , Table II.

Therefore, to find now the several abscisses or the length of the suspending bars for every division or hundredth part of the ordinate y , since the whole value of a is 1610, our unit (one hundredth of this) is 16·1, we have therefore only to multiply the several numbers in the column x by 16·1 for the lengths required.

For more on the subject the reader is referred to an excellent Memoir on Suspension Bridges, by Mr. Eaton Hodgkinson, vol. v. of the Manchester Memoirs. See also, Drewry on Suspension Bridges.

TABLE I.

Table for the Computation of Suspension Bridges.

G	S	S	T	ANGLE.	
2000	2.500	100.0	2002	87	8
1950	2.564	100.0	1952	87	3
1900	2.632	100.0	1902	86	59
1850	2.703	100.0	1852	86	54
1800	2.778	100.0	1802	86	49
1750	2.857	100.0	1752	86	43
1700	2.942	100.0	1702	86	37
1650	3.031	100.0	1653	86	31
1600	3.125	100.0	1603	86	25
1550	3.226	100.0	1553	86	18
1500	3.334	100.0	1503	86	10
1450	3.449	100.0	1453	86	3
1400	3.572	100.0	1403	85	54
1350	3.705	100.0	1353	85	45
1300	3.847	100.0	1303	85	35
1250	4.002	100.1	1254	85	55
1200	4.168	100.1	1204	85	13
1150	4.350	100.1	1154	85	1
1100	4.548	100.1	1104	84	47
1050	4.765	100.1	1054	84	33
1000	5.004	100.1	1005	84	16
980	5.106	100.1	9851	84	9
960	5.213	100.1	9652	84	2
940	5.324	100.1	9453	83	54
920	5.440	100.1	9254	83	47
900	5.561	100.2	9055	83	38
880	5.687	100.2	8856	83	30
860	5.820	100.2	8658	83	21
840	5.959	100.2	8459	83	11
820	6.105	100.2	8261	83	1
800	6.258	100.2	8062	82	51
780	6.418	100.2	7864	82	40
760	6.588	100.2	7665	82	28
740	6.767	100.3	7467	82	16
720	6.955	100.3	7269	82	4
700	7.154	100.3	7071	81	50
680	7.366	100.3	6873	81	36
660	7.590	100.3	6675	81	21
640	7.828	100.4	6478	81	5
620	8.081	100.4	6280	80	47
600	8.352	100.4	6083	80	29
580	8.642	100.4	5886	80	10
560	8.952	100.5	5689	79	49
540	9.283	100.5	5492	79	27
520	9.645	100.6	5296	79	2

TABLE I.—(CONTINUED.)

<i>a</i>	<i>x</i>	<i>z</i>	<i>T</i>	ANGLE.
500	10·03	100·6	5100	78° 36′
480	10·45	100·7	4904	78 8
460	10·91	100·7	4709	77 38
440	11·41	100·8	4514	77 5
420	11·96	100·9	4319	76 29
400	12·56	101·0	4125	75 49
380	13·23	101·1	3932	75 5
360	13·97	101·2	3739	74 17
340	14·81	101·4	3548	73 32
320	15·75	101·6	3357	72 22
300	16·82	101·8	3168	71 14
280	18·04	102·1	2980	69 57
260	19·46	102·4	2794	68 29
240	21·12	102·8	2611	66 47
220	33·11	103·4	2431	64 48
200	25·52	104·2	2255	62 28
180	28·55	105·3	2085	59 39
160	32·28	106·6	1922	56 19
140	37·25	108·7	1772	52 10
120	44·13	111·9	1641	46 58
100	54·30	117·5	1543	40 23
95	57·67	119·5	1526	38 28
90	61·51	121·8	1515	36 26
85	65·85	124·6	1508	34 17
80	71·07	128·1	1510	31 58
75	77·14	132·3	1521	29 32
70	88·43	137·6	1544	26 57

TABLE II.

Table for the Computation of Suspension Bridges.

<i>y</i>	<i>x</i>	<i>z</i>	<i>T</i>	ANGLE.
1	·0049	1·000	1000	9° 25′
2	·0200	2·000	1000	88 51
3	·0450	3·000	1000	88 16
4	·0800	4·000	1000	87 42
5	·1250	5·002	1001	87 8
6	·1800	6·003	1001	86 33
7	·2450	7·005	1002	85 59

TABLE II.—(CONTINUED.)

y	x	z	T	ANGLE.	
8	·3201	8·008	1003	85°	25′
9	·4052	9·012	1004	84	51
10	·5004	10·01	1005	84	16
11	·6056	11·02	1006	83	42
12	·7208	12·02	1007	83	8
13	·8461	13·03	1008	82	34
14	·9815	14·04	1009	82	0
15	1·127	15·05	1011	81	26
16	1·282	16·06	1012	80	52
17	1·448	17·08	1014	80	18
18	1·624	18·09	1016	79	44
19	1·810	19·11	1018	79	10
20	2·006	20·13	1020	78	36
21	2·213	21·15	1022	78	3
22	2·429	22·17	1024	77	29
23	2·656	23·20	1026	76	56
24	2·893	24·23	1028	76	22
25	3·141	25·26	1031	75	49
26	3·399	26·29	1033	75	16
27	3·667	27·32	1036	74	42
28	3·945	28·36	1039	74	9
29	4·234	29·40	1042	73	36
30	4·533	30·45	1045	73	3
31	4·843	31·49	1048	72	30
32	5·163	32·54	1051	71	58
33	5·494	33·60	1054	71	25
34	5·835	34·65	1058	70	53
35	6·187	35·71	1061	70	20
36	6·550	36·78	1065	69	48
37	6·923	37·84	1069	69	16
38	7·307	38·92	1073	68	44
39	7·701	39·99	1077	68	12
40	8·107	41·07	1081	67	40
41	8·523	42·15	1085	67	8
42	8·950	43·24	1089	66	36
43	9·388	43·33	1093	66	5
44	9·837	45·43	1098	65	33
45	10·29	46·53	1102	65	2
46	10·76	47·63	1107	64	31
47	11·24	48·74	1112	64	0
48	11·74	49·86	1117	63	29
49	12·24	50·98	1122	63	59
50	12·76	52·10	1127	62	28
51	13·28	53·23	1132	61	58
52	13·82	54·37	1138	61	27
53	14·37	55·51	1143	60	57
54	14·93	56·66	1149	60	27
55	15·51	57·81	1155	59	57

TABLE II.—(CONTINUED.)

y	x	z	T	ANGLE.	
56	16.09	58.97	1160	59°	28'
57	16.68	60.13	1166	58	58
58	17.29	61.30	1172	58	29
59	17.91	62.48	1179	58	0
60	18.54	63.66	1185	57	31
61	19.18	64.85	1191	57	2
62	19.84	66.04	1198	56	33
63	20.51	67.25	1205	56	4
64	21.18	68.45	1211	55	36
65	21.87	69.67	1218	55	7
66	22.58	70.89	1225	54	39
67	23.29	72.12	1232	54	11
68	24.02	73.36	1240	53	44
69	24.76	74.60	1247	53	16
70	25.51	75.85	1255	52	48
71	26.28	77.11	1262	52	21
72	27.05	78.38	1270	51	54
73	27.84	79.65	1278	51	27
74	28.65	80.94	1286	51	0
75	29.46	82.23	1294	50	34
76	30.29	83.53	1302	50	7
77	31.13	84.83	1311	49	41
78	31.99	86.15	1319	49	15
79	32.86	87.47	1328	48	49
80	33.74	88.81	1337	48	23
81	34.63	90.15	1346	47	57
82	35.54	91.50	1355	47	32
83	36.46	92.86	1364	47	7
84	37.40	94.23	1374	46	42
85	38.35	95.61	1383	46	17
86	39.31	96.99	1393	45	52
87	40.29	98.39	1402	45	27
88	41.28	99.80	1412	45	3
89	42.28	101.2	1422	44	39
90	43.30	102.6	1433	44	15
91	44.34	104.0	1443	43	51
92	45.39	105.5	1453	43	27
93	46.43	106.9	1464	43	4
94	47.53	108.4	1475	42	40
95	48.62	109.9	1486	42	17
96	49.72	111.4	1497	41	54
97	50.85	112.9	1508	41	31
98	51.98	114.4	1519	41	8
99	53.14	115.9	1531	40	46
100	54.30	117.5	1543	40	23

On the Transverse Strength, &c., of Malleable Iron.

158. It has been already observed, that iron has been only lately employed to any extent to resist a transverse strain; and writers, therefore, who have undertaken experiments to investigate the strength of materials, have hitherto passed over those inquiries which relate to the transverse strength of this metal.⁶ The extraordinary extent, however, to which malleable iron is now applied to resist transversely a passing load, renders it highly essential that this resistance, and its other properties, should be fully investigated; for it is obvious, that every additional weight of metal, beyond that which is requisite for perfect safety and durability, is not only uselessly, but injuriously employed,—it being generally admitted that bars beyond a certain weight cannot be so well nor so cheaply manufactured as those of less dimensions; and it is no less certain, that by a proper disposition of the metal in the sectional area

⁶ Some few experiments on the transverse strength of malleable iron have certainly been made. I have given three in my 'Essay on the Strength of Materials.' Mr. Hodgkinson has also glanced at this subject in his valuable paper of 'Experiments on Cast Iron,' published in the 'Memoirs of the Manchester Philosophical Society,' and M. Duleau has treated of the subject in his 'Essai Théorique et Experimental,' &c.; but those points of greatest importance connected with the application of this metal to the purposes of railways have never formed the subject of inquiry.

of the bar, (which depends on the data in question,) a greater strength may be obtained with a given weight of iron, than with a greater weight injudiciously disposed. Under these impressions, the following experiments have been undertaken, and to these inquiries they have been principally directed; but as there will be found references to some other matter connected with the practical application of malleable iron, &c., to railways, it may be well to state the circumstances under which the experiments were undertaken, in order to render some remarks and observations the more intelligible to the reader. These were as follow :

The Board of Directors of the London and Birmingham Railway Company, desirous of carrying on the great work in which they were engaged on the most scientific principles; and, if possible, to avoid the enormous cost of repairs which has attended some large works of a similar description; offered, by public advertisement, a prize of one hundred guineas “for the most improved construction of railway bars, chairs, and pedestals, and for the best manner of affixing and connecting the rail, chair, and block to each other, so as to avoid the defects which are felt more or less on all railways hitherto constructed;” stating, that their object was to obtain, with reference to the great momentum of the masses to be moved by locomotive steam engines on the railway,—

1. "The strongest and most economical form of rail.
2. "The best construction of chair.
3. "The best mode of connecting the rail and chair; and also the latter to the stone blocks or wooden sleepers. And that the railway bars were not to weigh less than fifty pounds per single lineal yard."

In consequence of this advertisement, a number of plans, models, and descriptions were deposited with the company within the time limited by the advertisement; and others were received afterwards, which, although not entitled to the prize, were still eligible to be considered with reference to their adoption for trial. On the 24th of December, 1834, a resolution was passed at a meeting of the Directors, appointing J. U. Rastrick, Esq., of Birmingham, N. Wood, Esq., of Newcastle, civil engineers, and myself, to examine and report upon the same, with a view to awarding the prize; and, at the same time, we were requested to recommend to the Directors such plans, whether entitled to the prize or not, as might be considered deserving of a trial. We met accordingly in London; and, after a long and careful examination of the several plans, drawings, and written descriptions, recommended those we thought entitled to the prize, which was awarded by the Directors accordingly. But that part of our instructions which required us to recommend one or

more rails for trial, we were unable to fulfil to our satisfaction, principally for want of data to determine which of the proposed rails would be strongest and stiffest under the passing load, and whether permanently fixing the rail to the chair, for which there were several plans, would be safe in practice; and as no experiments on malleable iron had ever been made, bearing on these points, it was considered better to leave the question unanswered, than to recommend, on no better ground than mere opinion, an expensive trial, which might ultimately prove a failure.

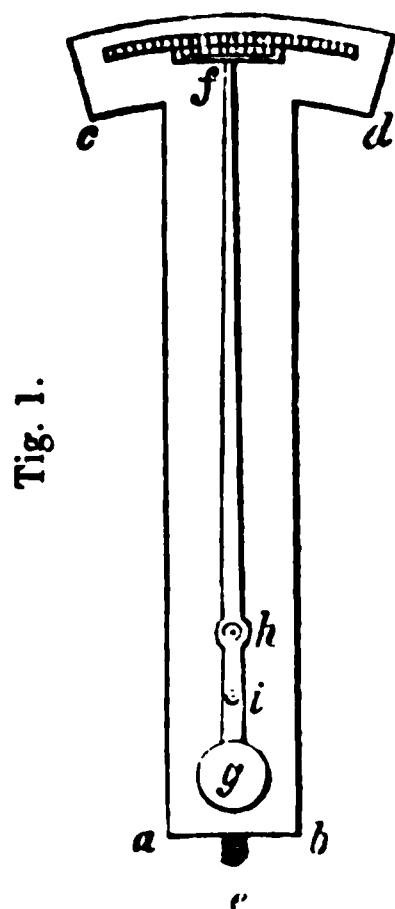
Seeing, however, how desirable it was that such data should be obtained, I proposed to the Directors to undertake a course of experiments, which should be conducted on a scale adequate to the importance of the subject, provided my Lords Commissioners of the Admiralty would allow me the conveniences His Majesty's Dockyard at Woolwich afforded, (which I had every reason to hope they would do, from the liberality I had so frequently experienced from that Board on similar occasions,) and that the Directors would supply such instruments, material, and workmanship, as might be required for the purpose.

The Admiralty, as I had anticipated, immediately granted my request; and, at a public meeting of the proprietors, held at Birmingham, a resolution was passed embodying my proposition. I accordingly commenced, and continued my experiments,

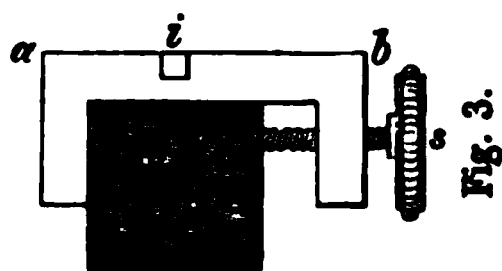
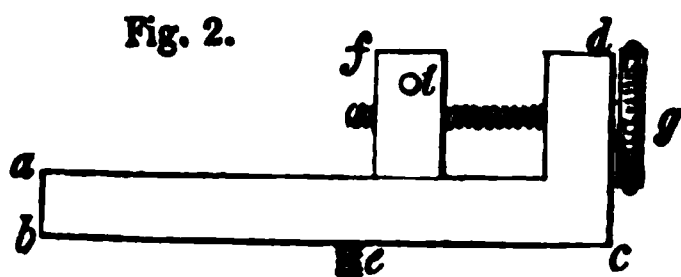
till I had elicited such facts as I thought necessary ; and having arranged them, I delivered the results, with a report founded upon them, to the secretary of the London Committee, to lay them before the Board; which being done, the Directors were pleased to express their approbation of my labours, and their wish that their results should be made public. They were accordingly printed and very generally circulated, nearly in the form in which they are given in the following pages ; such experiments, however, being added, as I have since made for other railway companies, and such remarks and observations as have arisen out of a more extended examination of the subject.

Experiments to determine the quantity which Iron extends under different degrees of Tension.

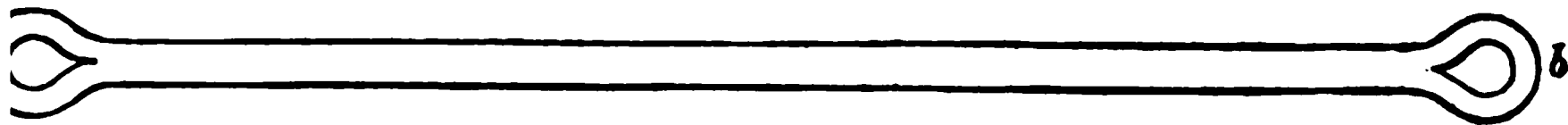
159. With a view to this inquiry, an instrument was made as in the annexed sketch.—*a b c d* is a piece of brass, about one-fifth of an inch thick, having an arc at top, divided into tenths of inches ; *h f g* is a hand, with a vernier, turning freely on a centre *h* ; and *i* is a steel pin, about half an inch long, projecting perpendicularly forward; the distances *f h* to *h i* being as 10 to 1.



e is a small end with a screw, for the purpose described below; *abcd* is another piece of brass, having a screw *e*; *f* is a piece working in a dovetail, adjustable for position by the screw *g*, and *i* is another steel pin projecting forward. *ab*, fig. 3, is an iron saddle-piece, with a set screw *s*; and at *i* a hole is tapped to receive the screw *e*, fig. 2; and another saddle-piece, exactly like this, is made to receive the screw *e*, of fig. 1.



The iron bars intended to be experimented on were made of the annexed form,⁶ about ten feet in length;



these, by proper bolts and shackles, were fixed at *a* and *b* in the proving machine already described (Art. 141); the two saddle-pieces were then fixed on at the exact distance of 100 inches; the instruments, figs. 1 and 2, screwed into their respective saddle-pieces, and a light deal rod hung, by means of two small holes formed in it, (also at the distance of 100 inches,) upon the two pins *ii*; and then by means of the set screw, fig. 2, the vernier of fig. 1 was adjusted exactly to zero. The pump of the press was now put in action, and after one, two, or more tons' pressure were on, according to the size of

⁶ Mr. Kingston's nippers (described in Art. 147) were not made when these first experiments were carrying on.

the bar, and every thing brought well to its bearing, the hand was again adjusted to zero, after which the index was read for every additional ton. Here it will be seen, that whatever the bar stretched between the two instruments, the lower pin of fig. 1 was drawn forward, and the index end thrown back ten times that amount, consequently to ten times the actual amount of the quantity stretched.

It has been observed, that after one, two, or more tons' strain were applied, to bring every thing well to its bearing, the index was adjusted to zero, and its reading afterwards carefully registered as each additional ton was added. The strain during the experiment was repeatedly let off, and the index was found to return to zero, till the strain amounted to about nine or ten tons per inch, when the stretching became greater for each ton, and the bar did not any longer retain its original length when the strain was removed, its elasticity with this tension being obviously injured.

These experiments required more attendance than it was possible for one person to give; the adjustment of the weights, the reading and registering the index, required each the undivided attention of one individual; the pumping also required to be watched with care. And I have great pleasure in acknowledging the ready assistance I received from Messrs. Lloyd and Kingston, the engineers of the yard; from Mr. P. W. Barlow, civil engineer; as also from Lieutenant Lecount, who came from Birmingham to witness and assist in the experiment.

EXPERIMENTS.

160. *On the Longitudinal Extension of Malleable Iron Bars, under different degrees of direct Tension.*

TABLE I.

Bar No. 1, 1 inch square. February 21st.			Bar No. 2, 1 inch square. February 21st.		
Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.	Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.
2	zero		2	zero	
3	·0625	·0000625	3½	·11	·0000733
4	·156	·0000935	4	·15	·0000800
5	·265	·0001090	5	·24	·0000900
6	·375	·0001100	6	·35	·0001100
7	not observed.	mean.	7	·44	·0000900
8	·562	·0000935	8	·52	·0000800
9	not observed.	mean.	9	·62	·0001000
10	·750	·0000940	10	·70	·0000800
11	·875	·0001250	11	·81	·0001100
			12	1·13	{ Elasticity injured. }
Bar No. 3, 1 inch diameter. February 23rd.			Bar No. 4, 1 inch diameter. February 23rd.		
Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.	Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.
1	zero		1	zero	
2	·16	·0001600	2	·15	·0001500
3	·31	·0001500	3	·28	·0001300
4	·44	·0001300	4	·42	·0001400
5	·56	·0001200	5	·56	·0001400
6	·67	·0001100	6	·69	·0001300
7	·79	·0001200	7	·79	·0001000
8	·91	·0001200	8	·97	·0000800
9	·103	·0001200	9	·116	{ Elasticity destroyed }
<p>Mean extension per ton, per square inch.</p> <p>Bar No. 1. ·0000982</p> <p>No. 2. ·0000903</p> <p>No. 3. ·0001010</p> <p>No. 4. ·0000976</p> <p>Mean of the four . . ·0000967</p>					

TABLE II.

Bar No. 5, 2 inches square. February 28th.			Bar No. 6, 2 inches square. February 28th.			Bar No. 7, 3 inches square. March 7th.		
Weight in tons.	Index readings.	Parts of the whole bar extended by each 4 tons.	Weight in tons.	Index readings.	Parts of the whole bar extended by each 4 tons.	Weight in tons.	Index readings.	Parts of the whole bar extended by each 4 tons.
4	zero		4	zero		4	zero	
6	100		6	100		6	065	
8	180	000120	8	150	000150	8	110	000125
10	240	000140	10	210	000120	10	175	000110
12	290	000110	12	270	000100	12	230	000090
14	350	000110	14	290	000090	14	280	000050
16	400	000110	16	335	000085	16	335	000050
18	450	000110	18	375	000080	18	385	000050
20	500	000100	20	410	000075	20	440	000050
22	550	000100	22	445	000070	22	490	000045
24	600	000100	24	485	000075	24	530	000045
26	650	000100	26	525	000080	26	575	000045
28	695	000095	28	565	000080	28	625	000045
30	740	000090	30	620	000095	30	670	000045
32	790	000090	32	660	000095	32	715	000090
34	825	000090	34	730	000110	34	755	000085
36	870	000075	36		{ Fall elasticity. }	36	805	000090
38	920	000095	38			38	845	000090
40	105	000115 { Elasticity exceeded. }	40			40	900	000095 { Elasticity perfect. }
Mean extension per ton, per square inch.								
						No. 5.	0001082	
						No. 6.	0000957	
						No. 7.	0000841	
						Mean	0000946	
						Mean of preceding Table	0000967	

Collecting the results of these seven experiments, and reducing them all to square inch sections, we find that the strain which was just sufficient to balance the elasticity of the iron, was in—

Bar No. 1, re-manufactured iron,	10 tons.
„ 2, ditto,	11 „
„ 3, New Bolt,	11 „
„ 4, ditto,	10 „
„ 5, re-manufactured,	9·5 „
„ 6, ditto, from old furnace bars, . .	8·25 „
„ 7, New bar, by Messrs. Gordon . .	10 „

We may consider, therefore, that the elastic power of good medium iron is equal to about ten tons per inch, and that this force varies from ten to eight tons in indifferent and bad iron. It appears, also, (considering $\cdot 000096$ as representing in round numbers $\frac{1}{10000}$ th,) that a bar of iron is extended one ten-thousandth part of its length by every ton of direct strain per square inch of its section; and consequently, that its elasticity will be fully excited when stretched to the amount of one thousandth part of its length.

Remarks on the foregoing Experiments.

161. These results have an important bearing on the question of railway bars. We shall see, in the following section, how they become applicable to the investigation of the transverse strain; but, at

present, I shall only speak of them as they apply to the fixing of the rail to the chair. Amongst the numerous models which the Directors did Messrs. Rastrick, Wood, and myself, the honour to submit to our inspection, for the purpose of awarding their prize, there were several in which it was intended to fix the rail permanently to the chair—a very desirable object, if it could have been safely adopted; and it was the want of data to enable us to decide on this point, which first led me to propose this course of experiments. The question is now satisfactorily answered. We have seen that, with about ten tons per inch, a bar of iron is stretched $\frac{1}{10000}$ th part of its length, and its elasticity wholly excited or surpassed. Again, admitting 76° to be the extreme range of the thermometer in this country, between summer and winter, it appears, from the very accurate experiments of Professor Daniell,⁷ that a bar of malleable iron will contract with this change $\frac{1}{5000}$ th part of its length. And hence it follows that if the rails were permanently fixed to the chair in the summer, the contraction in the winter would bring a strain of five tons per inch upon the bar, and a strain of twenty-five tons upon the chair, (the bar being supposed of five-inch section,) thereby deducting from the iron more than, or full half, its strength, and submitting the chair to a strain very likely to destroy it. Every

⁷ See Philosophical Transactions, 1831.

proposition, therefore, for permanently attaching the rail to the chair is wholly inadmissible.

These remarks may also be carried farther, for if it be dangerous to attach the rail *directly* to the chair, it must be bad in practice to affix it *indirectly* by wedges, cotters, or otherwise, beyond what is absolutely essential to give it steadiness under the passing load; for it is evident, that if by these means we could prevent any motion taking place, we should fall into the same evil as by the permanent attachment; and if, as most probably will happen, we fail of entirely accomplishing this, still all the friction which is produced must be overcome by the contracting force of the iron, and be so much strength deducted from its natural resisting power.

The problem, therefore, which engineers have to solve is, "To find a mode of fixing the rail to the chair, which shall give sufficient steadiness to the former; but which, at the same time, shall produce the least possible resistance to the natural expansion and contraction of the bar."

The quantity of motion which thus takes place is certainly but small, viz. about $\frac{1}{16}$ th of an inch between summer and winter, with a fifteen-foot bar; but the force of contraction is great, amounting to five tons per sectional inch for the annual extremes, and frequently to not less than two and a half tons, between the noon and night of our summer season, while the whole power of iron within the limits of its elasticity does not exceed nine or ten tons.

This is an important consideration, and for want of attention to it, or rather in consequence of its amount not having been ascertained, a practice of wedging or fixing the rails has prevailed, which must necessarily have been the cause of great destruction to the bars.

I would also state here, a suggestion by Mr. Woodhouse, one of the candidates for the prize, as a matter deserving the attention of practical men, that as the bar must necessarily contract, it will draw from that side which is least firmly fixed, and hence all the shortening will most probably be exhibited at one end, however slight the hold on either may be; and when it happens that the adjacent ends of two bars both yield, the space between the two is rendered double that which is necessary. To avoid this evil, one of the two middle chairs in each bar might be permanently attached to the rail, in which case the contraction must necessarily be made from each end, and the space occasioned by the shortening of the bars would then be uniform throughout, and much unnecessary and injurious concussion thus saved both to the rail and to the carriage.

Experiments to determine the comparative Resistance of Malleable Iron to Extension and Compression, and the position of the neutral axis in bars submitted to a transverse strain.

162. It has been already demonstrated, (Art. 46,) that if the length of a bar of any kind, supported at both ends and loaded in the middle, be denoted by l , the depth by d , the depth of tension and compression by d' and d'' , the tension per square inch by t , and the weight by w , then will

$$\frac{1}{2} (d' + d'') d' t = \frac{1}{2} l w, \text{ or}$$

$$\frac{1}{2} d \cdot d' t = \frac{1}{2} l w;$$

d being the whole depth, and d' the depth of tension: whence, for any given breadth a ,

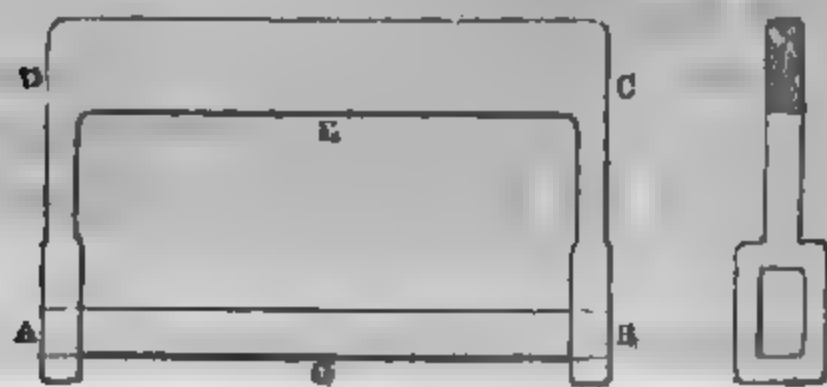
$$d' = \frac{3 l w}{4 d a t} = \text{depth of tension, and}$$

$$d'' = d - d' \text{ the depth of compression;}$$

consequently, $\frac{d'}{d - d'}$ the ratio, in which the neutral axis divides the sectional area in rectangular bars.

163. In order to submit this formula to practical results, a strong iron frame was forged, of the form shown in the figure, p. 320: D C is 36 inches long, 6 inches broad, by 2 deep; the two arms 2 inches square, and the ends of proportional dimensions to

those represented. The other view of the apparatus is shown in the side figure, with an opening 6 inches by 3, in which the bars for experiment were placed, as represented by AGB; the space between is 33 inches. The shackles were applied at E and G, and connected by strong iron cables to the press; the strain was then brought on, and the results recorded.



In order to measure with every requisite accuracy, the deflections which the bar sustained,



as different weights were applied, an instrument of the form shown in the annexed figure was neatly and accurately made in iron, having two feet, AD, BC; the centre was tapped to receive the brass screw, HS, of twenty threads to the inch, and the head was divided into five equal parts, and by again subdividing these divisions into ten, a deflection of $\frac{1}{1000}$ of an inch might be measured with great ease.

The method of applying it was to rest its feet on the bar, and then to retain it in its place by cramps and screws. The micrometer screw was then run down till it was in contact with the bar, and the divisions read and registered, either before any strain was on, or when the first slightest strain could be estimated, as stated in the following Table.⁸

The first six experiments were made on different parts of the bars, Nos. 5, 6, and 7, without cutting them, by introducing them into the iron frame above described (having 33 inches clear bearing) and straining them till the successive deflections showed a tendency to increase in amount, which was taken as a sign of the elasticity being injured; and the amount of this strain having been previously ascertained by the former experiments, they furnish the best possible data to apply to the formula for determining the position of the neutral axis.

⁸ As the numbers in the second column of the following Table have been misunderstood by a reviewer of my Report, it may be well to observe, that the reader must not understand them to be actual deflections, as it was quite accidental what the index read at the commencement. The actual deflections are given in the adjacent column.

Experiments made to ascertain the Deflections due to different Transverse Strains, and the Weight which first produces a Strain equal to the Elastic Power, and thence the position of the Neutral Axis.

TABLE III.

PART 1. Bar No. 5. Bearing 33 inches. 2 inches square.			PART 2. Bar No. 5. Bearing 33 inches. 2 inches square.		
Weight in tons.	Readings by scale.*	Deflections for each half ton.	Weight in tons.	Readings by scale.	Deflections for each half ton.
No weight.	1.96		No weight.	1.96	
.875	1.92	.023	.750	1.92	.020
1.00	1.90		1.00	1.91	.020
1.50	1.90	.016	1.50	1.89	.020
2.00	1.88	.020	2.00	1.86	.030
2.50	1.86	.020	2.50	1.84	.020
Weight removed.	} returned to 1.96		Weight removed.	} returned to 1.95	
3.00	1.80		3.00	1.67	
Weight removed.	} 1.88	Elasticity injured.	Weight removed.	} 1.81	Elasticity injured.
PART 1. Bar No. 6.			PART 2. Bar No. 6.		
Weight in tons.	Readings by scale.	Deflections for each half ton.	Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.			No weight.	.025	
.50	1.56?		.50	.043	.018
1.0	1.50		1.0	.068	.025
1.5	1.48	.020	1.5	.091	.023
2.0	1.45	.030	2.0	.128	.037 inj ^d .
2.5	1.24	.210	2.25	.178	.100
3.0		} Elast. inj ^d .	2.50	.313	.185

* In the first of these experiments the deflections were measured by a scale in front of the bar, the micrometer screw not being ready.

TABLE III.—CONTINUED.)

PART 1. Bar No. 7.			PART 2. Bar No. 7.		
Weight in tons.	Readings by micro. screw.	Deflections for each half ton.	Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.	·031		No weight.	·025	
·50	·053	·022	·50	·056	·031
1·0	·077	·024	1·0	·077	·021
1·5	·096	·019	1·5	·098	·021
2·0	·126	·030	2·0	·109	·011
2·5	·147	·021	2·5	·137	·028 inj ^d .
3·0	·211	·064 inj ^d .	3·0	·180	
PART 3. Bar No. 7.			PART 2. Bar No. 7. Reversed.		
Weight in tons.	Readings by micro. screw.	Deflections for each half ton.	Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.	·075 ?		No weight.	·025	
·50	·130		·50	·054	·029
1·0	·153	·023	1·0	·092	·038
1·5		·023	1·5	·153	·061
2·0	·199	·023	2·0	·235	·082
2·5	·220	·021	Elasticity clearly injured by the former experiment.		
3·0	2·90	·070 inj ^d .			

It appears from these experiments, that both parts of the bar No. 5, (whose direct elasticity was 9·5 tons,) had their restoring power just preserved with a transverse strain of two and a half tons on a bearing length of 33 inches. Hence in the formula

$$d' = \frac{3lw}{4d\alpha t}$$
 we have $l=33$, $w=2\frac{1}{2}$, $d=2$, $\alpha=2$, $t=9\cdot5$,

and $d'=1\cdot62$ inches, depth of tension.

Consequently $d'=.38$ inches, depth of compression, and the ratio of the area of compression to tension 1 : 4·3

In the first part of bar No. 6, w is not quite 2 tons, and $t=8\cdot5$ tons; and hence the ratio 1 : 2·7

In the second part of the same bar, ditto 1 : 2·7

In the first, second, and third parts of bar No. 7, $w=2\frac{1}{2}$ tons, and $t=10$ tons . 1 : 3·4

As far as these experiments are authority, therefore, the neutral axis divides the sectional area of a rectangular bar in about the ratio of one to three and a half.

In the following experiments, the iron was all supplied by Messrs. Gordon, and was of the same quality as the bar No. 7,—its elasticity may therefore be taken as ten tons, but it was not determined by testing, as in the preceding experiments.

TABLE IV.

BAR No. 8.

Distance of bearing.	Breadth.	Depth.	Weights.	Deflections	Deflections each half ton.	REMARKS.		
inches. 33	inches. 1.9	inches. 2	tons. -125 -250 -500 1.00 1.50 2.00 2.25 2.50 2.75	-034 -046 -060 missed. -098 -120 -134 -151 -176	-019 -019 -022 -028 -034 -044	} Mean -024 w = 2.25. Neutral axis 1 : 3.4 Elasticity injured with 2.50 T.		
BAR No. 9.								
33	1.9	2	-250 -500 1.00 1.50 2.00 2.25 2.50 2.75 3.00	-047 -055 -077 -097 -123 -132 -145 -164 -210	-016 -022 -020 -026 -018 -026 -092		} Mean -021 w = 2.25. Neutral axis 1 : 3.4 Elasticity injured with 2.50 Ditto destroyed with 3.00	
BAR No. 10.								
33	1.9	2	missed. 1.00 1.50 2.00 2.50 3.00	-056 -076 -095 -111 -151	-020 -019 -029 -027			} Mean -024 w = 2.5. Neutral axis 1 : 4.2

Deductions from the last three Experiments, confirmed by direct Observation of the place of the Neutral Axis.

164. These experiments, like the former, imply, according to the formula, that the neutral axis lies at about one-fourth or one-fifth of the depth of the bar from its upper surface; but a method was adopted in these to discover, if possible, its position mechanically. With this view, a key-way, or groove, was cut in the side of the bar, one inch broad and one-tenth of an inch deep,—thus reducing the breadth to 1·9 inch. To this key-way, or groove, was fitted a steel key, which might be moved easily; and when the strain was on, the key was introduced, which it was expected would be stopped at the point where the compression commenced, and this was accordingly found to be the case in two out of the three bars, but not in the third, the fitting not being sufficiently accurate. The other two, however, showed obviously a contraction of the groove, at about half an inch from the top, agreeing with the preceding computations. To make the results more certain, three other bars, exactly like the former, had deeper grooves cut, and the key more exactly fitted, and with these the results were as definite as could be desired. The key, as above stated, moved smoothly and easily before the experiment; but when two tons' strain were on, and the

key applied, it was stopped, and stuck at a definite point. The strain being then relieved, the key fell out by its own weight: the strain was again put on, the key sticking as before; the strain being relieved, the key again fell, and so on, as often as repeated. Precisely the same happened with all the three bars. One of them was then reversed, so that the part which had been compressed was now extended, and exactly the same result followed; showing, most satisfactorily, that our former computed situation of the neutral axis was very approximative: the measurements obtained in these experiments being tension 1·6, compression ·4, giving exactly the ratio of 1 to 4 in rectangular bars. These results seem the most positive of any hitherto obtained: still there can be little doubt this ratio varies in iron of different qualities; but looking to the preceding experiments, it is probably always between 1 to 3, and 1 to 5 in rectangular bars.

On the Stiffness of Rectangular Iron Bars, and their Deflections under different Weights.

165. Although it is necessary to know the actual resisting power of bars in their ultimate state of strain, in order to determine the relative strengths of differently shaped bars, yet the question of most practical importance is, the stiffness they exhibit when loaded with smaller weights; for we ought

never to strain a bar so nearly to its full power of bearing, as to make the ultimate strength the immediate subject of inquiry.

The experiments recorded in the last section are applicable to this purpose ; but as these are all of the same depth, it was thought more satisfactory to make a few other experiments on bars of different breadths and depths. These are given in the following page. They were performed precisely like the last, and therefore require no particular description.

EXPERIMENTS

On the Deflection of Malleable Iron Bars, under different Strains.

BAR No. 11.

Distance of bearing.	Breadth.	Depth.	Weight.	Deflections.	Deflections for each half ton.	REMARKS.
inches. 33	inches. 1·5	inches. 3	tons. ·125 ·500 1·00 1·50 2·00 2·50 3·00 3·50 4·00 4·50	·043 ·059 ·074 ·083 ·095 ·101 ·109 ·120 ·131 ·148	 ·015 ·009 ·012 ·006 ·008 ·011 ·011 ·017	 } Mean ·0103 w=4½. Neutral axis 1 : 4·9 Elasticity preserved at 4½ tons.

BAR No. 12.

33	1·5	3	0 ·50 1·00 1·50 2·00 2·50 3·00 3·50 4·00 4·50	0 ·017 ·037? ·052 ·061 ·064 ·078 ·089 ·102 ·124	 ·015 ·009 ·003 ·014 ·011 ·013 ·022	 } Mean ·0108 w=4½. Neutral axis 1 : 4·9 Elasticity injured.
----	-----	---	--	--	--	---

BAR No. 13.

33	1·5	2·5	0 ·50 1·00 1·50 2·00 2·50 3·00 3·50 7·5	·006 ·003 ·050 ·060 ·074 ·093 ·110 ·149 Bent 8 inches.	·024 ·020 ·010 ·015 ·019 ·017	 } Mean ·0173 w=3. Neutral axis 1 : 4·9 Elasticity preserved, 3 tons.
----	-----	-----	---	--	--	--

To obtain the law of deflection from these results, we may have recourse to two well-known and well-established formulæ (Arts. 28 and 66), viz.

$$\frac{l w}{4 a d^3} = S, \text{ and } \frac{P w}{32 a d^3 \delta} = E,$$

which are both constant quantities for the same material, w being the greatest weight the bar will bear without injuring the elasticity; consequently, when l is also the same in both, $d \delta$ will be also constant, a being the breadth, d the depth, and δ the deflection. That is, all rectangular bars having the same bearing length, and loaded in their centre to the full extent of their elastic power, will be so deflected, that their deflection (δ) being multiplied by their depth (d), the product will be a constant quantity, whatever may be their breadths or other dimensions, provided their lengths are the same.

Let us see how nearly our several results agree with this condition.

In the several bars, Nos. 8, 9, 10, 11, 12, 13, multiplying the mean deflection for each half ton, by the number of half tons which excited its whole elasticity, and this again by the depth of the bar, we find

	Depth.
No. 8, ultimate deflection	$\cdot 108 \times 2 = \cdot 2160$
No. 9	$\cdot 094 \times 2 = \cdot 1880$
No. 10	$\cdot 120 \times 2 = \cdot 2400$
No. 11	$\cdot 0876 \times 3 = \cdot 2628$
No. 12	$\cdot 0918 \times 3 = \cdot 2754$
No. 13	$\cdot 1038 \times 2\frac{1}{2} = \cdot 2595$
	<hr/>
	6)1·4417
	<hr/>
Mean	$\cdot 2403$

There is rather a large discrepancy in bar No. 9; the others are as approximative to the mean as can be expected in such cases.

If we make the same trial on the three parts of bar No. 7, we have

1st part	$\cdot 116 \times 2 = \cdot 2320$
2nd part	$\cdot 105 \times 2 = \cdot 2100$
3rd part	$\cdot 115 \times 2 = \cdot 2300$
	<hr/>
	3)·6720
	<hr/>
Mean	$\cdot 2240$
Former mean	$\cdot 2403$
	<hr/>
	2)·4647
	<hr/>
General mean	$\cdot 2323$

We may therefore say, that any malleable iron bar, of 33 inches' bearing, being strained to its full elasticity, will be so deflected, that its depth, multiplied by the deflection due to 30 inches, will produce the decimal $\cdot 23$; consequently $\frac{\cdot 23}{d} =$ the deflection, d being the whole depth in inches.

In this form, however, it applies only to rectangular bars. To make it general, we must estimate it from the neutral axis, which, in rectangular bars, being $\frac{1}{4}$ th of the depth below the upper surface, the above constant, when thus referred, becomes $\cdot 2323 \times \frac{1}{4} = \cdot 1857$. But, on the other hand, our instrument for measuring the deflection was but 30 inches long; it has therefore to be increased again in the ratio $30^3 : 33^3$, or as $10^3 : 11^3$ on this account; so that, ultimately, the formula is $d'' \delta = \cdot 22$, d'' denoting the depth of the bar below the neutral axis; and in this form it is general for parallel rails of any section whatever.

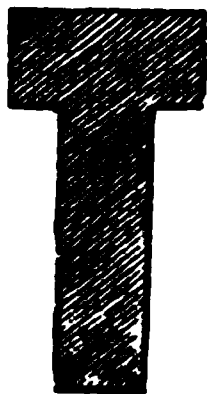
A curious circumstance was observed in these experiments, which, although it has no immediate bearing on the subject in question, it may be well to notice, and which is, I apprehend, characteristic of good malleable iron, viz. that the resistance to compression, although so much greater than the resistance to extension, is the first of the two which loses its restoring power; for if we so far increased the strain as to overcome the elastic power, the point of compression descended to nearly the middle of the depth, proving that the tensile force, although so much less, is the most tenacious.

Comparative Strength of Parallel Rails of equal Sections but of different Figures.

166. Various figures have been proposed for the transverse section of railway bars, some with a view to a more convenient and efficient mode of fixing the rail to the chair, and others with a view to greater strength : at present the question is the comparative strength of different sections ; the best form for fixing will be considered afterwards.

It would be useless in this inquiry to go generally into the section of greatest strength, as we must have regard to certain conditions and limitations ; we must, for example, necessarily have a head to the rail of a certain breadth and not less than a certain depth. The depth of the whole rail must also be confined within certain limits. We shall not, therefore, treat this question generally, but only as applicable to practical sections, which may be stated to be comprised under the four following forms :—

1. The plain T shaped rail, fig. 1.



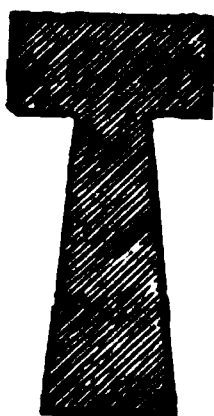
2. The H or double T formed rail, with a lower table, as fig. 2.



3. The inverted T rail, having a broad lower table, as fig. 3.



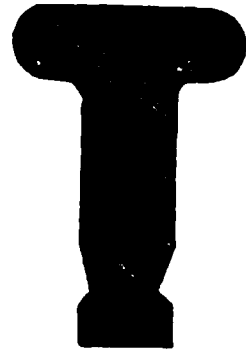
4. The trapezoidal rail, as fig. 4.



Each of which will admit of various changes of proportions, without altering the general character of the section ; indeed, the second figure may be considered as comprehending also fig. 1 and fig. 3. We have only to suppose the lower flanch as having no projection to give fig. 1, and to have it more extended to give fig. 3.

The upper and the lower tables are here repre-

sented as rectangular, with sharp edges. In practice these are rounded off, the metal thus displaced furnishing a sort of bracket between the table and stem, or rib, as shown in the annexed figure ; but to treat of them in this form would introduce great intricacy into the calculation, without much affecting the results. It will therefore be sufficient to consider them as rectilinear, but preserving the same area.



167. I would here observe, also, that some projectors have made the upper and lower tables of equal figure, upon the distant contingency, that when the upper table has been worn down, the rail may be turned, and the lower table made the upper. But this is certainly providing without foresight ; for the bottom table is the most efficient for strength, and it would be a very dangerous experiment, after one side of a bar has been submitted for many years to a high compressing force, and its substance (by the hypothesis) greatly worn, to turn the rail, and expose this worn part to a still greater strain, but tensile instead of compressive, which could not fail very soon to destroy it. Instead of this, therefore, I should certainly recommend to work whatever metal is introduced into the lower table or web, into that form which is most efficient for present purposes, without regard to the contingency alluded to above.

That the rail is deteriorated by exposure and wear is undoubtedly true, although, perhaps, the amount is not yet well ascertained. Amongst the papers submitted to Messrs. Rastrick and Wood, with whom I was associated, we found it estimated at the rate of $\frac{1}{8}$ th of a pound per yard per annum ; but I have since seen it stated, in a letter from Mr. Dixon to Mr. Bidder, at $\frac{1}{10}$ th of a pound per yard per annum. This was determined by taking up three rails, having them well cleaned and weighed, and then putting them in their places, and afterwards washing and re-weighing them at the end of a twelvemonth, when two of them were found to have lost $\frac{1}{2}$ lb. in weight for the 5 yards length, and the third $\frac{3}{4}$ lb., which last was taken up from a particular situation where it was more exposed to friction ; but even this does not prove that the whole loss of weight is in the upper face of the rail ; and if it did, it would be, as I have before observed, a stronger reason for not turning the rail : and, on the other hand, should the waste not be on the upper surface, the provision alluded to is unnecessary. Mr. Rastrick informs me, that even the small fins left at the meeting of the rolls are still quite distinctly seen on the face of the upper table. And Mr. Stevenson states, that the marks of the tools left in turning the flanches of the wheels are seldom obliterated ; which proves, at all events, that there is no side wear.

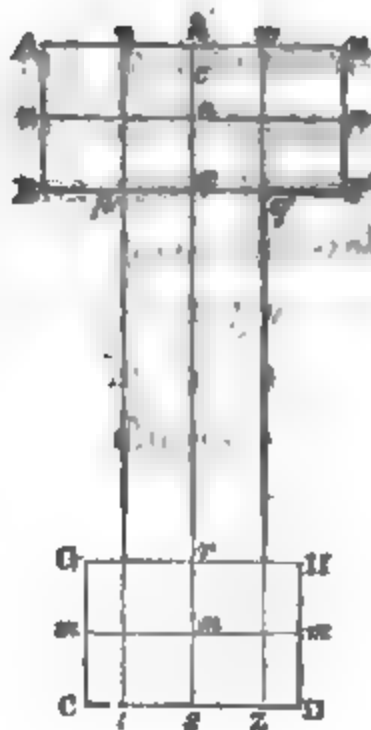
Mr. George Bidder, who attributes all the waste

to the wear on the upper surface, estimates the annual reduction at $\frac{1}{90}$ th part of an inch; in which case the rails would not last more than thirty years before they would require to be replaced. And it then becomes a question, whether, in point of economy, it would not be better to lay an additional third of an inch upon the upper table, which would, by this reckoning, make the rail last sixty years. This increase of $\frac{1}{3}$ rd of an inch would call for an additional expense, to the amount of about $7\frac{1}{2}$ per cent. on the present cost; and this $7\frac{1}{2}$ per cent., at compound interest, would amount to about thirty per cent. in thirty years. If, therefore, a charge of 30 per cent. at the end of thirty years, would meet the amount of re-manufacture, and supply the waste, the two accounts would be about balanced. In this case, I must consider the latter as preferable. 1st. Because the other plan would increase the weight of the bar, and the difficulty of the manufacture, and probably diminish its soundness. 2nd. Because thirty years' experience may introduce improvements, of which, at the end of that period, it would be desirable to take advantage. And, lastly, because I do not (judging from the opinions of different practical men) think that it has yet been clearly determined what part of the waste is due to wear on the upper face.

To return again to the subject of the best-formed section, I beg to repeat, that whatever figure the above, or other considerations, may lead practical

men to adopt in the upper or lower table and rib; it will be fully sufficient for the purposes of calculation to consider them as rectilinear, which will greatly facilitate the investigation, without sensibly affecting the results.

168. Let $ABCD$, in the annexed figure, represent any rectangular rail with a bottom table; nn its neutral axis; s the centre of compression, on being $\frac{1}{2}$ of hn . Now, the tension of each fibre being as its distance from the neutral axis, and that of the lower fibre being given equal to t , the tension at any variable distance x , will be $\frac{tx}{d}$ (d being taken to denote the whole depth ns), and therefore the sum of all the tensions will be



$$\frac{t}{d} \int x \cdot dx, \quad (1)$$

which, therefore, become known, x being taken within its proper limits, according to the figure of the section.

But as the effective resistance of each fibre is also as its depth below the line nn , the sum of all the resistances will be

$$\frac{t}{d} \int x^2 \cdot dx, \quad (2)$$

x being taken here also within its proper limits. And then to find the centre of tension, or that point into which, if all the tensions were collected, the whole resistance would be the same as in the actual case, this would be given by the formula

$$\frac{\int x^2 . dx}{\int x . dx}, \quad (3)$$

which is precisely the expression for the centre of oscillation of a disc of the same figure.

We have hence the following general rule for finding the resistance or the weight which any given bar or rail will support at its middle point, within the limits of its elastic power, that is,

Calling the integral of formula (1)=A

Ditto ditto ditto (2)=B

Ditto ditto ditto (3)=D

And the distance $cn=C$

then, referring the sum of all the resistances B to the centre of compression, we have

$$D :: D + C :: B : \frac{B(D+C)}{D},$$

which is the whole effect.

169. For those who understand the integral calculus, this solution is sufficient; but as this work will probably be consulted principally by practical men, it may be convenient to show the origin of these formulæ, particularly the third, which is not investigated in the preceding pages, except that it has been shown generally, that if d' denote the

depth of the lower fibre below nn , and its tension be made $=t$, and any variable distance $=x$,

That $\frac{t}{d} \int x dx = \text{sum of all the tensions to a unit of breadth}$:

That $\frac{t}{d^3} \int x^2 dx = \text{sum of all the resistance referred to the axis } n$; and

$$\frac{\frac{t}{d^3} \int x^2 dx}{\frac{t}{d} \int x dx}, \text{ or } \frac{\int x^2 dx}{\int x dx} = \delta', \text{ distance of centre of tension.}$$

From which it follows, that $\frac{t \delta'}{d^2} \int x dx = \text{sum of all the resistances for a unit of breadth, } x \text{ being taken in its ultimate state.}$

Now, in the rib, when

$$x = d', \delta' = \frac{2}{3} d', \text{ and } \int x dx = \frac{1}{2} d'^2,$$

whence the above becomes $\frac{1}{3} d'^2 t$; but to refer this to the centre of compression c , we have (calling the whole depth d)

$$\frac{2}{3} d' : \frac{2}{3} d :: \frac{1}{3} d'^2 t : \frac{1}{3} d d' t;$$

and introducing the breadth pq , it becomes

$$\frac{1}{3} h s . n s . p q . t = \text{resistance of the rib.}$$

In the same way, calling the tension at $x = t'$, and the breadth $(nn - pq)$, we have for the resistance of the head $\frac{1}{3} h x . n x . (nn - pq) t'$; but the tension at $x = \frac{nx}{ns} t$; therefore, substituting this for t' , we have

$$\frac{1}{3} h x . n x . \left[\frac{nx}{ns} \right]^2 \frac{(nn - pq)}{ns}, \text{ or}$$

$$\frac{1}{3} h x . n x . (nn - pq) \frac{n^2 x}{ns^2} t = \text{resistance of the head.}$$

For the lower flanch,

$$\frac{\int x^2 dx}{\int x dx} = \delta'.$$

Calling $nr = d''$, and x any variable distance below r , it becomes

$$\frac{\int (d'' + x)^2 dx}{\int (d'' + x) dx} = \delta';$$

which, when $x = rs$, gives

$$\delta' = nm + \frac{rs^2}{12mn},$$

$$\text{and } \frac{t}{d'} \int (d'' + x) dx = \frac{t\delta'}{d'} nm \cdot rs,$$

whence the resistance referred to nn is, for the breadth $(mm - pq)$

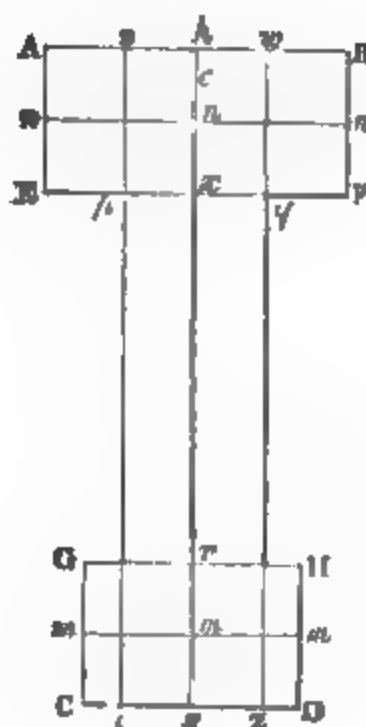
$$nm \cdot rs (mm - pq) \frac{t\delta'}{d'};$$

and calling $\delta' + nc = \delta''$, it is, when referred to c ,

$$nm \cdot rs (mm - pq) \frac{\delta'' t}{d'} \text{ resistance lower flanch,}$$

which is the formula in question.

Let now ABCD, in the annexed figure, represent a section, of which all the dimensions are given, as also the position of nn the neutral axis, the point c which is the centre of compression, cn being $\frac{2}{3}$ rds of nh , and the point m , which is in the centre of rs . The breadths nn and mm are also known. Then the resistance of the whole section referred to the common centre of compression c , may be considered to be made up of the three resistances.



1st. Of the middle rib, continued through the head and foot tables, $vtzw$.

2nd. Of the head AEFB, minus the breadth of the centre rib.

3rd. Of the lower web, GCDH, also minus the continuation of the centre rib.

Now, t being taken to represent the tension of iron per square inch, just within its limits of elasticity, we shall have

$$1. \text{ Resistance of } vtzw = \frac{1}{2} hs . ns . pq . t.$$

$$2. \text{ Resistance of AEFB} = \frac{1}{2} hw . ns . (nn - pq) \frac{ns}{ns} t.$$

Now, let $nm + \frac{rs^2}{12nm} = \delta'$, and $\delta' + cn = \delta''$, then

$$3. \text{ Resistance of GCDH} = nm . rs . (mm - pq) \frac{\delta''}{s} t.$$

These three resistances being computed, let their sum be called s , and the clear bearing l ; then $\frac{4s}{l} = w$, the load the bar ought to sustain at its middle point, for an indefinite time, without injury to its elasticity.

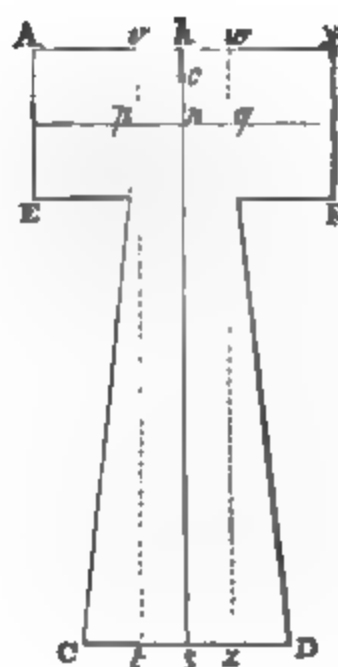
Note.—In the case of the plain T rail, the formula, No. 3, vanishes; the flanch having no thickness.

Trapezoidal Rail.

170. Produce the sloping sides till they intersect the neutral axis in p, q . Then the rule for the head AEFB, and middle rib, $vtzw$, will be the same as given above; and for the two sides, pCt, qDz , the formula is

$$\frac{1}{2} (\frac{1}{2}ns + cn) \times (CD - pq) \text{ \³}$$

The sum of this, with that of the head and middle rib, multiplied by 4, and divided by l , as before, will be the weight required.

*Mechanical Solution.*

171. Another general and very curious mechanical method of finding the resistance of a railway bar, is suggested by the remark in p. 339, viz., that the centre of tension is the same as the centre of oscillation of a disc of the form of the section, cut off at its neutral axis, which in words may be given as follows:

* This includes the small dotted part of the triangular sides in the head and in the sides, but the amount is so very inconsiderably in error, as to be nearly or wholly insensible in the result.

Find the centre of oscillation, and the centre of gravity of the area below the axis, by the usual mechanical methods, and call the distance of the former below the neutral axis o , that of the latter g , the area a , the depth d' , and the distance $cn=c$, the tension t , and bearing l , as usual, then the weight the bar will support will be

$$w=4 \frac{(o+c) agt}{ld'}$$

The following numerical rules, however, will be generally more convenient, particularly when some of the dimensions become fixed, as necessarily happens in such cases as we are considering. For instance, whatever figure may be given to the transverse section, the head may generally be supposed to occupy $\frac{2}{5}$ ths of it, and therefore, in the larger rails, to have about two inches section, and to be one inch deep; that the lower web, when there is one, is the same depth as the head, and that the neutral line bisect the head, or upper table.¹⁰ With these, as fixed dimensions, the preceding formulæ, Art. 169, are reducible to words at length. They apply, however, only to the larger rails; for other cases, it will be best to have recourse to the general formulæ.

¹⁰ The correct rule is, that the area of compression into the distance of its centre of gravity from the neutral axis, is to the area of tension into the distance of its centre of gravity from the same line, as $1^2 : 4^2$, or as 1 to 16.

172. Resistance of the Head or Upper Table.

1. Subtract the thickness of the middle rib from 2 inches, and multiply the remainder by 10.

2. Subtract $\frac{1}{2}$ an inch from the whole depth, and multiply the remainder by 12.

Then the former product divided by the latter will be the resistance in tons due to the head, not including the continuation of the middle rib.

Resistance of the Centre Rib.

Multiply the whole depth of the rail by the whole depth, minus $\frac{1}{2}$ an inch, and that product by 10 times the thickness of the rib ; and the last product, divided by 3, will be the resistance in tons of the middle rib continued through the whole depth, *i. e.* through the upper and lower tables.

Resistance of the Lower Web.¹¹

(1.) Multiply the whole depth of the rail, minus 1 inch, by the breadth of the bottom web, minus the thickness of the rib, and that product by 10.

¹¹ This rule only applies when the depth of the head and lower web are each 1 inch. In other cases, recourse must be had to the general formulæ.

2. From the whole depth of the rail subtract 1 inch, and to 12 times the square of the remainder add 6 times the remainder, and call this the first number. From this subtract twice the remainder, and add 1, and call this the second number. Then say, as the first number is to the second, so is the product obtained in the former part of the rule to the resistance of the lower web, not including the continuation of the middle rib.

Lastly, the sum of these three resistances, multiplied by 4, and divided by the clear bearing length, will be the weight the rail will sustain without injury. A few examples worked at length are given below, to illustrate the rules.

173. Examples.

(1.) Let the depth of the rail be 5 inches, with a plain rib whose thickness is $\cdot 9$ of an inch. Here

$$\begin{array}{rcl}
 \text{Resistance of head} & \left\{ \begin{array}{l} (2 - \cdot 9) \times 10 = 11 \\ (5 - \frac{1}{2}) \times 12 = 54 \end{array} \right\} \frac{11}{54} & \overset{\text{Tons.}}{=} 0\cdot 20 \\
 \text{Ditto of rib} & \cdot \cdot \cdot \frac{4\frac{1}{2} \times 5 \times \cdot 9 \times 10}{3} & \cdot \cdot \cdot = 67\cdot 50 \\
 & & \hline
 & & 67\cdot 7
 \end{array}$$

$$\text{And } \frac{4 \times 67\cdot 7}{33} = 8\cdot 21 \text{ tons, the greatest weight.}$$

$$\text{Deflection with this weight } \frac{\cdot 22}{4\cdot 5} = \cdot 05 \text{ nearly.}$$

(2.) Parallel rail of the same depth, the thickness of the centre rib being $= \cdot 78$. Hence,

$$\begin{array}{rcl}
 \text{Resistance of head} & \left\{ \begin{array}{l} (2 - .78) \times 10 = 12.2 \\ (5 - \frac{1}{2}) \times 12 = 54 \end{array} \right\} \frac{12.2}{54} & = \overset{\text{Tons.}}{0.225} \\
 \text{Ditto rib} & \frac{4\frac{1}{2} \times 5 \times .78 \times 10}{3} & = 58.5 \\
 & & \hline
 & & 58.725
 \end{array}$$

And $\frac{4 \times 58.725}{33} = 7.11$ tons, the greatest weight.

Deflection with this weight $\frac{.22}{4.5} = .049$.

(3.) Parallel rail with bottom web, the depth being still 5 inches, the thickness of rib .6 of an inch, thickness or breadth of section of lower web 1.32, the weight being 50 lbs.

$$\begin{array}{rcl}
 \text{Resistance of head} & \left\{ \begin{array}{l} (2 - .6) \times 10 = 14 \\ (5 - \frac{1}{2}) \times 12 = 54 \end{array} \right\} \frac{14}{54} & = \overset{\text{Tons.}}{0.26} \\
 \text{Ditto of rib} & \frac{4\frac{1}{2} \times 5 \times .6 \times 10}{3} & = 45.00 \\
 \text{Lower web} & \left\{ \begin{array}{l} (5 - 1) \times .72 \times 10 = 28.8 \\ 12 (5 - 1)^2 + 24 = 216 = \text{1st No.} \\ 216 - 7 = 209 = \text{2d No.} \end{array} \right\} & \\
 \text{As } 216 : 209 :: 28.8 : 27.94 & & 27.94 \\
 & & \hline
 & & 73.20
 \end{array}$$

And $\frac{73.20 \times 4}{33} = 8\frac{2}{3}$ tons, the greatest weight.

Deflection with this weight $\frac{.22}{4.5} = .048$.

(4.) As another example, let us take a parallel rail of 50 lbs. per yard, depth $4\frac{1}{2}$ inches, thickness of rib $\frac{7}{10}$ ths of an inch, and of the bottom web 1.39.

$$\text{Resistance of head } \left\{ \begin{array}{l} (2 - .7) \times 10 = 13 \\ (4\frac{1}{2} - \frac{1}{2}) \times 12 = 48 \end{array} \right\} \frac{13}{48} = 0.27 \text{ Tons.}$$

$$\text{Ditto of rib } \frac{4 \times 4\frac{1}{2} \times .7 \times 10}{3} = 42.00$$

$$\begin{array}{l} \text{Do. of lower } \left\{ \begin{array}{l} 3\frac{1}{2} \times (1.39 - .7) \times 10 = 24.15 \\ 12 (3\frac{1}{2})^2 + 21 = 168 = 1\text{st No.} \end{array} \right. \\ \text{web } \left\{ \begin{array}{l} 168 - 6 = 162 = 2\text{d No.} \end{array} \right. \end{array}$$

$$\text{As } 168 : 162 :: 24.15 : 23.28 = 23.28$$

$$\underline{65.55}$$

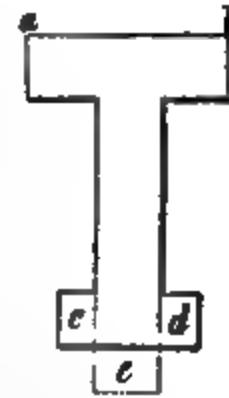
$$\frac{4 \times 65.55}{33} = 8 \text{ tons nearly, the greatest weight.}$$

$$\text{Deflections with this weight } \frac{.22}{4} = .055.$$

Rail of Maximum Strength.

174. The preceding rules and examples will enable any one to estimate the strength of any proposed rail; but the question of the strongest rail with a given quantity of metal, remains still to be decided. In this, of course, we must limit ourselves to practical forms; but even under this limitation, considerable difference of opinion exists. Thus, while one party contend that the strongest form is that which has the lowest and broadest flanch, others maintain, that if the flanch was wholly removed, and the metal placed so as to continue the centre rib to a greater depth, a considerable additional strength would be obtained. The argument advanced in support of the latter doctrine is this:— Suppose *a b c d* to denote a rail with a double flanch, the lower one being marked *c* and *d*, it is maintained, if these parts were removed, and placed in

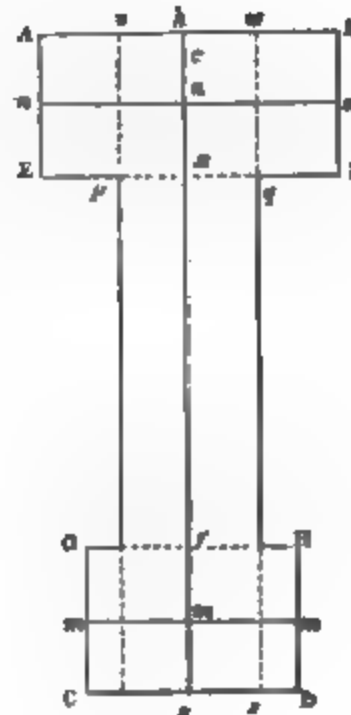
continuation of the centre rib, as shown at *e*, that these fibres being now placed further from the neutral axis than in their former position, they would become more effective. So far this is true, but then the part of the centre rib between *c* and *d*



would become less effective; because, in the first form, this part is amongst the lower fibres, which are all exerting their full tension of 10 tons, while, in the second form, the parts between *c* and *d* are no longer found amongst the lower fibres, and it is the lower ones only in which this full tension can be exerted.

It is clear, therefore, that this is a question which comes immediately under the class of *maxima et minima*, the solution of which is as follows; viz.

Given the area of section of a railway bar below the neutral axis, to find the dimensions of the lower flanch, so that the strength shall be a maximum; the breadth of the middle rib, and the depth of the lower flanch, being also given. Referring to the annexed figure; let the whole of the sectional area below



nn = *a*
 the breadth of rib *pq* . . . = *b*
 the depth *ns* = *d'*
 the depth of lower flanch *rs* = *e*
 the tension of the lower fibre = *t*

1. To find first the expression for the strength of the middle rib; take any variable distance x , then

$$d' : t :: s : \frac{s^2}{d'} = \text{tension of fibre at } s.$$

Multiply by the distance x , and breadth b , and we have for the sum of all the resistances

$$\int \frac{1}{2} b x^2 ds = \frac{1}{3} \frac{b}{d'} s^3;$$

this when $s = d'$ becomes $\frac{1}{3} d'^2 b t$.

2. To find an expression for the strength of the lower flanch: Let the breadth $= b'$; any distance from $nn = x$,

$$\text{then } d' : t :: s : \frac{s^2}{d'} = \text{tension of fibre at } s.$$

Multiply by the distance x , and breadth b' .

We have for the sum of all the resistances

$$\int \frac{1}{2} b' x^2 dx.$$

This taken between the values

$$x = d', \text{ and } x = d' - e,$$

gives: Resistance,

$$\frac{3 d'^2 e - 3 d' e^2 + e^3}{3 d'} b' t = \left(d' e - e^2 + \frac{e^3}{3 d'} \right) b' t.$$

Therefore, the resistance of the rib and lower flanch is

$$\frac{1}{3} d'^2 b t + \left(d' e - e^2 + \frac{e^3}{3 d'} \right) b' t.$$

And the question is, to determine what value must be given to d' , that this expression may be a maximum.

To effect this it is only necessary to consider d' as

variable, to denote it by x , to find the value of the dependent quantity b' in terms of x , to substitute these quantities in the preceding expression, and to make its differential equal to zero.

Now, since the depth of the middle rib is x and breadth b , the area is $b x$, and consequently the area of the lower flanch $= a - b x$, and its depth being e , its breadth

$$= \frac{a - b x}{e}, \text{ that is, } b' = \frac{a - b x}{e}.$$

Substituting, therefore, $\frac{a}{e} - \frac{b}{e} x$ for b' , and x for d , in the preceding expression, it becomes, rejecting t , which is common,

$$\frac{1}{3} b x^2 + \left(e x - e^2 + \frac{e^3}{3 x} \right) \left(\frac{a}{e} - \frac{b}{e} x \right) = \text{a max.} \quad \text{Or,}$$

$$d \left(\frac{1}{3} b x^2 \right) + d \left(e x - e^2 + \frac{e^3}{3 x} \right) \cdot \left(\frac{a}{e} - \frac{b}{e} x \right) + d \left(\frac{a}{e} - \frac{b}{e} x \right)$$

$$\left(e x - e^2 + \frac{e^3}{3 x} \right) = 0. \quad \text{Or } \frac{2}{3} b x d x$$

$$+ \left(e - \frac{e^3}{3 x^2} \right) \cdot \left(\frac{a}{e} - \frac{b}{e} x \right) d x$$

$$- \frac{b}{e} \left(e x - e^2 + \frac{e^3}{3 x} \right) d x = 0. \quad \text{Or,}$$

$$\frac{2}{3} b x + \left(e - \frac{e^3}{3 x^2} \right) \cdot \left(\frac{a}{e} - \frac{b}{e} x \right) - \frac{b}{e} \left(e x - e^2 + \frac{e^3}{3 x} \right) = 0.$$

$$\text{Or, } \frac{2}{3} b x + a - b x - \frac{e^2 a}{3 x^2} + \frac{e^2 b}{3 x} - b x + b e - \frac{b e^2}{3 x} = 0.$$

Reducing every term to the denominator $3 x^2$, and rejecting it, this becomes

$$2bx^3 + 3ax^2 - 3bx^3 - e^2a + be^2x - 3bx^3 +$$

$$3ebx^2 - be^2x = 0. \text{ Or}$$

$$-4bx^3 + (3a + 3eb)x^2 - e^2a = 0. \text{ Or}$$

$$x^3 - \frac{3(a + eb)x^2}{4b} = \frac{-e^2a}{4b}$$

$$x^3 - \frac{3}{4} \left(\frac{a + eb}{b} \right) x^2 = \frac{-e^2a}{4b}.$$

From which x may be determined for any given values of a , b , and e .

As an example, take a rail in which the middle rib is .78 inch, or $b = .78$, as given in Example 2, p. 346, to find what flanch must be given, and the corresponding depth of rail to produce the maximum strength.

The rail being $4\frac{1}{2}$ inches below the neutral axis, and its breadth $b = .78$, its area is $.78 \times 4\frac{1}{2} = 3.51 = a$; and it is required to distribute this area so as to produce a rail of maximum strength, the depth of the proposed flanch being 1 inch. Substituting

$$a = 3.51, b = .78, e = 1,$$

the foregoing equation becomes

$$x^3 - 4.11x = -1.12.$$

Whence $x = 4.04$ the depth of the rail required.

Now, $4.04 \times .78 = 3.15$ area middle rib,

$$a - bx = 3.51 - 3.15 = .36 = b'.$$

Or, .36 the area of the lower flanch, which is also its breadth, its depth being 1.

The strongest rail, therefore, of this weight, whose

breadth is $\cdot 78$, is that whose depth is $4\cdot 04$ inches, and the breadth of the lower flanch, including the middle rib, is $\cdot 78 + \cdot 36 = 1\cdot 14$ inch.

It will be observed, that in this solution, for the sake of simplification, the strength or resistance of the head, which is very little, has been neglected. Nor is the resistance transferred to the centre of compression; but it is obvious, that when the strength is a maximum estimated from the neutral axis, it must be so also when referred to the centre of compression, or very nearly so.

Strength of the Rail computed.

The whole depth $4\cdot 54 = h s$, $n s = 4\cdot 04$, $p q = \cdot 78$.

$$\text{Head} \quad \left\{ \begin{array}{l} (2 - \cdot 78) \times 10 = 12\cdot 2 \\ (4\cdot 54 - \frac{1}{2}) \times 12 = 48\cdot 48 \end{array} \right\} \quad . \quad . \quad . = \cdot 25$$

$$\text{Middle rib} \quad \frac{4\cdot 54 \times 4\cdot 04 \times \cdot 78 \times 10}{3} \quad . \quad . \quad . = 47\cdot 69$$

$$\text{Lower flanch} \quad \left\{ \begin{array}{l} (4\cdot 54 - 1) \times \cdot 36 \times 10 = 12\cdot 7 \\ 12 (4\cdot 54 - 1)^2 + 4 \times 3\cdot 54 = 171\cdot 6 \\ 171\cdot 6 - \frac{2 \times 3\cdot 54}{1} + 1 = 165\cdot 6 \end{array} \right\}$$

$$171\cdot 6 : 165\cdot 6 : 12\cdot 7 : . \quad . \quad . \quad . = 12\cdot 25$$

60·19

4

33) 240·76

nearly 7·3 tons.

If we take the area the same 3·51, with the flanch 1 inch deep, and the rib ·6 thick, the general equation becomes

$$x^3 - 5·137 x^2 = - 1·46.$$

This gives $x = 5$ very nearly.

So that the depth assumed in Example 3, p. 347, is for the thickness ·6 of an inch the rail of maximum strength.

On the Longitudinal Figure of Rails.

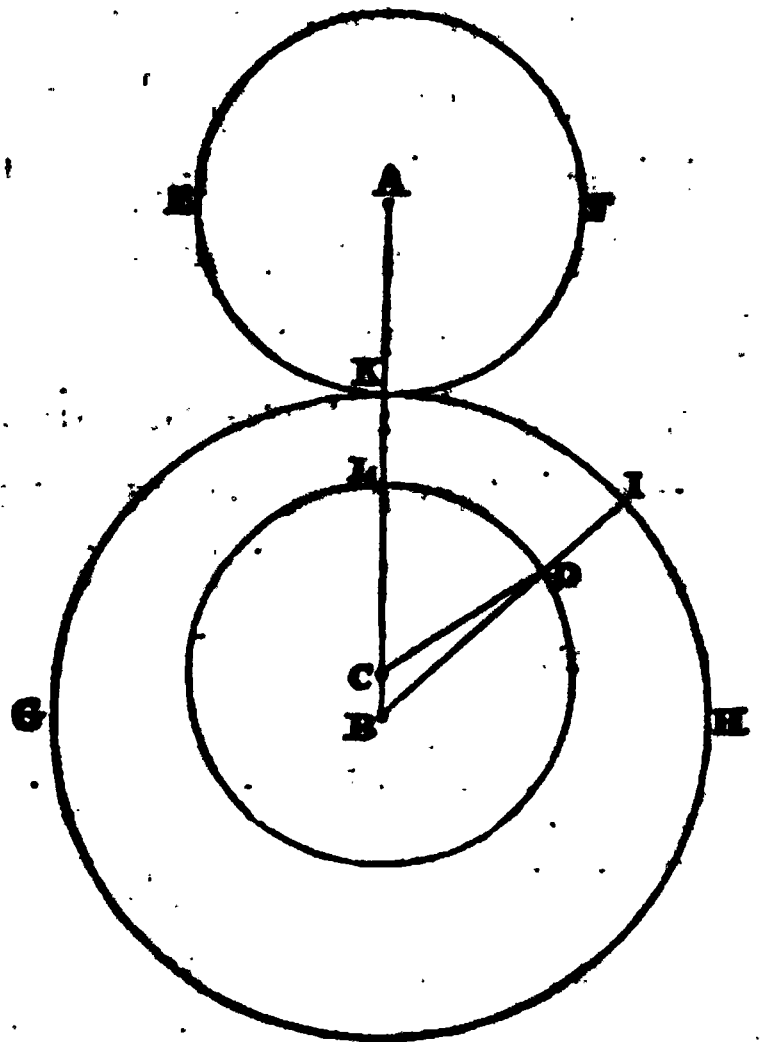
175. To produce the lightest kind of rail, having a given strength, was in the infancy of railway practice considered an object of importance; and as the strain upon a rail is greater in the middle than elsewhere, it was considered that it would be economical to have the rail of such a figure longitudinally, that the depth should be every where proportional to the strain; and we have seen (Art. 21) that the strain is proportional to the rectangle of the two parts into which the length is divided by the load, and the resistance is as the square of the depth. What is required, therefore, for a rail to be of equal strength throughout, is to have one in which the depth is every where as the square root of the rectangle of the two parts, which is the property of an ellipse. The bars, therefore, according to this view of the subject, ought to be elliptical, the length forming the transverse axis and the depth the semi-conjugate axis. In cast iron such

a form is commonly given in buildings, &c., with great advantage, and it was thought that it would be equally advantageous here also. Great ingenuity was accordingly displayed by Mr. Birkenshaw, in contriving a pair of rollers that would produce this form, if not exactly, at all events pretty nearly; and before we enter upon an examination of the comparative advantages of this and the parallel rail, it may be interesting to some readers to be informed of the means by which this figure is obtained in rolled iron.

176. This is done by a pair of finishing rollers, of the kind shown in Plate VI., fig. 3. The iron is first drawn down to a square bar of a proper size; it is then passed successively through the rollers, as numbered in the figure. First passing through the grooves No. 1, it takes the form shown in that figure, its sides being parallel; then the form No. 2, still parallel; it then passes through the edging groove No. 3, in which it will be observed, the lower cylinder is turned eccentric to the axis of the rollers, so that as the iron passes on it is rendered of different depths, as shown in fig. 4; but it has not yet received its finished form; this is obtained by passing it successively through the grooves No. 4 and No. 5, by which it obtains its final thickness and shape, fig. 4. This is not, however, as we have said, strictly elliptical, as will be seen by examining what takes place in the edging rollers, an enlarged

section of which is represented in the annexed diagram.

EF is the section of the upper roller; GH the section of the other. This latter being hung on a false centre C, is turned down, leaving a groove of varying depth as represented in the figure. The cylinder GH being now again placed on its proper centre B, the bars,



as have been stated, are introduced between the two rolls at KL; and as the iron passes through, it acquires the variable depth intended. The inner circle, or bottom of the groove, is generally 1 foot in diameter, and the upper 3 feet in circumference; consequently, the figure is completed in a length of 3 feet, and there are commonly five such lengths in a bar.

177. The computation of the ordinates to the curve thus formed is by no means difficult; for, calling the radius of the cylinder $CD = r$, and the distance of the centres $BC = d$, and x any angle LCD , we find the ordinate,

$$ID = BI - \sqrt{(r^2 + d^2 - 2rd \cos x)}.$$

And by this formula the ordinates of the curves have been computed for two different fish-bellied rails; the extreme depth in both being 5 inches, but the lesser depth in one 3 inches, and in the other $3\frac{3}{4}$ inches, the latter being that proposed by Mr. Stephenson for the London and Birmingham Railway. The ordinates are taken for each 10° , or for every inch of the half-length, and in the last column are given the ordinates of the true ellipse.

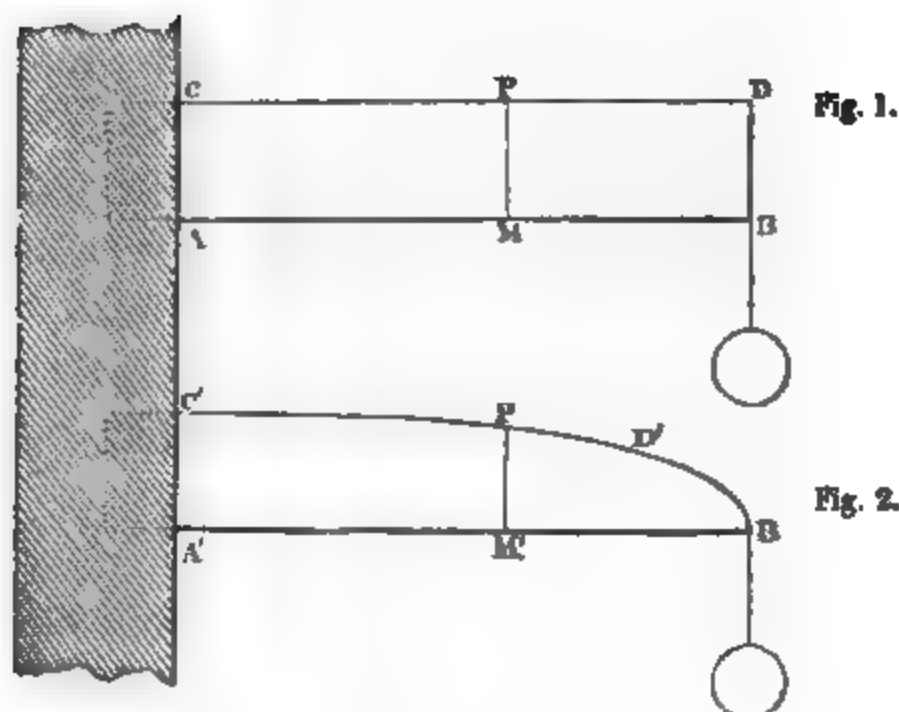
TABLE OF ORDINATES.

ABSCISSAS.			Ordinates in fish-bellied rail. Greatest depth 5 in. Least do. 3 in.	Ordinates in Mr. Stephenson's rail.	Ordinates in the ellipse.
Deg.		Inch.			
0	=	0	3.00	3.75	0
10	or	1	3.01	3.76	1.64
20	..	2	3.05	3.78	2.29
30	..	3	3.12	3.82	2.76
40	..	4	3.21	3.88	3.14
50	..	5	3.31	3.95	3.46
60	..	6	3.44	4.04	3.72
70	..	7	3.59	4.14	3.96
80	..	8	3.75	4.23	4.16
90	..	9	3.92	4.34	4.33
100	..	10	4.09	4.45	4.48
110	..	11	4.27	4.55	4.61
120	..	12	4.43	4.66	4.71
130	..	13	4.59	4.75	4.80
140	..	14	4.72	4.84	4.87
150	..	15	4.84	4.91	4.93
160	..	16	4.93	4.95	4.97
170	..	17	4.98	4.99	4.99
180	..	18	5.00	5.00	5.00

We see by this Table, (although it is impossible, with any proportions or degrees of eccentricity, to work out a true elliptic figure by this method,) that we may approximate towards it sufficiently near for any practical purpose, as Mr. Stephenson has done; while, on the other hand, without due precaution, we may so far deviate from it as to render the bar dangerously weak in the middle of its half-length.

As far as relates to ultimate strength, there can be no doubt Mr. Stephenson's rail is equal to that of an elliptic rail, and consequently to that of a rectangular rail of the same depth; but there is still an important defect in all elliptical bars, viz., that although this form gives a uniform strength throughout, it is by no means so stiff as a rectangular bar of a uniform depth, equal to that of the middle of the curved bar, and it is the stiffness rather than the strength that is of importance; for the dimensions of the rail must so far exceed those which are barely *strong enough*, as to put the consideration of ultimate strength quite out of the question. The object, therefore, with a given quantity of metal, is to obtain the form least affected by deflection; and unfortunately the elliptical bar, although equally as strong as the rectangular bar of the same depth, as far as regards its ultimate resistance, is much less stiff.

178. This will appear from the following investigation :



The deflections which beams sustain when supported at the ends and loaded in the middle, is the same as the ends would be deflected, if the beams were sustained in the middle, and equally loaded at the ends, each with half the weight; and the *law* of deflection is the same in the latter case, as when the beam is fixed in a wall and loaded at its end, although the *amount* is greater. At present, however, our inquiry is not the actual, but the relative deflection in two beams, one elliptical, and the other rectangular, of the same length, and of the same extreme depth—the breadth and load being also equal in each. It is quite sufficient, therefore, to consider the corresponding effects on two half-beams, each fixed in an immoveable mass, as represented in the preceding figures.

Now, in the first place, the elementary deflection at C is the same in both beams, because the lengths and loads are the same, and the depths at C are equal; but the whole deflection at any other point P, will be directly as MB^2 , and inversely as MP^3 . If, therefore, we call $MB=x$, and $MP=y$, the sum of all the deflections in the two beams will be $\int \frac{x^2}{y^3} \cdot dx \Delta$, Δ being the sine of deflection at C.

But in fig. 1, y is constant and equal to d (the depth), while in the latter,

$$y = \frac{d}{l} \sqrt{(2lx - x^2)},$$

l being the semi-transverse or length, and x any variable distance.

The whole deflections, therefore, in the two cases are, fig. 1,

$$\text{Deflection} = \int \frac{x^2 dx}{d^3} \Delta = (\text{when } x=l) \frac{1}{3} \frac{l^3}{d^3} \Delta;$$

and in fig. 2,

$$\text{Deflection} = \int \frac{x^2 dx}{\frac{d^3}{l^3} (2lx - x^2)^{\frac{3}{2}}} \Delta.$$

This is best integrated in parts; thus in the expression,¹²

$$\int \frac{x^2 dx}{(2lx - x^2)^{\frac{3}{2}}},$$

¹² In my former Reports I had found the integration by a series. I am indebted to my colleague, S. H. Christie, Esq., for the above complete integration by parts.

Let $x=l-y \therefore x^2=l^2-2ly+y^2$ and $2lx-x^2=l^2-y^2$

$$\therefore \int \frac{x^2 dx}{(2lx-x^2)^{\frac{3}{2}}} = \int \frac{-l^2 dy}{(l^2-y^2)^{\frac{3}{2}}} + \int \frac{2ly dy}{(l^2-y^2)^{\frac{3}{2}}} - \int \frac{y^2 dy}{(l^2-y^2)^{\frac{3}{2}}}$$

$$\int \frac{l^2 dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{Ay}{(l^2-y^2)^{\frac{1}{2}}} + B \int \frac{dy}{(l^2-y^2)^{\frac{1}{2}}}$$

$$\therefore \frac{l^2 dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{A dy}{(l^2-y^2)^{\frac{1}{2}}} + \frac{A y^2 dy}{(l^2-y^2)^{\frac{3}{2}}} + \frac{B dy}{(l^2-y^2)^{\frac{1}{2}}}$$

$$\text{And } l^2 = A(l^2-y^2) + A y^2 + B(l^2-y^2)$$

$$B y^2 - (A+B-1) l^2 = 0$$

$$\therefore B=0, A=1.$$

$$\text{And } \therefore \int \frac{l^2 dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{y}{(l^2-y^2)^{\frac{1}{2}}}$$

$$\int \frac{2ly dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{2l}{(l^2-y^2)^{\frac{1}{2}}}$$

$$\int \frac{y^2 dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{y}{(l^2-y^2)^{\frac{1}{2}}} - \int \frac{dy}{(l^2-y^2)^{\frac{1}{2}}}$$

$$= \frac{y}{(l^2-y^2)^{\frac{1}{2}}} - \sin^{-1} \frac{y}{l}$$

$$\therefore \int \frac{x^2 dx}{(2lx-x^2)^{\frac{3}{2}}} = \frac{2l-y}{(l^2-y^2)^{\frac{1}{2}}} + \sin^{-1} \frac{y}{l} + C$$

$$= \frac{2x}{(2lx-x^2)^{\frac{1}{2}}} + \sin^{-1} \frac{l-x}{l} + C.$$

Which taken from $x=0$ to $x=l$, will be

$$2 - \frac{\pi}{2} = 2 - 1.5707963 = .4292037.$$

That is, the deflection is here

$$.4292037 \frac{l^3}{d^3} \Delta.$$

The deflections, therefore, in the two cases are,

with the same weights, as 33 to 43,¹⁸ or nearly as 3 to 4, a result fully borne out by subsequent experiment. It is to be observed also, that this investigation applies only to the deflection when the weight is in the middle of the bar, and that it would be much greater in comparison with the parallel rail towards the middle of its half-length.

179. This want of stiffness is, I should imagine, but badly compensated by the trifling saving of metal thus effected; for I find that an addition of little more than four pounds per yard, would convert this rail into a rectangular one of the same depth, which would have one-third more stiffness at its middle point, and probably one-half more, a little beyond the middle of the half-lengths. I am aware objections are made to rectangular bars having so much depth of bearing in their chairs, and this may be a practical defect; at all events, however, it is well to estimate properly both evils, and then to choose the least.

For my own part I have not the least hesitation in stating, after having well considered every point that has been advanced in favour of the fish-bellied rail, that the parallel rail is the best.

¹⁸ Experiments have been made from which it has appeared, that the fish-bellied rail was stiffer than the parallel rail, which is certainly possible, if the parallel rail be of inferior metal or of injudicious figure; but it is mechanically impossible if the parallel bar be made of the figure here assumed.

First.—Because, although it is not stronger in the middle point than the former, it is both stronger and stiffer in a very sensible degree in every other point.

Secondly.—The deflection of a parallel rail, during the passage of a load, is less every where than in the middle, which is not the case in the fish-bellied rail. The fall and rise of a carriage, therefore, after passing over a support, is more rapid in one case than in the other; and to this I am induced in part to attribute many of the fractures which have occurred in these curved rails within a short distance of their point of support, but, perhaps, more are caused by the unequal drawing of the iron through the rolls, as well as by some of the early rails of this kind deviating too much from the real elliptical figure. Mr. Stephenson, by a judicious and scientific distribution of the metal, has now avoided this latter evil, and no doubt such fractures will be with his rail less common; but the objection I offer above, applies not merely to the fish-bellied rail, but to the truly elliptical form itself, if it were possible to arrive at it.

Thirdly.—The parallel rail is the best, because it enables the engineer to keep the blocks and chairs of the two rails directly opposite to each other, so that the wheels of the carriage shall pass over both supports at the same time,—a point, I believe, not hitherto much attended to, but which is, I conceive, of great importance. There can be no doubt

the motion of a locomotive carriage consists of a succession of ascents and descents; and it must be evident how much easier and better the motion would be, to have the opposite wheels both rising and both falling together, than to have one always rising while the other is falling, and the contrary. The difference is similar to that of a vessel keeping her head to the waves, and another crossing their direction obliquely. And every one who has never been further than Margate must have experienced this difference.

It may be said, that the waves of the railway, or the deflections of the rails, are very small; but I would observe also, that the weights and velocities of the carriages are very great, and that it is very desirable every possible cause of momentum should be removed, particularly when it is as easy to do it as not to do it, as is the case with parallel rails, because they can always be cut to determinate lengths, which is not so easily done in the fish-bellied rail, in consequence of the occasional slipping of the bar in the rolls, as already mentioned.¹⁴ At all events, their length cannot be varied at pleasure, which the former will admit of, and which is necessary, in going round sweeps, to preserve the blocks

¹⁴ I ought to mention, that Mr. Blackmore, the engineer of the Carlisle and Newcastle Railway, by a judicious selection of long and short rails, has preserved the parallelism of the blocks; and it is the only case that I am aware of, in which this has been attended to, not only in the general line, but in the curves.

always parallel. For example, in going round a sweep of 800 feet, to keep the supports parallel, the rails of the inner course require to be about an inch shorter than the outer ones, and they are as easily cut into lengths of 14 feet 11 inches as of 15 feet, which is not practicable in the other form.

Having arrived at these conclusions, I delivered in the following Report to the Directors of the London and Birmingham Railway.

“ Report addressed to the Directors of the London and Birmingham Railway Company.

180. “ The accompanying paper contains the details of the experiments I have made, in conformity with the resolution of the General Meeting of the Proprietors of the London and Birmingham Railway Company, held at Birmingham, on the 13th of February, 1835, and I am in hopes several important data and rules have been thus elicited. These will be found in the paper referred to; but it may be convenient to state the results in this place, referring to the proper pages for the experiments and investigation on which they are founded.

“ It has been ascertained, page 315, that a malleable iron bar of any length is extended $\frac{1}{10000}$ th part of its length by a direct strain of a ton per inch on its sectional area; and that, when strained with ten tons per inch, or when stretched $\frac{1}{1000}$ th part of

its length, its elasticity is injured, and the bar will not return to its original state.

“Now, as the contraction of iron, between summer and winter, amounts to ~~eight~~ ^{seven}th part of its length, it follows, that the bars cannot be fixed permanently to the chairs and blocks, without great danger of drawing so much upon their strength, as materially to impair their efficiency for bearing a great passing load.

“It follows also as a consequence, that if the rails and chairs must not be permanently fixed to each other by direct means, it ought not to be attempted by indirect means, viz., by cotters, keys, or wedges; for, either these will hold the rail to the chair, or they will not. If they do hold fast, they produce all the mischief which permanent fixing would occasion; and if they draw, then they do no good, although they may still do mischief. Whence I am led to conclude that the rails should have no greater attachment to the chairs than is sufficient for preserving them steady while the load is passing.

“My next experiments were directed to finding the position of the neutral axis in malleable iron, for without this datum, the strength of rails, of differently formed transverse sections, cannot be computed and compared with each other, at least without the expensive mode of having them first made, and then their strengths found by experiment. In this inquiry, as in the preceding, I have succeeded to my entire satisfaction; and, by the

results obtained, have formed rules of very simple character, which will enable any person to compute with great precision the bearing strength of a bar of any proposed section within the limits of its elastic or restoring power, and also the amount of the deflection it will sustain under this or any lesser load. I have demonstrated by these means, that we may find certain practicable forms of parallel rails, which shall be, weight for weight, equally as strong as the fish-bellied rail, when loaded at their middle point, and of course stronger in every other part. For which reasons, and for others I have explained in page 362, I feel fully convinced that the parallel rail, formed according to the requisite proportions, is decidedly the best.

“Such are the results of my experimental researches on this subject ; and here, perhaps, I ought to close my Report, leaving it to practical men to work out the conditions I have shown to be necessary ; but I hope I may be allowed to offer some suggestions on a few practical points,—a task for which I feel myself the better qualified, by being a week associated with Messrs. Rastrick and Wood, in examining the models sent in for the prize, and thus benefiting by their practical skill and remarks. To which I may also add, the advantage I derived from examining so many models, several of them exceedingly ingenious, and accompanied with descriptions, containing very sensible remarks on different modes of practice.

“In the first place, as I have already stated, I am decidedly convinced that the rail should be parallel; that its whole depth should not be less than $4\frac{1}{2}$ or $4\frac{3}{4}$ inches; that the thickness of the middle rib should not exceed that—which is essential to the perfect manufacture of the bar; and that the lower web (without any reference to the distant contingency and dangerous proposition of turning the rail) should be made of the best form for present purposes, viz., for giving to it a steady bearing in its seat.

“With respect to the joint chair, I do not think any better form can be devised, than the whole chair proposed by Mr. Daglish, viz., in which he uses no filling-up piece, but with a different wedge. It is my opinion, that by well gauging both the ends of the rails and chairs, and then leaving the former free, we should best comply with the conditions I have endeavoured to show to be desirable, if not absolutely necessary.¹⁵

“For the intermediate chairs, I think a slight modification of Mr. Stephenson’s would best answer the purpose; that is, I would support the rail in the chair simply by the ends of two plain-ended pins, so as to give it the requisite steadiness with as little friction as possible. Of course, I would have these pins pointing horizontally, or upwards, instead

¹⁵ I have omitted here a paragraph on the fixing of the blocks and chairs, which is only applicable to a particular model, and would not be intelligible to the general reader.

of downwards, as they do in the chair in question.¹⁶ I do not conceive such pins would be necessary in the joint chair, but provision might be made for them, and they could be applied, if necessary.

“ I have no doubt, practical men, who have taken a different view of this subject, and have thought it necessary to fix every thing as fast as possible, will see objections to the light fixing I have proposed, but without attempting to anticipate and answer those objections, I can only say, after having, I believe, heard every thing that can be advanced on the subject, that my opinion is such as I have stated.

“ I have, above, alluded to the gauging the ends of the rails and the openings in the joint chair, and I have also spoken in the description of my experiments, of the advantage of keeping the blocks of the two lines of rails parallel. On all these points it is probable I shall be considered by many as entering into refinements neither called for nor practicable, in the case of railways; but I would ask, why is it found that so much breakage takes place, and that so many repairs are rendered necessary? There is no theoretical reason why a heavy load, passing with great velocity, should cause more damage than the same load passing slowly, if the road were perfect;

¹⁶ It may be worth consideration, whether, if this mode of fixing were adopted, it would not be practicable and advantageous to introduce pieces of felt, or other substance, within the seat of the chair, which would greatly subdue the jars that take place between metal and metal.

the mischief, therefore, is in the imperfect practical execution, and the disregarding small inequalities, as we would disregard them in common cases. It has perhaps never occurred to such persons, that a difference of level at a joint chair between the two abutting rails of only $\frac{1}{16}$ th of an inch will, when the carriage is moving from the higher to the lower level at its greatest speed, cause the wheel to pass the distance of a foot without pressing on the rail, and consequently throwing the whole weight, which ought to be borne equally by the two rails, wholly upon one; yet this is a fact resting on a natural law, and cannot be otherwise. To fall $\frac{1}{16}$ th of an inch by the action of gravity requires $\frac{1}{4}$ th part of a second, and in that time the carriage will have advanced a foot; consequently, for that space the whole weight has been borne by one rail only. It may be said there are springs provided, which assist gravity to bring down the wheel. I am afraid, however, after allowing for their inertia, that such aids are very inefficient; at all events, they furnish no arguments against having every thing as accurate as possible. Again, with reference to the abutting rails, I was certainly surprised when a gentleman officially attached to the Manchester and Liverpool Railway, informed me, that in some parts of their line the rails were half an inch apart, and that it was not thought injurious. But why, I would ask, whether injurious or not, have them half an inch apart, when they never need be open

above $\frac{1}{10}$ th of an inch ; and for more than half the year not above $\frac{1}{20}$ th of an inch, if proper care be taken in laying them down ? Hitherto an idea has prevailed, that in laying down the rail, $\frac{1}{8}$ th or $\frac{1}{10}$ th of an inch must be left for expansion, and whether hot weather or cold, the same allowance is made ; consequently, if the rail is laid in the summer, the $\frac{1}{8}$ th of an inch becomes nearly a quarter of an inch in the winter, provided the contraction takes place in the same direction in two adjacent rails ; but if in a contrary direction, it becomes half an inch, or nearly so, as my informant states the fact occasionally to be. To prevent this, I would, as stated, have each rail fixed to one chair, and to one chair only ; and I would have three steel plates, the thicknesses of the proper spaces, to be left between the rails, according to the temperature—one between 15° and 35° , another between 35° and 65° , and another for all temperatures above 65° , whereby to regulate the distances of the rails. This again will, I have little doubt, be considered an unnecessary refinement ; but to such objections I reply, that this accuracy costs nothing additional in the execution, and may therefore, at all events, be as well attended to as not.

“ It only remains now for me to make a few observations upon the absolute requisite strength of the bars, and the tests of strength to which they ought to be submitted, before they are reported, and received as efficient.

First.—With respect to their absolute strength, the amount of this will depend upon the weight of the locomotive intended to be employed, which I shall here assume at 12 tons; and, notwithstanding six wheels may be used, I shall, for the sake of being on the safe side, consider only four, or that each wheel bears one-fourth of the whole weight, or three tons. I will also suppose that, whether my suggestions are acted on or not, cases may occasionally occur, when the weight, which ought to be borne equably on two rails, is, from irregularities in the road, thrown all on one. This gives the greatest bearing load 6 tons, and I would then have a surplus strength of 50 per cent., making 9 tons; that is, I would have rails whose absolute elastic or restoring power should be 9 tons, and I would test every sample of rails to $7\frac{1}{2}$ or 8 tons. Such a test would be perfectly harmless on bars of good iron; and, unless it is carried thus high, it is impossible to detect bars made of an inferior quality, which show more stiffness in the commencement than the best iron: but their elastic power at length yields suddenly, and the bar becomes useless. Such iron should, of course, be excluded, unless indeed it be contracted for, and the rails proportioned accordingly. This testing should be carried on in the line of works by a proper person, and the manufacturer left free to use his own plans without superintendence; as practised by the Admiralty in the receipt of their iron cables. There

can be no doubt, that if the cables were sent to sea without proof, and every failure of a link attributed to a want of sufficient dimensions, before this time we should have had cables for the several rates of vessels of much larger bolts than at present, thereby adding, at a great expense, much unnecessary and even injurious weight, as appears to be the tendency of the present practice in railway bars.

“The proof I would recommend is as follows:— On the line, near the place where the work is going on, all the intermediate chairs in one length of rail should be removed, and over this space the bar for trial should be placed. A carriage, then, rightly adjusted for weight, and with wheels at a proper distance to bring the requisite strain on the metal, should be passed over twice; when, if no permanent deflection be observed, the bar is to be considered sound, and removed, and its place supplied by another, to undergo the same test.¹⁷

“For the gauging, I would recommend an over and under gauge, according to the plan followed by the Ordnance Board, in the receipt of shot and shell.

“I think it is possible, by a slight modification of the form of rail I have comprised in my fourth example, p. 347, to give to it a strength of 9 tons, without any increase of weight. I have allowed

¹⁷ A paragraph is here omitted which had reference to the testing every individual bar; one or two, however, of each sample delivered might be sufficient.

rather more metal for the head, I believe, than is generally employed, which, if transferred to the lower web, would give all the additional strength required; or, perhaps, the centre rib might bear a slight reduction. At all events, leaving every thing as it is, except adding 2 lbs. per yard to the bottom web, the rail would come up to the whole strength of 9 tons, as required. And here, I would observe, is the great advantage of working by rule rather than by opinion, for if we had only the latter to guide us, we should be hard to believe that an increase of $\frac{1}{8}$ th in the weight could be made to add about $\frac{1}{8}$ th to the strength and stiffness of the bar; yet such is unquestionably the case.

“In conclusion, I feel it my duty to state, that through the liberal permission of my Lords Commissioners of the Admiralty, I have had every convenience I could desire for conducting the experiments; that the London Committee have caused to be supplied every instrument and material I had occasion for; and that I have been much indebted to Messrs. Lloyd and Kingston for their cheerful assistance, and ingenious suggestions on various occasions. On my own part, I will only say, that I entered upon the inquiry without prejudice; that I have made the best use in my power of the means placed at my disposal; have faithfully recorded every result as it was noted down at the moment of observation; and that I am in hopes the laws and rules I have deduced are legitimate, and may be

found useful, by enabling practical men to compute results which they have hitherto been only able to conjecture."

Experiments on the actual Strength of Railway Bars of various forms and dimensions.

181. Having in the preceding pages investigated every circumstance which has a theoretical bearing on the question of the strength of malleable iron generally, and as applied to railway bars in particular, the following trials on the bars themselves will be useful as offering the best means of comparing the rules with actual experimental results.

Experiments on the Resistance and Deflection of Railway Bars, without a Lower Web.

Mr. Stephenson's Fish-bellied Rail, 50 lbs. per yard.

Greatest depth 5 inches, less depth $2\frac{1}{2}$ inches, thickness of centre rib $\frac{1}{4}$ inch.

BAR No. 1.			BAR No. 2.		
Weights.	Deflections by index.	Deflections for each ton.	Weights.	Deflections by index.	Deflections for each ton.
1	·033		1	·014	
2	·045	·010	2	·022	·008
3	·053	·010	3	·030	·008
4	·063	·010	4	·042	·012
5	·071	·006	5	·050	·008
6	·076	·006	6	·062	·012
7	·087	·011	7	·075	·013
7½	·093	·018	8	·088	·016
			9	·101	·016
			10	{ Elasticity injured.	
			11		·200

BAR No. 3.			BAR No. 4.		
Weights.	Deflections by index.	Deflections for each ton.	Weights.	Deflections by index.	Deflections for each ton.
1	·018		1	·045	
2	·025	·007	2	·050	·011
3	·039	·013	3	·060	·008
4	·054	·016	4	·075	·010
5	·062	·008	5	·084	·009
6	·069	·007	6	·095	·011
7	·080	·011	7	·105	·010
8	·094	·014	8	·110	·005
8½	·100	·012	9	·116	·006
9	·112	·018	10	·125	·009
9½	·118	·019	11	·135	
10	·126	·014			
11	·160	·034			
17	Destroyed.				

Mean deflection per ton, Bar No. 1.	·0097
No. 2.	·0101
No. 3.	·0110
No. 4.	·0100
Mean	·0100

TABLE — (CONTINUED).

Bar No. 5, Fish-bellied. Great depth, 5 inches. Less ditto, 3½. Thickness of rib, ⅞. Head estimated, 2 by 1. Weight, 50 lbs.			Bar No. 6, Fish-bellied. Great depth, 3½ inches. Less ditto, 2½. Thickness of rib, ⅞. Head estimated, 2 by ¾.			Bar No. 7, Fish-bellied. Great depth, 3 inches. Less ditto, 2. Thickness of rib, ⅞. Head estimated, 2 by ½.		
Weight in tons.	Deflection by index.	Deflection for each ton.	Weight in tons.	Deflection by index.	Deflection for each half ton.	Weight in tons.	Deflection by index.	Deflection for each half ton.
1	·030		0·5	·120		0·5	·033	
2	·260		1·0	·140	·020	1·0	·060	·027
3	·270	·010	1·5	·170	·030	1·5	re-adjusted ·062	
4	·290	·020	2·0	·180	·010	2·0	·090	·028
5	·300	·010	2·5	·200	·020	2·5	·120	·030
6	·320	·020	3·0	·230	·030	3·0	·155	·035
7	·335	·015	3·5	·280	·050	3·5	·240	
8	·410	·060	4·0	·420	·140	4·0		
Mean deflection per ton to 7 tons		} ·015	Mean deflection per ¼ ton to 3 tons		} ·022	Mean deflection per ¼ ton to 3 tons		} ·030
Do., with 7½ tons ·107			Do., with 3 tons ·066			Do., with 2 tons ·060		

Comparison of the above Results with the Formulæ,¹⁸
p. 342.—viz.

Rib ⅓ h s . n s . p q . t

Head ⅓ h x . n x]² $\frac{n n - p q}{n s} t$

BARS Nos. 1, 2, 3, 4, 5.

Here . . . { h s = 5, n s = 4·5, p q = ·9, t = 10

{ h x = 1, n x = ·5, n n - p q = 1·1

Hence . . ⅓ × 5 × 4·5 × 9 = 67·5

¹⁸ The same formulæ of course apply to the fish-bellied as to the parallel rail, but for the deflection we must multiply the result by ¼. See p. 362.

$$\frac{1}{2} \times 1 \times \overline{0.5^2} \times \frac{11}{45} = .20$$

$$\frac{67.7 \times 4}{32} = 8\frac{1}{11} \text{ tons}$$

$$\frac{.22}{4.5} \times \frac{4}{3} \times .066 \text{ deflection.}$$

BAR No. 6.

$$\text{Here } \begin{cases} hs = 3.25, ns = 2.88, pq = .7, t = 10 \\ hx = .75, nx = .375, nn - pq = 1.3 \end{cases}$$

$$\text{Hence } \frac{1}{2} \times 3.25 \times 2.88 \times 7 = 21.84$$

$$\frac{1}{2} \times .75 \times \overline{.375^2} \times \frac{13}{2.88} = .15$$

$$21.99 = s$$

$$\frac{4s}{33} = 2\frac{1}{3} \text{ tons}$$

$$\frac{.22}{2.88} \times \frac{4}{3} = .092 \text{ deflection.}$$

BAR No. 7.

$$\text{Here } \begin{cases} hs = 3, ns = 2.75, pq = .6, t = 10 \\ hx = .5, nx = .25, nn - pq = 1.4 \end{cases}$$

$$\text{Hence } \frac{1}{2} \times 3 \times 2.75 \times 6 = 16.50$$

$$\frac{1}{2} \times .5 \times \overline{.25^2} \times \frac{14}{2.75} = .05$$

$$16.55 = s$$

$$\frac{4s}{33} = 2.06 \text{ tons}$$

$$\frac{.22}{2.75} \times \frac{4}{3} = .106 \text{ deflection.}$$

The following experiments have been made subsequently to the publication of my Report to the London and Birmingham Directors.

NORTH UNION OR GRAND JUNCTION RAIL.

182. *Experiments, with the Proving Machine in Woolwich Dockyard, to ascertain the Strength and Stiffness of the Parallel Rail, with double flanch, for the North Union Railway Company. Weight, per yard, 60 lbs.; Area of Section, $6\frac{1}{8}$ inches; Depth, $4\frac{1}{2}$ inches.*

Figure of Section as in the following page.

Note.—The dotted line shows the assumed equivalent right-lined section.

Results obtained from three single Experiments.								Results obtained from the Means of three Experiments.									
Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.			
1	.028	.003	1	.035	.004	1	.027	.004	1	.021	.005	1	.018	.006			
2	.031	.005	2	.039	.004	2	.031	.005	2	.026	.005	2	.024	.004			
3	.036	.002	3	.044	.005	3	.036	.003	3	.031	.005	3	.028	.005			
4	.038	.005	4	.048	.004	4	.039	.005	4	.036	.003	4	.033	.005			
5	.043	.003	5	.054	.006	5	.044	.004	5	.041	.005	5	.037	.004			
6	.046	.004	6	.059	.005	6	.048	.004	6	.044	.003	6	.040	.003			
7	.050	.004	7	.064	.005	7	.052	.004	7	.048	.004	7	.044	.004			
8	.055	.005	8	.069	.005	8	.057	.005	8	.053	.005	8	.048	.004			
9	.060	.005	9	.076	.007	9	.063	.006	9	.059	.006	9	.053	.005			
10	.066	.006	10	.082	.006	10	.079	.007	10	.064	.005	10	.059	.006			
11	.074	.008	11	.086	.004	11	.077	.007	11	.071	.007	11	.067	.008			
12	.084	.010	12	.096	.010	12	.087	.010	12	.081	.010	12	.077	.010			
Mean deflection, } with 11 tons.			.051			.056			.051			.055			.054		

*Computed Strength and Deflection by the rule,
p. 345. See also Example, p. 348.*

$$\begin{array}{ll}
 \text{Head} \dots \left\{ \begin{array}{l} (2.25 - .65) \times 10 = 16 \\ (4\frac{1}{2} - \frac{1}{2}) \times 12 = 48 \end{array} \right\} \dots = 0.33 \\
 \text{Middle rib} \quad \frac{1}{2} \times 4 \times 4\frac{1}{2} \times .65 \times 10 \dots = 39.00 \\
 \text{Lower web} \quad \left\{ \begin{array}{l} 3\frac{1}{2} \times (2.25 - .65) \times 10 = 56 \\ 12 \times 3\frac{1}{2}^2 + 21 \dots = 168 \\ 168 - 6 \dots = 162 \end{array} \right\} \\
 168 : 162 :: 5.6 : 54 \dots = 54.00 \\
 \hline
 93.33
 \end{array}$$

$$\frac{4 \times 93.33}{33} = 11\frac{1}{2} \text{ tons, computed weight or strength;}$$

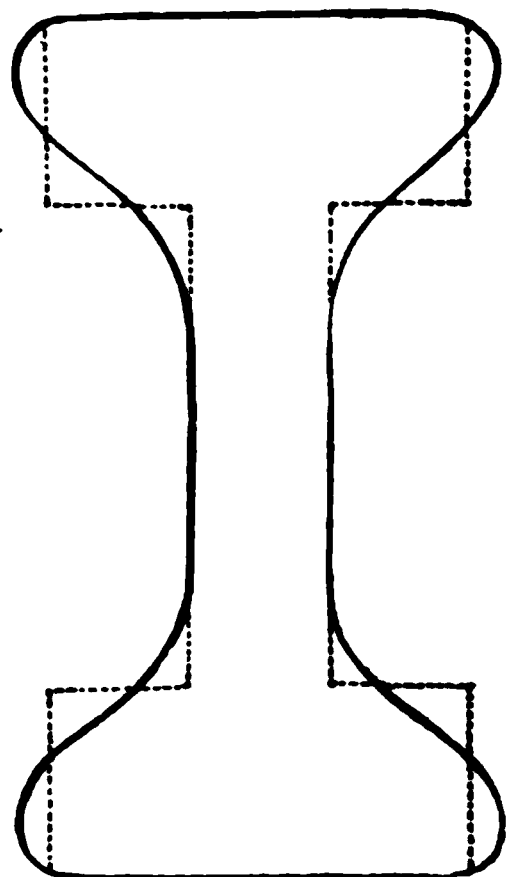
$$\frac{.22}{4} = .055 \text{ computed deflection.}$$

Section of Equivalent Straight-lined Rail.

Weight 60 lbs. per yard.

$$\begin{array}{l}
 nn = 2.25 \\
 nn - pq = 1.6 \\
 ns = 4 \\
 nx = .5 \\
 hx = 1 \\
 hs = 4.5
 \end{array}$$

(See fig. Art. 168.)



183. *Report of Experiments made with the Proving Engine in the Royal Dockyard, Woolwich, to ascertain the Strength and Stiffness of Three Specimens of Railway Bars designed for the Southampton Railway.*

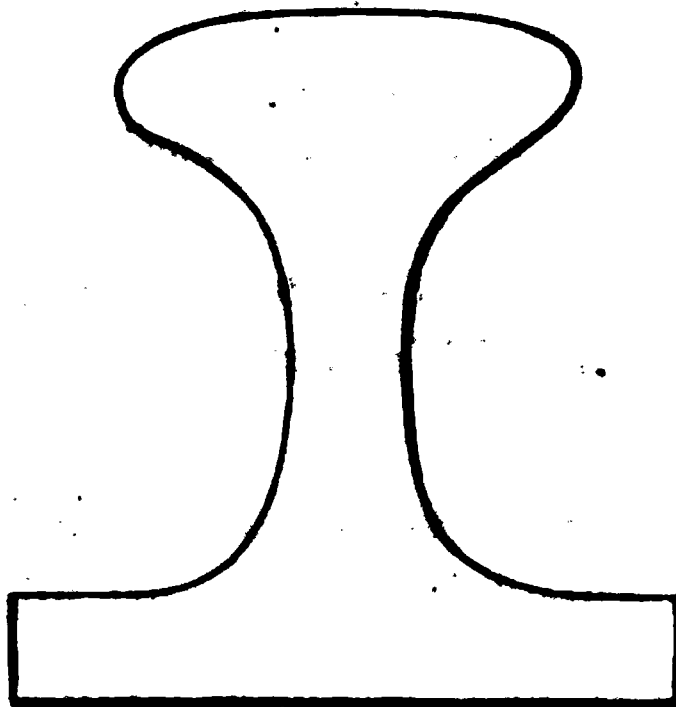
Fig. of Section as in following page.

Present,—COL. HENDERSON, Acting Director : MR. GILES, Engineer ; and WM. REED, Esq., Secretary.

The experiments were made precisely in the same way as is described in my Report addressed to the Directors of the London and Birmingham Railway Company, except that, in consequence of the greater breadth of the lower flanch, the frame I had hitherto used was too narrow to admit the Southampton rail. Another frame was therefore made by Messrs. Gordon and Company for the purpose ; like the other frame, except in the above particular, and that the opening of the frame to form the points of bearing, was by mistake made 34 inches instead of 33 inches. For the sake of comparison, I have therefore reduced the observed strength to 33 inches' bearing ; and also, as the engineer proposes, to have the chairs 5 inches long, giving only a bearing of 31 inches clear. I have also reduced the strength to this bearing. The deflection requires no correction, being measured by the same instrument ; and the observed deflections are those which take place be-

tween the feet of the instrument, independently of the points of bearing. The following are the detail of the experiments.

Section.



Depth of rail, $3\frac{1}{2}$ inches.

Thickness, centre rib, $\cdot 8$ inch.

Breadth, lower flanch, $3\frac{1}{2}$ inches.

Depth of ditto, $\cdot 6$ inch.

Weight, 57 lbs. per yard.

Bar No. 1.			Bar No. 2.			Bar No. 3.		
Strain.	Index readings.	Deflection per ton.	Strain.	Index readings.	Deflection per ton.	Strain.	Index readings.	Deflection per ton.
tons.			tons.			tons.		
2	$\cdot 076$		2	$\cdot 043$		2	$\cdot 030$	
3	$\cdot 087$	$\cdot 011$	3	$\cdot 052$	$\cdot 009$	3	$\cdot 040$	$\cdot 010$
4	$\cdot 097$	$\cdot 010$	4	$\cdot 066$	$\cdot 014$	4	$\cdot 052$	$\cdot 012$
5	$\cdot 110$	$\cdot 013$	5	$\cdot 077$	$\cdot 011$	5	$\cdot 065$	$\cdot 013$
6	$\cdot 122$	$\cdot 012$	6	$\cdot 094$	$\cdot 017$	6	$\cdot 076$	$\cdot 011$
7	$\cdot 137$	$\cdot 015$	7	$\cdot 109$	$\cdot 015$	7	$\cdot 093$	$\cdot 017$
8	Quite destroyed.		8	$\cdot 137$	$\cdot 026$	8	$\cdot 016$	$\cdot 023$
						9	$\cdot 167$	$\cdot 051$

The above bars were in 7½ feet lengths, and the experiments were all made on their middle point. In the following, the experiment was first made on the middle of the length, and then on the middle of one half-length.

Middle, Bar No. 4.			Half-length, Bar No. 4.		
Strain.	Index readings.	Deflection per ton.	Strain.	Index readings.	Deflection per ton.
tons.			tons.		
2	·041		2	·014	
3	·053	·012	3	·024	·010
4	·063	·010	4	·030	·006
5	·071	·008	5	·041	·011
6	·077	·006	6	·054	·013
7	·083	·006	7	·070	·016
8	·108	·015	8	·094	·024
			9	·166	·076

From the above results, it appears, that the mean strength of the bars cannot be stated at more than 7 tons, four out of the five bars showing indications of weakness with that weight. But this is with 34 inches' bearing.

	Tons.
{ This reduced to 33 inches, gives	7½
{ and reduced to 31 inches	7½
{ Mean deflection, estimated per ton	·011
{ Deflection with 3 tons	·033

Computed Strength.

Here the equivalent right-lined section may be taken as follows:

$$hs=3.5 \quad ns=3 \quad pq=.8 \quad nn=2.25 \quad ks=1 \quad ns=\frac{1}{2} \quad nn-pq=1.45 \\ nm=2.7 \quad mm=3.5 \quad rs=.6$$

Hence by the rule, p. 342.

$$\begin{array}{rcl} \text{Resistance of rib } \frac{1}{2} hs \cdot ns \cdot pq \cdot t & \dots\dots\dots & \text{Tons.} \\ & & = 28.0 \end{array}$$

$$\text{Head } \frac{1}{2} ks \cdot ns^2 \cdot \frac{nn-pq}{ns} t \dots\dots\dots = 0.6$$

$$\text{Lower web } \left\{ \begin{array}{l} nm + \frac{rs^2}{12 nm} + ns = \delta' \\ nm \cdot rs \cdot (mm-pq) \frac{\delta''}{\delta'} t = 43.7 \end{array} \right.$$

72.8

Whence $\frac{72.8 \times 4}{.34} = 8\frac{1}{2}$ tons, the computed strength for 34 inches. Whereas the experiment shows a strength of only 7 tons.

I had no hesitation on this ground of reporting the iron bad; and that I was justified in so doing, is shown by the following experiments, which were other specimens from a different maker. Bars 1 and 2, of good medium quality; and bar No. 3, a higher priced iron of superior quality. The character of the section the same, but the centre rib $\frac{1}{2}$ inch more between the flanches.

184. *Report of Experiments made with the Proving Machine in His Majesty's Dockyard, Woolwich, on Three Bars of Iron sent as Specimens for the Railway Bars of the London and Southampton Railway. Dec. 26, 1835.*

Equivalent rectilinear dimensions of the section.	{	Head $2\frac{1}{4}$ inch by 1 inch deep. Whole depth 4 inches. Thickness, middle rib $\frac{3}{4}$ inch. Lower web { Depth $\frac{8}{16}$ do. Breadth $3\frac{1}{2}$ inches.
Mean weight per yard 60 lbs.		

The experiments were performed exactly in the same manner as described in my former Report, in the presence of Col. Henderson, R. E., P. Giles, Esq., Engineer, and W. Reed, Esq., Secretary.

The bearing distance in the frame made for the London and Southampton Railway experiments being 34 inches, and the frame on which my other experiments were made being only 33 inches, I have determined the strength for 33 inches by computation, that these strengths may be more readily compared with the bars, of which the experimental results are given in my printed Reports. I have also found the strength at 31 inches, the bearing proposed by Mr. Giles. The deflections require no correction.

BAR No. 1.

Weight of 9 feet 170 lbs.

Position of bar direct. First trial.			Strain left on 2 hours; experiment repeated in the same place.		
Strain in tons.	Index readings.	Deflections with each ton.	Strain in tons.	Index readings.	Deflections with each ton.
2	·030		2	·037	
3	·0325	·0025	3	·042	·005
4	·034	·0015	4	·045	·003
5	·037	·003	5	·0515	·0065
6	·043	·006	6	·056	·0045
7	·0475	·0045	7	·063	·007
8	·057	·0095	8	·070	·007
9	·065	·008	9	·075	·005
10	·0765	·0115	10	·083	·006
			11	·092	·009
			12	·122	·046

It appears, from these experiments, that although the bar shows great stiffness with the first strains, it yields considerably to the last strains, and that it had taken a permanent set with 10 tons.

The mean deflection per ton of this bar, taken between

5 and 10 tons, 1st experiment ·0079
Ditto ditto 2nd experiment ·0063

Mean ·0071

Mean strength . . 10 tons at 34 inches.

10½ tons at 33 inches.

11 tons at 31 inches.

To try the effect of the lower web, the bar was reversed in position, and another part submitted to the strain.

POSITION REVERSED.		
Strain in tons.	Index readings.	Deflections with each ton.
2	·012	
3	·016	·004
4	·024	·008
5	·027	·003
6	·031	·004
7	·036	·005
8	·041	·005
9	·051	·010
10	·067	·016
11	·082	·015
12	·125	·043
Mean between 5 and 10 tons . .		·008

EXPERIMENTS ON BAR No. 2.

DIRECT AND REVERSED.

Weight of 9 feet 181 lbs.

POSITION DIRECT.			POSITION REVERSED.		
Strain in tons.	Index readings.	Deflections with each ton.	Strain in tons.	Index readings.	Deflections with each ton.
2	·035		2	·041	
3	·040	·005	3	·047	·006
4	·045	·005	4	·052	·005
5	·049	·004	5	·061	·009
6	·055	·006	6	·0655	·0045
7	·062	·007	7	·072	·0065
8	·072	·010	8	·077	·005
9	·0795	·0075	9	·0825	·0055
10	·086	·0065	10	·0905	·008
11	·0905	·0045	11	·099	·0085
12	·105	·0145	12	·113	·014
13	·127	·022	13		
Mean between 5 and 10 tons		·0074	Mean between 5 and 10 tons		·0059

EXPERIMENTS ON BAR No. 3.

BOTH DIRECT.

Weight of 9 feet 179 lbs.

POSITION DIRECT.			POSITION DIRECT.		
Strain in tons.	Index readings.	Deflections with each ton.	Strain in tons.	Index readings.	Deflections with each ton.
2	·037		2	These readings were missed, the strain being brought on too quickly.	
3	·044	·007	3		
4	·052	·008	4		
5	·058	·004	5		
6	·064	·008	6	·031	·009
7	·070	·006	7	·040	·006
8	·0765	·0065	8	·046	·006
9	·084	·0075	9	·052	·006
10	·089	·005	10	·060	·008
11	·096	·007	11	·0685	·0085
12	·107	·011	12	·077	·0085
13	·127	·021	13	·084	·007
Mean between 6 and 11 tons }		·0064	Mean between 6 and 11 tons }		·0074

GENERAL MEAN RESULTS.

	Mean strength at			Mean deflection.
	34 in. bearing.	33 inches.	31 inches.	
	Tons.	Tons.	Tons.	Per ton.
BAR No. 1.	10	10½	11	·0071
No. 2.	11	11½	12	·0074
No. 3.	12	12½	13½	·0069

To the Directors
of the London and Southampton
Railway Company.

Computed Strength.

Here the equivalent right-lined figure gives

$$hs = 4 \quad ns = 3.5 \quad pq = .75 \quad nn = 2.25$$

$$hx = 1 \quad nn - pq = 1.5 \quad nn = 3.2 \quad mm = 3.5 \quad rs = .6$$

Hence by the rule, page 342,

$$\text{Resistance of rib } \frac{1}{3} hs . ns . pq . t \quad . \quad . \quad . \quad = \overset{\text{tons.}}{35.0}$$

$$\text{Do.} \quad \text{head } \frac{1}{3} . hx . nx^2 . \frac{nn - pq}{ns} . \quad . \quad . \quad = 0.7$$

$$\text{Lower web} \begin{cases} nm + \frac{rs^2}{12 nm} + cn = \delta'' \\ nm . rs . (mm - pq) \frac{\delta''}{d'} t \quad . \quad . \quad = 52.8 \end{cases}$$

$$88.5$$

And then $\frac{88.5 \times 4}{34} = 10\frac{1}{2}$ computed strength, which agrees with the mean of the two medium bars as nearly as possible.

The third bar, as has been stated, was of a superior description of iron.

185. Report of Experiments made on Three Bars, for the Southampton Railway Company, from the same Iron Works as the first set; March 12th, 1835.

Present, W. RENN, Esq., Secretary.

Inches.			Inches.		
Depth	34	Breadth of centre rib	34		
Depth of lower flanch	6	Breadth of lower flanch	34		
Weight 57 lbs. per yard.					
Strain in tons.	Index readings.	Deflection for each ton.	Strain in tons.	Index readings.	Deflection for each ton.
1	0575		1	0050	
2	0680	0105	2	0150	0100
3	0790	0110	3	0250	0100
4	0900	0110	4	0360	0110
5	0970	0070	5	0450	0090
6	106	0090	6	0540	0090
7	120	0140	7	0660	0120
8	128	0080	8	0880	0220
9	149	Destroyed	9	109	Destroyed

By comparing the above results with those obtained on the bars first tested, the strength and stiffness will appear to be very nearly the same, except bar No. 1, which retained its elasticity with 8 tons. Bars No. 2 and No. 3, cannot be said to have borne more than 7 tons at 34 inches' bearing; but reduced to a bearing of 31 inches, the strengths will be as follow :

Bar No. 1. Strength at 31 inches' bearing	tons.	84
No. 2. do. do.		74
No. 3. do. do.		74
Bar No. 1. Deflection 3 tons		024
No. 2. do. do.		023
No. 3. do. do.		020

It appears, therefore, that No. 1 is the strongest bar, and No. 3 the stiffest. Upon the whole, the bars are nearly the same as those first sent; as will be observed in referring to my Report on them.

I believe that some improvement was attempted to be made in the manufacture of these bars, but it is clear that the metal itself is defective. And nothing, perhaps, could have better proved the accuracy of the rules I have given, nor the propriety of testing the bars when delivered from the maker, as recommended in my first Report to the Directors of the London and Birmingham Railway, than the preceding experiments.

186. The following are experiments made on two specimens of iron in bars of 75 to 77 lbs. per yard, intended for 5 feet bearings.

Report of Experiments made on the Testing Machine in His Majesty's Dockyard, Woolwich, on two Specimens of Railway Bars, viz.:

Two bars, maker not known.

Two bars from Messrs. Solly, best patented.

First specimen.	Section, double flanch with centre rib, similar to fig. Art. 182.
	Greatest breadth of flanch 2·6 inches.
	Mean depth $1\frac{1}{2}$ inch. Whole depth of rail 5 inches.
	Mean breadth of flanch 2·125 inches.
	Thickness of centre rib ·85 inch.
	Weight not stated, but about 75 lbs. per yard.
Best patented.	The same dimensions, rather more full.
	Thickness of centre rib ·9 inch.
	Weight of one of these bars 3 cwt. 1 qr. 20 lbs., or 77 lbs. per yard; of the other, 3 cwt. 1 qr. 12 lbs., or $75\frac{1}{2}$ lbs. per yard.

The experiments were performed as before, except, that in consequence of these bars being intended for 5 feet bearings, the iron frame was obliged to be altered; and that it might answer both for those bars designed for 4 feet bearings as well as 5 feet, it was lengthened to 4 feet 6 inches, and proportionally strengthened, which, as I understand the experiments to be only comparative, seemed to answer both cases without having a new frame made.

The difference in the strength of the two specimens, it will be seen, is very considerable, although

the stiffness at first is nearly the same: the first specimen is, however, rather the stiffest, but the other much the strongest; the elasticity or restoring power being preserved up to a strain of 10 tons in the latter, and only to 8½ tons in the former, at a bearing of 4 feet 6 inches. Or reducing both to 5 feet bearing, we have for the greatest load that can be safely borne,

	Tons.
First specimen	7·65
Best patented	9·00

But the deflection per ton with

	Inches.
First specimen	·0165
Best patented	·0175

In the computation I made in my last Report, it was intended the bars should bear 8 tons, at 5 feet bearings. It appears, therefore, that the strength of the former is rather less than ought to be expected of good medium iron, and that the other is in excess of strength 1 ton.

The following are the experiments from which these deductions have been made:

FIRST SPECIMEN.

BAR No. 1.

Strain in tons.	Index readings.	Deflection per ton.
1		
2	·050
3	·067
4	·075
5	·092
6	·107
7	·122
8	·142
9	·165
	 Injured very little.
10	.. { Elasticity quite destroyed.	·016 Mean deflection per ton.

BAR No. 2.

1		
2	·032
3	·045
4	·062
5	·085
6	·102
7	·121
8	·136
	 Mean per ton ·017.
9	·171
10	·255

BEST PATENTED.

BAR No. 1.

Weight, 3 cwt. 1 qr. 20 lbs.

Strain in tons.	Index readings.	Deflections per ton.
1		
2	·036	
3	·045	·009
4	·066	·021
5	·086	·020
6	·096	·010
7	·110	·014
8	·128	·018
9	·149	·021
10	·168	·019
11	·188	·020
Mean per ton ·017.		
12	·210	·023

BAR No. 2.

Weight, 3 cwt. 1 qr. 12 lbs.

1		
2	·054	
3	·064	·010
4	·084	·020
5	·105	·021
6	·120	·015
7	·140	·020
8	·161	·021
9	·180	·019
Mean per ton ·018		
10	·207	·027
11	·244	·037
12	·315	·071

*To the Directors
of the London and Birmingham
Railway Company.*

Woolwich, Oct. 31st, 1836.

187. *Report of Experiments on Two Railway Bars received October 27th; manufacturer's name not stated, nor the weight, but by the section about 65 lbs. per yard. Double flanch, whole depth $4\frac{1}{2}$ inches, intended for 4 feet bearings.*

Tested at $4\frac{1}{2}$ feet bearings, the same as those tested on the 26th and 27th instant.

Bar No. 1.			Bar No. 2.		
Strain in tons.	Index readings.	Deflection per ton.	Strain in tons.	Index readings.	Deflection per ton.
1			1		
2	·048		2	·064	
3	·072	·024	3	·082	·018
4	·091	·019	4	·106	·023
5	·110	·019	5	·125	·020
6	·131	·021	6	·145	·020
7	·153	·021	7	·165	·020
8	·177	·024	8	·186	·021
9	·199	·022	9	·211	·022
10	·222	·023			
11	{ Elasticity destroyed.		10	·275	·065 { Elasticity gone.
Mean deflection per ton. Bar No. 1, ·022 inch.			Mean deflection per ton. Bar No. 2, ·021 inch.		

Mean strength of the two bars $9\frac{1}{2}$ tons, at 4 feet 6 inches' bearing, or 10·2 tons at 4 feet.

The Directors cannot but observe the striking fact elicited by these and the preceding experiments on the bars Nos. 1 and 2: viz.

That 65 lbs. per yard is 1 ton stronger at the same bearing distance with these bars than with the other at 75 lbs. per yard; that is, with $13\frac{1}{2}$ per cent. less weight there is 12 per cent. very nearly more strength. Now whether this proceeds from a difference of the ore, a difference in the mode of

manufacture, or from the difficulty of manufacturing such large bars, I cannot tell; but it is a question which appears to me to be very deserving the attention of the Committee.

Taking into account the difference in the depth of the two specimens, the proportional stiffness is very nearly the same.

These experiments, again, as compared with the preceding, show the strong necessity of some mode of testing; as a Company may otherwise be liable to purchase bars at a great expense actually weaker than others of less cost, not only in the gross, but per ton; for I have since learned that these latter bars were bought at less per ton than the former.

MISCELLANEOUS EXPERIMENTS

**CONNECTED WITH RAILWAYS, BEING THE SUBSTANCE OF A SECOND
REPORT ADDRESSED TO THE DIRECTORS OF THE LONDON AND
BIRMINGHAM RAILWAY COMPANY.**

188. As explanatory of the object of the following experiments, it may be well to state the resolution of the general meeting which gave rise to their being undertaken.

EXTRACT.

“RESOLVED UNANIMOUSLY—That Professor Barlow be requested to visit the Liverpool and Manchester Railway, to view that line, and advise this Board as to the weight of rails, the description of chairs and fastenings, the distance of the supports, and the size of the blocks that he would advise the Directors to adopt; and to accompany such advice with any observations generally on the subject.”

It was accordingly arranged, conformably with this resolution, that I should visit Liverpool with the Chairman, Isaac Solly, Esq., and Thomas Tooke, Esq., one of the London Directors, and there meet T. W. Rathborn, Esq., and Edward Cropper, Esq., two of the Liverpool Directors; that they should

accompany me in my inspection, and while making such experiments as that inspection might render desirable;—the Liverpool and Manchester Directors, on their part, most handsomely offering every assistance and facility in their power, by placing at the disposal of the deputation the *Swiftsure* locomotive engine, with such carriages as might be required for our purpose, and every other accommodation we might desire.

We met, as appointed, at the Liverpool station of the Liverpool and Manchester line, and employed the first day in examining the state of the rails, chairs, and blocks, modes of fixing, and other particulars. In the course of this examination, I took the opportunity of inquiring, on the spot, the opinion of the resident engineers, contractors for repairs, workmen, and others, relative to these several points; but I was much disappointed to find those opinions, in most instances, discordant, and in many directly contradictory; a circumstance the more remarkable, as one would have thought that five years' incessant practice would have been sufficient to eradicate many early erroneous ideas.

I am not myself a practical man, but from my situation and pursuits I have been for nearly thirty years in almost constant intercourse with two of the largest and most varied mechanical establishments in the kingdom, and have, during that time, witnessed or superintended a vast number of experiments and trials on various mechanical subjects, many of which

I have afterwards been enabled to examine in the works at large : I am, therefore, to a certain extent, acquainted with what theory gives, and what practice requires, and the limits it prescribes ; as I am also with the views and arguments of practical men, who, I know, sometimes, like other persons, in their anxiety to avoid one evil, lose sight of other collateral evils, which their remedy increases or creates ; but I must say that I never saw this so strongly marked as on the present occasion, nor such a diversity of conflicting opinions on what appears so simple and plain a case. This is a circumstance much to be regretted, not only as regards the doubts which it naturally throws upon the minds of proprietors, embarking large amounts of capital in the undertaking, but also in respect to practical men themselves, whose judgment must suffer depreciation by such discordance. Opinions derived from long experience are exceedingly valuable, and outweigh all others, while they are consistent with facts and with each other ; but they are worse than useless when they lead, as in this instance, to directly opposite conclusions.

In making these remarks, I beg to be understood as intending no disrespect to the opinions of practical men generally, but simply to show that it was impossible, in this case, for me to be guided by them ; and thereby to justify the plan I soon determined to adopt, viz., to avoid, as far as possible, argument founded on mere hypothesis, and to substitute for the latter, facts drawn from actual experiments, which

should be made publicly, registered generally, and witnessed by any one interested in the decision; and moreover, as I intended to rest my Report entirely on these data, I resolved to offer no opinion, till I had time to analyze and compare my results.

EXPERIMENTS.

189. The first and most important point which required to be decided was, the strength of iron necessary to insure the most ample safety, at any practicable speed, with any given load and given length of bearing. The strain which any quiescent load impresses on a bar, is, I think, now well known; but what is the effect of velocity? This was one of those questions on which I found opinions greatly divided; and it was a question, perhaps, considered merely hypothetically, in which there was great room for doubt. My first object, therefore, was to reduce it to a matter of experimental fact: this rendered it necessary to construct an instrument for the purpose, and I feel myself much indebted to Mr. King, of the Liverpool Gas Works, for the ready attention he paid to my suggestions, and for the ingenuity he exercised in giving it its first form, the whole of which was left to his own invention, after being simply informed of its object, and the general mode of its intended operation.

This instrument, which it is proposed to call a

deflectometer, is represented in plan and elevation in the annexed diagrams. A B is a plain board about 47 inches long and 6 inches broad, with two pillars or standards, one of which is seen in the elevation; and between them is suspended the lever D E by screw points, divided in C, in the proportion of 10 to 1; G H is a slightly inclined stout wire, on which slide the two indexes i, i, but with sufficient friction to remain in their places.

DEFLECTOMETER.

ELEVATION.



PLAN.



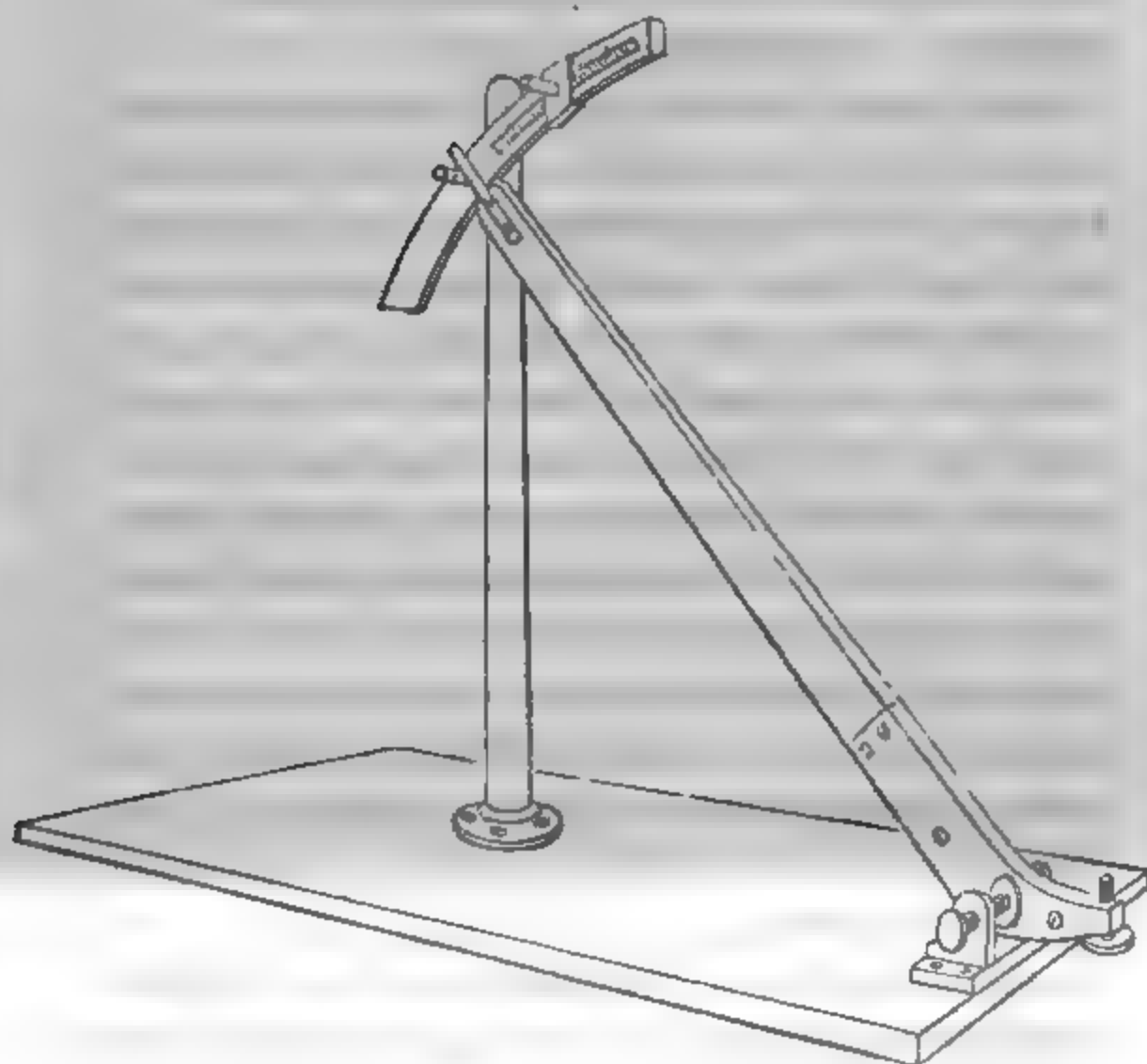
The manner of using the instrument is by leveling the ground under the centre of the rail, and placing the point E under its lower edge; the preponderance then being on the side of the long arm, the point E is kept in contact with the lower edge of the bar, and the lower index *i* is moved up to the metal plate *k*; the upper one is then, in like manner, brought down and placed in contact also. It is obvious, now, that whatever deflection the rail may sustain during the passage of an engine, or a train of waggons, the index *i* will be lifted ten times the quantity the bar is deflected, and remaining in its place, the greatest deflection the bar has sustained will be truly and distinctly indicated.

The first instrument having been constructed, (the men working all night to finish it,) and my intention being known of using it in the morning, I was much gratified to meet on the ground, on that and the following day, several directors and proprietors, engineers, and practical men interested in the inquiry. Of the former I may name Isaac Solly, Esq., Chairman; Messrs. Thomas Tooke, Henry Rowles, Theodore W. Rathborn, Edward Cropper, Robert Garnet, Edward Wilson, Hardman Earle, David Hodson, Directors, or very extensive Proprietors; and of the latter, Messrs. R. Stephenson, J. Locke, C. Vignoles, Civil Engineers; Mr. Dixon, Resident Engineer; Captain Moorsom, one of the Secretaries of the London and Birmingham Company; and

Mr. Booth, the experienced and intelligent Secretary of the Liverpool and Manchester line.

Our first experiments were only tentative, with a view to try the instrument, but even in these, much was very distinctly shown: when, for example, a train passed over, we could see clearly the operation of each wheel upon the rail, which, where these were well laid, and the joints and blocks secure, were only of a certain amount, but when the rails were unlevel, or other irregularities occurred, some lurch would take place, towards the middle or end of the train, which would strike the rail with sufficient force to throw up the index to nearly double its previous amount, indicating, of course, that it had, in the case in question, sustained a deflection nearly double what it would have done with the same weight in a quiescent state.

An improved form of this instrument is represented in the following page, but the principle of its action is the same. We found in the first instrument an inconvenience from the index being so near the ground, and in order to avoid this, Mr. W. Gilbert, 148, Leadenhall Street, gave it the form shown in the figure. The register here is by a sliding vernier on an arc; the latter also being raised, the result may be read with great ease and convenience. The upright stand carrying the arc is a brass tube which fits tightly over a brass pin on the base board. It may, therefore, be easily removed, and the whole packed very close for convenience of carriage.



190. *Experiments made with a view to ascertain the Strain which a Load in rapid Motion produces upon the Rail over which it passes, in order to compare the same with the known Strain produced by an equal quiescent Load.*

It may be proper to state in this place, that the Directors of the Grand Junction Railway Company having resolved that their rails should not weigh less than 60 lbs. per yard, and their engineer, Mr. J.

Locke, finding that with this weight of iron, or rather with bars of 62 lbs. per yard, and with 3 feet bearing, there would be much more strength than the present or any contemplated future practice would require, very properly took advantage of the superabundant strength which the bar possessed to increase the distance, and thereby to reduce the number of the bearing blocks; but he would not, as a practical man, whose character might suffer by a failure, undertake to recommend, with the rail in question, a longer bearing than 3 feet 9 inches, which of course saved one-fifth of the usual number of blocks. But an experiment which it was unadvisable for an engineer to undertake, might be made without scruple by a private gentleman (no immediate mischief being to be apprehended); and accordingly, at the suggestion of Hardman Earle, Esq., another block was dispensed with, and the bearings extended to 5 feet: several yards of this rail are thus laid in the main line near the Liverpool end, and the same with 3 feet 9 inches bearing, as also other experimental rails of different forms and weights. The rails at 3 feet 9 inches, and at 5 feet bearing, have been laid down since May last,¹ without having sustained any observable injury; but it was desirable to examine their strength and stiffness, and the strains to which they are exposed, before such bearings

¹ The experiments were made the latter end of the August following, *i. e.* in 1835.

could be safely recommended for general adoption. All the other rails throughout the line are laid on 3 feet bearings, and vary in weight from 35 lbs. to 60 lbs. per yard.

The little instrument of Mr. King's was admirably suited to this inquiry, for by this the greatest deflection the bar sustains, from whatever cause it proceeded, was accurately registered, and by comparing this deflection with the experiments made on the same bar with quiescent loads, the effects due to velocity, and those proceeding from irregularities in the joints, &c., became known, at least in the aggregate, and this aggregate is of course the strain against which it is necessary to provide.

Experiments on the central Deflection of Railway Bars during the passage of a heavy Load at different Degrees of Speed; and on different Lengths of Bearings.

191. Our observations were commenced in and near the cutting at Wavertree Hill, in rock cutting, the ground being as sound, and the bearings as firm, as in any part of the line.

The first trials were made on the Grand Junction Rail laid down in May, on 3 feet 9 inches' bearings. The weight of rail 62 lbs. per yard. A deflectometer was accurately placed under each of four

bearing lengths—one having been selected next the bearing end, the other three were middle lengths. The following were our recorded observations :

FIRST EXPERIMENT.

With the passage of the *Speedwell* engine and train, at a medium velocity, or about 20 miles per hour : this showed—

Deflection of joint length	.	.	.	·0625	inch.	
Ditto	middle length	.	.	·0425		} mean ·0408
Ditto	ditto	.	.	·0400		
Ditto	ditto	.	.	·0400		

SECOND EXPERIMENT.

With the *Swiftsure* engine, furnished for the experiments : weight on driving wheels, 5 tons 16 cwt.; velocity about 20 miles per hour.

Deflection of joint length	.	.	.	·0800	inch.	
Ditto	middle length	.	.	·0320		} mean ·0380
Ditto	ditto	.	.	·0400		
Ditto	ditto	.	.	·0420		

THIRD EXPERIMENT.

The same engine, very slow :

Deflection of joint length	.	.	.	·040	inch.	
Ditto	middle length	.	.	·024		} mean ·027
Ditto	ditto	.	.	·025		
Ditto	ditto	.	.	·032		

FOURTH EXPERIMENT.

One trial, quite at rest ·040

The mean of the above three means is ·0353

To compare this with the mean deflection of such a bar, with a quiescent load, I may refer to the experiments on the same bars at Woolwich, forwarded for the purpose by the Directors of the Grand Junction line, (Art. 182,) by which it appears that the mean deflection per ton, at 33 inches clear bearing, was $\cdot 0050$; consequently, for three tons, $\cdot 0150$; and reducing this to the clear bearing of $45 - 3 = 42$ inches, we have as $33^3 : 42^3 :: \cdot 0150 : \cdot 0314$, the deflection with three tons at rest; and the mean of the preceding deflections in motion is $\cdot 0353$, a close agreement, which shows, that when every thing is well fixed and secure, the deflection, and consequently the strain, is nearly the same, whether the load be in motion or at rest, and that each rail is only pressed with half the weight of one pair of wheels.

Experiments on the same Bars at Five Feet Bearing.

FIFTH EXPERIMENT.

SWIFTSURE ENGINE.—VELOCITY ABOUT TWENTY-TWO MILES.

	v. = 22.	v. = 22.	v. = 22.
Deflection middle length	$\cdot 093$	$\cdot 077$	$\cdot 080$
Ditto joint length	$\cdot 083$	$\cdot 080$	$\cdot 123$
Ditto ditto	$\cdot 108$	$\cdot 143$	$\cdot 130$
Ditto middle length	$\cdot 082$	$\cdot 070$	$\cdot 077$

WITH GREATER VELOCITIES.

	Speedwell. v. = 30.	v. = 32.	Fury train. v. = 23.
Deflection middle length	$\cdot 112$	$\cdot 122$	$\cdot 083$
Ditto joint length	$\cdot 080$	$\cdot 105$	$\cdot 085$
Ditto ditto	$\cdot 250$	$\cdot 120$	$\cdot 095$
Ditto middle length	$\cdot 091$	$\cdot 115$	$\cdot 085$

In obtaining a mean from these results, the deflections on the joint lengths are, as in the preceding case, rejected, being obviously in excess. The mean of the rest, that is, of the central length, is $\cdot 089$.

In my experiments at Woolwich, the deflection per ton at 33 inches' bearing being $\cdot 0050$, or, for 3 tons, $\cdot 0150$, we have, deducting 3 inches from 60, to obtain the clear bearing—

$$83^8 : 57^8 : : \cdot 0150 : \cdot 079,$$

while the mean determined by the deflectometer, as we have seen, is $\cdot 089$.

Nothing can be expected much more satisfactory; as it is thus proved, *independently of any opinion*, that while the blocks and fixings are secure, the strain from a passing load is but little in excess of that from a quiescent load: whereas the effect on the joint ends amounts, from a mean of the preceding, to $\cdot 121$, being in excess nearly 40 per cent. This, however, is not all strain, part being due to the looseness of the chair or block.

192. *Continuation of the Experiments on the Deflections of different Rails and Blocks on the Liverpool and Manchester Railway.*

DUBLIN AND KINGSTOWN PARALLEL RAIL.

Weight 45 lbs. per yard; with a lower web; bearing distance 3 feet; fixed by vertical keys; depth $3\frac{1}{8}$.

SWIFTSURE ENGINE.

	Deflections in parts of inches.						Means.
Joint length . .	·120	·120	·106	·167*	·177*	·106	} ·114
Ditto . .	·120	·084	·098	·090	·080	·098	
Middle length . .	·125	·110	·130	·130	·156*	·130*	} ·120
S Ditto . .	·110	·103	·106	·112	·120	·106	

The deflections marked with an asterisk are remarkable instances of the effect of the lurching of the engine and carriages, spoken of in the Report as amounting to nearly double the smaller and more natural deflections.

In the above experiments the blocks were sounded, and found firm; the fixings also appeared to be secure at the time of making the experiment; but generally the vertical keys used with this rail, require, according to the report of the workmen, incessant attention.

MR. STEPHENSON'S FISH-BELLIED RAIL.

Weight $43\frac{1}{2}$ lbs. per yard; bearings 3 feet, fixed by iron keys on the side; great depth, $4\frac{1}{2}$, less ditto, $3\frac{1}{2}$.

SWIFTSURE ENGINE.

	Deflections.					
1 Joint length . .	·032	·040	·038	·027	·045	
2 Ditto . .	·070	·170	·068	·130	·077	
3 Middle length . .	·125	·130	·130	·170	·093	
4 Joint length . .	·030	·025	·030	·028	·056	

The blocks of Nos. 2 and 3 were loose.

The mean of the other deflections is $\cdot 034$, but we have no experiments to compare with.

*The same Experiments repeated on four other Rails :
velocities not recorded.*

Middle length	. $\cdot 105$	$\cdot 135$	$\cdot 100$	$\cdot 150$	} Mean $\cdot 062$
Ditto . . .	$\cdot 035$	$\cdot 050$	$\cdot 047$	$\cdot 053$	
Ditto . . .	$\cdot 075$	$\cdot 075$	$\cdot 070$	$\cdot 085$	
Ditto . . .	$\cdot 065$	$\cdot 060$	$\cdot 070$	$\cdot 060$	

The great discrepance between the means in these two sets of experiments is very remarkable; I am quite unable to explain the cause from any fact I am acquainted with.

THE RAILS ON THE ST. HELEN'S LINE.

Parallel, with lower bead; weight, 43 lbs. per yard; bearings, 3 feet.

SWIFTSURE ENGINE.

Joint length	. $\cdot 110$	$\cdot 092$	$\cdot 115$	$\cdot 095$
Middle ditto	. $\cdot 060$	$\cdot 075$	$\cdot 100$	$\cdot 068$
Joint ditto . .	$\cdot 070$	$\cdot 080$	$\cdot 148$	$\cdot 135$
Middle ditto . .	$\cdot 082$	$\cdot 045$	$\cdot 063$	$\cdot 045$

Mean deflection of joint length, $\cdot 105$; of middle lengths, $\cdot 067$.

MR. BOOTH'S NEW RAIL.

Parallel, with equal upper and lower flanch; weight, 60 lbs. per yard; depth, 4 inches; bearing distance, 3 feet.

94

SWIFTSURE ENGINE.

Middle length . .	·066	·062	·066
Joint ditto . . .	·038	·064	·050
Ditto ditto . . .	·100	·042	·144 inch.
Middle ditto . . .	·040	·042	·044

The deflectometers were removed from the above two joint lengths; the other two remained the same.

Middle length . .	·062	·064	·064
New joint ditto . .	·048	·064	·042
Ditto ditto . . .	·074	·062	·050
Middle ditto . . .	·056	·060	·064

Mean of the four middle lengths, ·056

Parallel Plain T Rail.—Huyton Plane.

Weight, 50 lbs. per yard; bearing, 3 feet; laid down ten months; depth $3\frac{1}{2}$ inches.

		Vesta train.	Sampson train.	
1st Middle length . .	·088	·070	} Mean	·067
2nd ditto . . .	·072	·066		
3rd ditto . . .	·052	·044		
4th ditto . . .	·068	·080		

SWIFTSURE ENGINE.

	Slow	Velocity 12.	V. 15.	
1st	·064	·084	·082	} Mean
2nd	·065	·080	·082	
3rd	·048	·060	·060	
4th	·072	·080	·086	

General mean, ·0695.

On Chat Moss.

MR. R. STEPHENSON'S FISH-BELLIED RAIL
CHAIR.

Weight, 44 lbs. per yard, 3 feet bearing on wooden sleepers. The four deflectometers were here applied to two blocks and two rails, but not adjacent, and the disturbance on the blocks and rails observed together as below :

SWIFTSURE ENGINE.

	Deflections.				Means.
1 Block . . .	·058	·060	·060	·060	·059
2 Middle rail .	·176	·178	·200	·198	·188
3 Block . . .	·030	·028	·040	·032	·032
4 Joint block .	·152	·160	·160	·170	·160

Experiments repeated.

The rails and blocks being now selected, so as to have one rail between the two blocks, and the other adjacent, the results were—

	Deflections.					Means.
1 Block . . .	·018	·018	·018	·022	·023	·019
2 Rail between .	·178	·195	·190	·194	·196	·191
3 Block . . .	·050	·056	·060	·056	·060	·056
4 Rail adjacent .	·136	·124	·154	·130	·124	·134

These last results, as in the other fish-bellied rails, are very anomalous. In the present instance, we may suppose a great deal is to be attributed to their

peculiar situation, as the whole road trembled under our feet as the engine passed; but still the great excess of deflection of the rail, beyond that of the disturbance shown by the block, is very unaccountable, although some of it may be due to the working of the segmental piece in this particular chair. Still, however, after every allowance, I must think there are obvious indications of the rails being much more strained in such a situation as this, than on a good bottom; and should this be verified by further observations, it would certainly be advisable in future, in such cases to strengthen the rails, either by enlarging them beyond the dimensions given in the other part of the line, or, which would amount to the same, preserving the dimensions, and reducing the bearing distance.

The speeds, in the last two sets of experiments, varied from 15 to about 21 miles per hour.

193. *Experiments on the lateral Deflection of Railway Bars.*

Having ascertained the deflection of the bars in a vertical direction, it occurred to me that it would be very desirable to determine also to what extent the rails were deflected laterally on the outer sweeps of curves, in order that I might, if it should be found necessary, increase the thickness in the longer bearing rails, beyond what mere strength required, in order

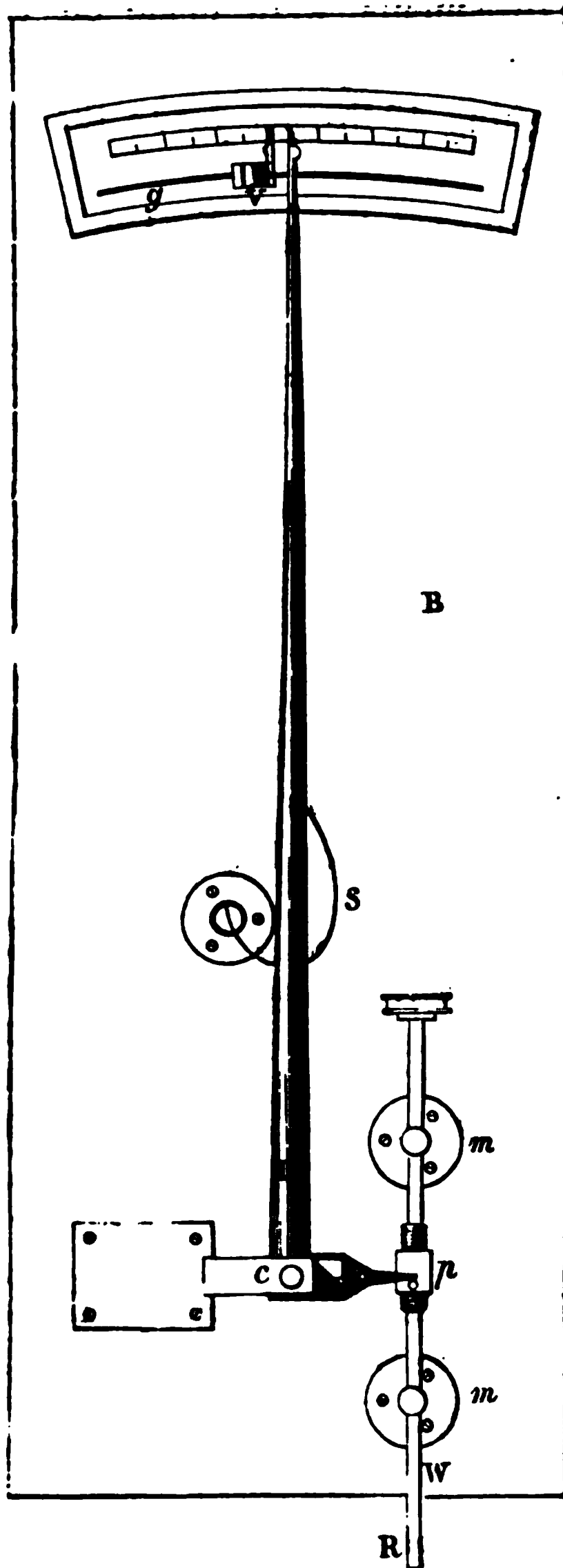
to counteract this necessarily greater strain. I accordingly wrote to T. W. Rathbone, Esq., to beg that he would have the goodness to get these experiments made, giving him a very rough sketch of the method I proposed to have employed; and I find myself again greatly indebted to Mr. King for working out my idea in a most satisfactory way, by the construction of the instrument described hereafter; as I am also to Mr. Edward Woods, for the detail of the series of experiments which follow.

The whole of these experiments have a tendency to show, that the stress which the bars have to sustain in this direction is not such as to require to be more amply provided for than the increased thickness the bar must have, to meet the greater vertical strain due to the longer bearing. In other words, the additional strength given to the bar, for the purpose of meeting the vertical strain, will be amply sufficient to meet and resist the lateral strain. It will therefore not be necessary, in proportioning the weights and sections of bars for different lengths of bearing, to attend to more than the vertical strength.

The following description of the instrument, and one set of experiments, will be sufficient for illustration.

Description of the Instrument.—In the annexed figure, L is a bent lever, turning on a centre *c*; V a vernier, sliding in the groove *g*; S a steel spring, to keep the short end of the lever in contact with the stud *p*, to a wire sliding in the standards *m*, *m*,

having an adjusting screw at *p*, to set the index to zero. The end *R* being now brought into contact with the rail, the stud *p*, on the passage of the engine, will press upon the short arm of the lever to the extent of its deflection, the amount of which, ten times multiplied, will be read on the scale or vernier at *V*.



"The experiments were made on the Wigan Railway, with the engine 'Experiment:' the rail parallel weighing 42 lbs. per yard; the bearing distances, 3 feet.

"The instrument being adjusted, the following results were obtained :

	Deflection.	Velocity.	Direction of the engine.
Exp. 1.	·047	8 miles per hour.	Back.
2.	·045	10 ..	Forward.
3.	·038	11 ..	B.
4.	·036	12 ..	F.
5.	·040	10 ..	B.
6.	·035	12 ..	F.

"The same experiment repeated, after the middle chair between two others was removed; the clear bearing now being 5 feet 10½ inches :

	Deflection.	Velocity.	Direction of the engine.
Exp. 1.	·070	4 miles per hour.	Back.
2.	·078	6 ..	Forward.
3.	·093	7 ..	B.
4.	·097	8 ..	F.

Continuation of the Experiments on Lateral Deflection, made on the Wigan Railroad, 10th September, 1835. By Mr. Edward Woods.

"The rails are of the parallel form, weight 42 lbs. per yard; bearings, 3 feet.

“ *1st Series.*—On the curve near the junction to the Liverpool and Manchester Railway.

Curve = 2 feet 4 inches per chain.
= to a radius of 622 yards.

“ The outer rail of the curve $1\frac{3}{8}$ inch higher than the inner rail, to counteract the centrifugal force of the trains.

“ Deflection (lateral) of an outside rail, 1 ft. 6 in. from the bearing. Engine, *Experiment*.

Defl. in inches.					
No. 1.	·040	10 miles per hour.	
2.	·024	8	ditto.
3.	·026	8	ditto.
4.	·022	14	ditto.
5.	·007	10	ditto.

“ *2nd Series.*—Another rail on the outside of the curve, same engine, &c. as before.

Defl. in inches. Miles per hour.					
No. 1.	·000	13 F.
2.	·018	10 B.
3.	·000	9 F.
4.	·023	9 B.
5.	·017	11 F.
6.	·060	8 B.
7.	·031	10 F.
8.	·055	9 B.
9.	·042	12 F.
10.	·086	11 B.

“ N.B.—The letters F. and B. denote whether the engine was working forwards or backwards.

“ *3rd Series.*—With a rail exactly opposite that of the second series, viz., on the inner rail of the curve.

“ In this and in all other experiments, the deflection was measured outwards from the centre of the road.

“ In this instance the deflection seemed to arise solely from the wedge-like action of the conical tire on the wheels, as some paint which had been smeared on a few yards on the inner side of the rail had not been wiped off; showing that the flange had not come into contact with the rail. Engine, the *Experiment*.

		Deflect. inches.		Miles per hour.		
No. 1.	·030	8	B.
2.	·030	9	F.
3.	·040	9	B.
4.	·040	10	F.
5.	·030	4	B.
6.	·000	2	F.
7.	·037	3	B.
8.	·002	2	F.
9.	·033	3	B.
10.	·001	2	F.
11.	·006	6	{ ‘Jupiter’ with a coach train.	

“ 4th and 5th Series are given in the Report.

“ 6th Series.—With a rail on the straight road. Engine, the *Experiment*.

		Deflect. inches.		Miles per hour.		
No. 1.	·010	8	B.
2.	·010	14	F.
3.	·010	15	B.
4.	·002	10	F.

“ 7th Series.—Another rail near the same place. Engine, the *Experiment*; weight of working wheels, 5 tons 15 cwt. 1 qr.

		Deflect. inches.		Miles per hour.		
No. 1.	·032	16	B.
2.	·032	12	F.
3.	·020	13	B.
4.	·010	5	F.
5.	·008	4	B.
6.	·010	4	F.
7.	·046	25	B.
8.	·020	18	F.

(Signed)

“ EDWARD WOODS.”

As the velocities are not the same in these experiments, except the first of the first series and the last of the second, we can only make this one comparison, and by this the deflection appears to be about double, which is certainly less than calculation would lead us to expect; but the amount is so far within the elastic power of the iron, and the strength of the rail experimented on so inferior to what will probably be adopted, that I am quite satisfied no additional strength will be required to meet this strain.

The above experiments were made by Mr. Edward Woods and Mr. King, in the presence of T. W. Rathbone, Esq., Dr. S. Trail, of Edinburgh, and J. Reynolds, Esq., of Swansea.

DEDUCTIONS.

194. It would be useless to go through a comparison of all the experiments noted in this and the preceding section; I shall therefore only observe, referring to the vertical deflections, that the obvious deduction from them is, that with firm blocks, chairs well fixed, and with joints well made, the road itself being firm, the rail is only deflected at the greatest velocity a little more than is due to a quiescent load equal to half the weight on the two wheels; but that in consequence of the imperfection of these parts, a strain is occasionally thrown on the rail which produces a deflection about double that which belongs to the load in question. This effect was frequently and obviously exhibited in the experiments with the trains. In many cases, the deflectometer showed only the common amount of deflection when the engine (by far the heaviest load) passed over; whereas, perhaps in the middle, or at the end of the train, a waggon would lurch over from some irregularities, and throw up the index to double its former amount. This effect was very particularly noticed by the Deputation, Directors, Proprietors, and other parties present. It follows, therefore, that till greater perfection can be obtained in railways, a strength of bar more than double that due to the mean strain must be provided. In my original Report, I have allowed 50

per cent. beyond the double, as a surplus ; but from these experiments, it appears this allowance is in excess, and that from 10 to 20 per cent. beyond the double will be sufficient ; that is, for a 12 ton engine, as the weight is at present distributed, a strength of 7 tons would be an ample provision, and with greater accuracy of construction, such as the care now taken may be expected to insure, a less strength would be sufficient ; or rather, allowing the same strength, an engine of 14 or 16 tons might be passed over with the greatest confidence.

By referring to the preceding results, it will be seen, that one rail is sometimes depressed by one wheel a quarter of an inch, while the other wheel is perhaps on a block ; and immediately after the high wheel is depressed, and the lower wheel raised, giving thus a rocking motion to the carriages, the effect of which was rendered remarkably obvious by the little instrument employed. No doubt much of this is due to a want of parallelism in the bearing blocks ; and therefore, as one step towards correction, I would recommend it to be made a special instruction, that the blocks shall in every case be placed immediately opposite to each other, which in parallel rails may always be effected without expense or inconvenience. Other corrections, however, are necessary, which will be noticed in their proper places. I shall therefore, pursuing the present subject, proceed to draw such results as the

foregoing experiments seem to justify; and one of the most important of these is, (being given not as an opinion or supposition, but as an experimental fact,) that with engines of 12 tons' weight, and with velocities not exceeding 32 or 35 miles per hour, it is not necessary, even as railways have been hitherto constructed, to provide for a strain of more than 7 tons, which is allowing a surplus strength of 16 per cent. beyond the double of the mean strain.

195. *On the proportional increased Section with increased Distance of Bearings.*

The above fact being established, it is clear that we may provide this strength for any length of bearing, by increasing the section of the bar proportionally as the distance of the props increase; but it may still be a question, which is the best length to adopt? or whether different lengths may not be advantageously employed, according to local circumstances? For example, in some places the stone blocks cost more than the iron rails they support, while in others, the blocks may be obtained at a cheap rate; therefore in the former case, if present cost only were considered, it might be advantageous to diminish the number of the blocks, and increase the weight of iron, and in the latter, to use less iron, and increase the number of bearing blocks.

Still, however, there are limits which cannot be conveniently exceeded: if the blocks are too close, the requisite quantity of iron in the bar may be so small as to give a very unsatisfactory section; and on the other hand, if the lengths be too much extended, the weight of the bar must be made inconveniently great. Thus restricted, therefore, it will be useless to examine any distance of bearing less than three feet, or more than six; and in apportioning the quantity of metal for each length, regard must of course be had to the limits prescribed by practice, that is, we must only employ such sections as may be subject to no substantial practical objections; but with this condition, the form of section is unlimited.

The first limitation which practice enforces is, that whatever be the bearing length and weight of rails, the head ought to have the same certain weight.

It is not necessary to go far along the Liverpool and Manchester line to see that the heads of the original 35 lbs. elliptical rails are far too small for the present weight of the engines, the outside flanch of the upper table being, in numerous instances, nearly separated from the central rib. The Dublin 45 lbs. parallel rail, which has a broader and somewhat larger head, does not show the same defects: still, however, it is generally, I find, considered too small. The 50 lbs. parallel plain T rail, and the

Grand Junction rail, are perhaps the best proportioned heads in the line ; their area of section to an inch deep, occupying about $2\frac{1}{2}$ square inches. In the following calculations, therefore, I shall lay it down as a practical limit, that the head ought not to occupy less than 2.25 inches' area, or, which is nearly the same, not weigh less than 22.5 lbs. per yard.

Another practical limit, in which I believe most engineers agree, is, that the depth of the rail ought in no case to be more than 5 inches.

Abiding, therefore, by these conditions, I propose to compute the weight of iron per mile, on four lines of rails, preserving in all cases a constant strength of 7 tons at the several bearing lengths of 3 feet, 3 feet 9 inches, 4 feet, 5 feet, and 6 feet, and the number of cubic feet of stone per mile required in each case ; distributing the iron in each bar most economically for strength.

The lightest rail in the line, which appears to approach towards the required degree of strength, is the Dublin parallel rail, of 45 lbs. per yard ; but as the head is lighter than the present practice seems to point out as the best, I would increase this by $2\frac{1}{2}$ or 3 lbs., and with a little addition to the rail itself, make the whole about 52 lbs., which is, perhaps, the least weight that ought to be given to a rail on 3 feet bearings ; and the best disposition of this weight, according to the solution of the problem on

the principle of *maxima et minima*, regard being had to the practical limits above stated, is given in Art. 174 ; and on similar principles, although not strictly following the minutia of the solution, have been arranged the proportions for the other bearings, the section at half size and the several particulars being as follow :

Section for a Three Feet Bearing.

ON A SCALE OF HALF THE LATERAL DIMENSIONS.

Head to 1 inch depth, 22·5 lbs.
per yard ; whole depth $4\frac{1}{2}$
inches.

Ditto bottom web, 1 inch.

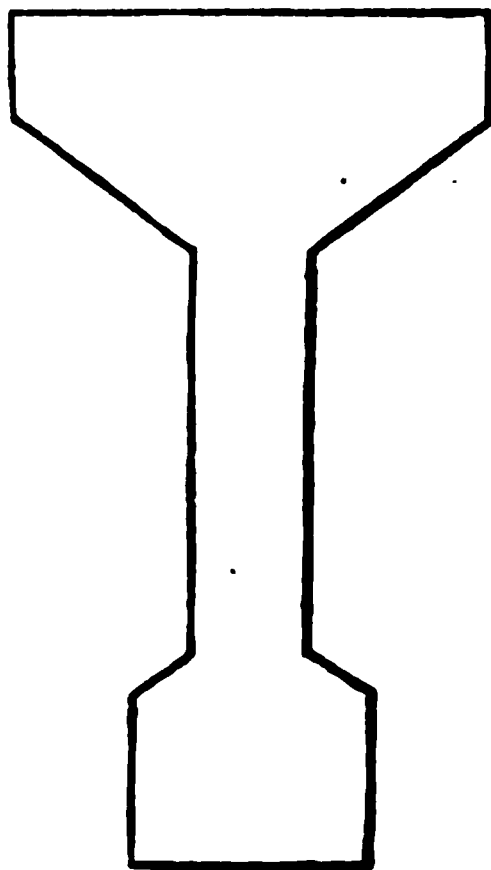
Breadth ditto, 1·25 inch.

Thickness of middle rib, ·6
inch.

Whole weight, 51·4 lbs. per
yard.

Strength, 7 tons.

Deflection with 3 tons, ·024
inch.



Section for a Three Feet Nine Inch Bearing.

Head to 1 inch depth, 22·5 lbs.
per yard.

Whole depth, $4\frac{1}{2}$ inches.

Ditto of bottom web, 1 inch.

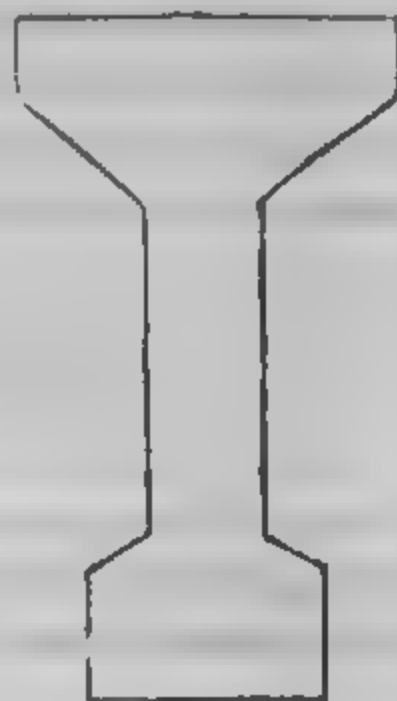
Breadth ditto, $1\frac{1}{2}$ inch.

Thickness, middle rib, ·75
inch.

Whole weight, 58·8 lbs. per
yard.

Strength, 7 tons.

Deflection with 3 tons, ·037
inch.

*Section for a Four Feet Bearing.*

Head to 1 inch depth, 22·5 lbs.
per yard.

Whole depth, $4\frac{1}{2}$ inches.

Ditto of bottom web, 1 inch.

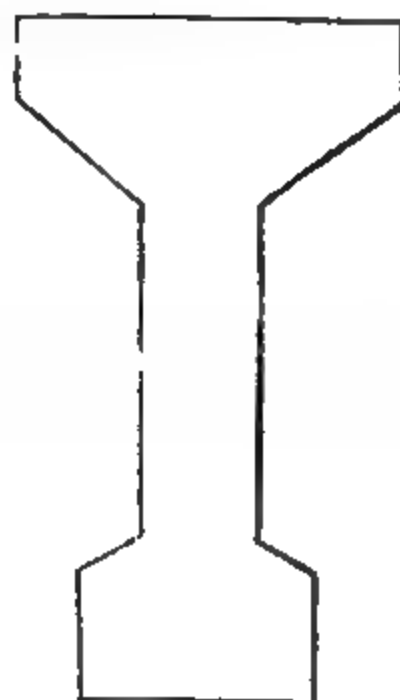
Breadth of ditto, $1\frac{1}{2}$ inch.

Thickness of middle rib, ·8
inch.

Whole weight, 61·2 lbs. per
yard.

Strength, 7 tons.

Deflection with 3 tons, ·041
inch.



Section for a Five Feet Bearing.

Head to 1 inch depth, 22·5 lbs.
per yard.

Whole depth, 5 inches.

Ditto of bottom web, $1\frac{1}{8}$ inch.

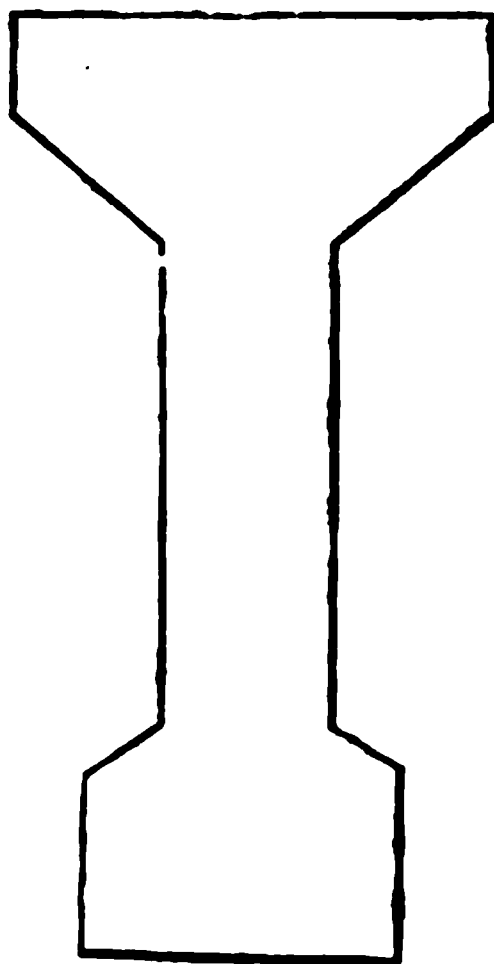
Breadth of ditto, 1·66 inch.

Thickness of middle rib, ·85
inch.

Whole weight, 67·4 lbs. per
yard.

Strength, 7 tons.

Deflection with 3 tons, ·064
inch.

*Section for a Six Feet Bearing.*

Head to 1 inch depth, 22·5 lbs.
per yard.

Whole depth, $5\frac{1}{10}$ inches.

Ditto of bottom web, $1\frac{1}{2}$ inch.

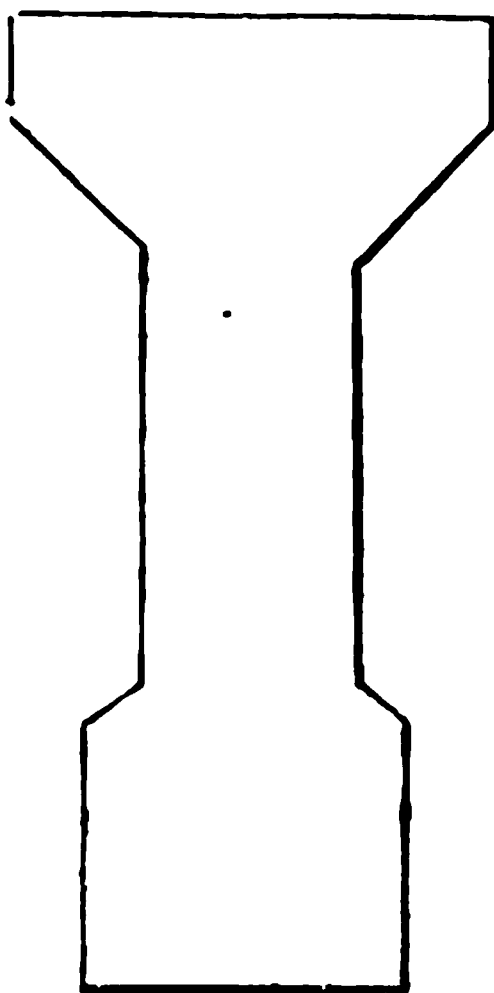
Breadth of ditto, 1·66 inch.

Thickness of the middle rib,
 $1\frac{1}{8}$ inch.

Whole weight, 79 lbs. per
ton.

Strength, 7 tons.

Deflection with 3 tons, ·082
inch.



It will be seen, by the above statement, that although I have preserved the same strength or resistance in each of the rails, the longer bearings are less stiff than the shorter; indeed, unless this increased deflection be allowed, all thoughts of greatly increasing the distance of the bearings must be given up; for, in order to preserve a proportional deflection, either the breadth of the rail must be so increased as to require a weight of iron altogether inadmissible, or the depth must be increased in the same proportion as the length of bearing, which is impracticable. The deflections, however, of the longer bearings, although greater than the shorter, do not amount to a large quantity; the deflection of several of the rails at present on the line being much greater, as may be seen by referring to the several experiments on this subject.

196. Assuming these dimensions, I have in the following Table computed the number of tons of bar iron per mile, requisite for four lines of rail at different bearing distances, also the weight of the requisite number of chairs, and the number of cubic feet of stone, in order thereby to facilitate a comparison of the expense, or first outlay, under these several circumstances; but the price of stone being very variable, according to its quality and locality, the pecuniary amount is not attempted.

It may be proper to state, that the weights I have taken for the chairs is somewhat arbitrary; on

weighing Mr. Stephenson's chairs, I find the joint chairs and pins to weigh 1 qr. or 28 lbs., and the intermediate chairs and pins 24 lbs.; and as the depth in his rail, including his segmental piece, is the same as that I propose for the 3 feet bearing, I take this as my guide, and I have slightly increased the weight of both chairs as the rail becomes deeper. That is, I make for the

3 feet bearing-joint chairs	28 lbs.	Intermediate do.	24 lbs.
3 feet 9 inches do.	do. 30	Do.	25
4 feet do.	do. 30	Do.	25
5 feet do.	do. 33	Do.	27
6 feet do.	do. 33	Do.	27

I was pretty generally informed, that by far the greater number of chairs that are broken, are destroyed in the wedging; and as no wedging is here intended, it is expected the above weights are sufficient.

In computing the quantity of stone, I have assumed the present sized blocks, of 4 cubic feet, sufficient at each distance, except the joint blocks, which, as they ought certainly to be larger than the others, I have taken at 5 feet.

While the blocks preserve a perfect level, I am not aware that a joint block, when the joint itself is perfect, has more to sustain than any other; but if one of them sink a little, the continuous rail over an intermediate block has a much greater sustaining power as the load passes, than the discontinued rails at the joints. Moreover, as these have been hitherto

made, a much greater shock is communicated to the support at the joints than to the other blocks; for both which reasons it becomes necessary that the joint blocks should be greater, but to what extent it is difficult to say; I am, however, disposed to think 1 foot additional a sufficient allowance.

TABLE I.
*Showing the Weights of Rails and Chairs requisite for forming a mile of Railway on Four Lines,
accordingly as the Bars are supported at greater or less distances.*

Length of bearing.		Length of rail.	Weight of one rail.	Number of rails in four lines per mile.	Weight of rail per mile.	Number of joint chairs per mile.	Weight of joint chair and pins per mile.	Number of intermediate chairs per mile.	Weight of intermediate chairs and pins per mile.	Total weight of chairs and pins per mile.
Pt.	In.	Feet.	lbs.		Tons.		Tons.		Tons.	Tons.
3	0	15	257	1408	161	1408	17½	5632	60½	78
3	9	15	294	1408	185	1408	19	4224	47½	66½
4	0	16	326	1320	192	1320	17½	3960	44½	62
5	0	15	337	1408	212	1408	20½	2816	34	54½
6	0	12	316	1760	248	1760	26	1760	21½	47½

TABLE II.

Showing the Number of Cubic Feet of Stone requisite for forming a Mile of Railway in Four Lines, according as the Bars are supported at a greater or less distance.

Length of bearing.	Number of joint blocks in four lines per mile.	Number of cubic feet in joint blocks per mile.	Number of intermediate blocks in four lines per mile.	Number of cubic feet in the intermediate blocks per mile.	Total number of cubic feet of blocks per mile.	Expense of ballasting and laying per mile, as depending on the number of blocks.
Pt. In. 3 0	1408	7040	5632	22,528	29,568	?
3 9	1408	7040	4224	16,896	23,936	?
4 0	1320	6600	3960	14,840	21,440	?
5 0	1408	7040	2816	11,264	18,304	?
6 0	1760	8800	1760	7,040	15,840	?

N.B.—The above calculation for the stone rests on the assumption, that the same sized blocks are applicable at all distances of bearing. Some remarks on this subject will be found in a subsequent page.

In the preceding Table I have only attempted to preserve the same strengths at all the different bearings; and I have observed, that to preserve the same proportional deflection would require, if greatly extended, an inadmissible weight of iron in the rails. It may, however, be well to state that amount, which is as below: viz.

	Ft.	In.	Weight of rail per yard.	Weight of rail.	Weight per mile.
At	3	0	51·4 lbs.	257 lbs.	161 tons.
	3	9	67·5	337½	212
	4	0	72·0	384	226
	5	0	92·0	460	289
	6	0	22·5	490	385

If (as was agreed upon by Mr. Stephenson and Mr. Locke) we take the present 50 lb. fish-bellied rail as the criterion for strength and proportional deflection, we must provide at least a strength of 8 tons, and not allow a greater slope than $\frac{9}{18}$. To insure these conditions the following weights and dimensions must be adopted: viz.

	Ft.	In.	Weight per yard.	Weight of rail.	Weight per mile.
At	3	0	55·5 lbs.	277·5 lbs.	174½ tons.
	3	9	64	320	201
	4	0	66·7	356	209½
	5	0	75·8	379	238
	6	0	100	400	314

MISCELLANEOUS EXPERIMENTS.

Remarks relative to the Mechanical Effect of different Bearing Lengths.

As above is, to the best of my judgment, an accurate statement of the weight of iron, and the quantity of stone, requisite to assure the same strength and security for the different bearing distances; and as far as mere present outlay is concerned, it may always be determined which is the most economical (rather, which will cost the least money) when the price of the stone, the expense of the labour in laying, and the price of iron, are given; but looking to the future expenses, I must certainly prefer the larger bars, and longer bearings, having due regard to the soundness of the bar when made. Whatever deterioration is going on upon the iron, it is on its surface, and consequently will proceed no faster, or very little faster, in the larger bars than in the smaller; and therefore we have a right to assume, that the larger bars will last serviceable longer than the smaller, although their strengths at the beginning are equal.

But this is not all the advantage that was claimed by some parties for the longer bearings. It was contended, that as it was the practice, generally, on railway lines, to allow the blocks, while they were in a state of subsidence, to settle only a certain quantity before packing them, that this quantity

being the same at long and short bearings, a central block sinking (say a quarter of an inch) would cause a less slope in long bearings than in the shorter; and thus, the slope being less, the action of the engine and carriages on the blocks would be less also, and to such an extent as to make it a consideration of economy, with regard to the maintenance of the road; that is, not only were the three blocks, for instance, in the five feet bearings, to perform their duty as well as the five blocks in the three feet bearings, each block being of the same size, but they were to perform it better, in consequence of the lesser slope produced by a central sinking block. Every one, during our experiments, took part in this discussion, but as is generally the case with such questions, without convincing or being convinced. It occurred to me, therefore, to submit it to the test of observations, which our little instrument enabled us to do. It appears, from the previous experiments, on 3 feet 9 inch, and 5 feet rails, that the deflection of the former was $\cdot 035$, and of the latter, $\cdot 089$; the slopes, therefore, were as $\frac{0.35}{0.89}$ to $\frac{0.89}{0.35}$, or nearly as 1 to 2: consequently, according to the principle maintained in the argument, the blocks of the five feet bearings ought to have been more rocked than the 3 feet in that proportion; whereas, on trial, they were found to be, of the two, rather less affected, but as nearly equal as possible. The error, as I must consider it to be, unquestionably arose from assuming that the pressure of a body on an inclined

plane was proportioned to the height of the plane, or to the sine of the angle; whereas, it is as the cosine, and as the cosines of small angles are nearly constant, so also are the pressures of the weight upon the plane.²

198. *Experiments made with a view to estimate the effect of the Carriages on the Blocks at different Bearing Distances.*

The manner in which these experiments were conducted was as follows:—A block being selected, a hole was drilled in it, and into this a strong cranked piece of iron was driven, and the tail of the deflectometer brought into contact with its lower edge. The effects of the passage of the engines were then read and registered, as in the case already described.

² Another idea may have led to this false conception, i. e. that the motion of the carriage "is naturally horizontal," whereby it may have been supposed, that it pressed harder on the plane, and of course on the prop, as the slope was greater; but this is not the case: the force which urges the body is tangential to the rail, and the pressure ought actually to be less as the slope is greater, as we found it to be. In my original Report, I used an expression with respect to the elliptical rails which seems to imply something like this; but what is meant there, is, that the change of direction of the tangent is more rapid in that part, and that this sudden change of direction was what produced the injurious effect spoken of.

The first trials were made on four of the blocks of the 3 feet 9 inch bearings; one was what is called a hanging block, viz., one under which the ground has so much subsided as to leave the block just suspended by the rail; two of the blocks were sounded, and considered quite firm and solid, and another suspected not quite firm.

Experiments with the 'Swiftsure,' 3 ft. 9 in. bearings.

		Velocity = 10 miles.	v. = 16.	v. = 20.	v. = 30.
Hanging block	disturbance	·060	·090	·080	·085
Firm block	ditto	·010	·020	·022	·032
Ditto	ditto	·000	·012	·017	·032
Supposed not quite firm	} ditto	·018	·028	·028	·032

Taking the mean of all but the hanging block, we obtain for the general disturbance ·021.

Experiments with the 'Swiftsure,' and two Trains, on the Blocks of the 5 feet Bearings; blocks all firm.

		SWIFTSURE.			
		Velocity = 15.	v. = 15.	v. = ?	v. = ?
Middle block, No. 1	disturbance	·014	·004	·004	·005
Joint ditto, No. 2	ditto	·024	·017	·012	·016
Middle ditto, No. 3	ditto	·017	·006	·004	·012
Middle ditto, No. 4	ditto	·030	·020	·018	·026

		Fury train.	Orion train.
Block No. 1	disturbance	·012	·018
No. 2	ditto	·046	·028
No. 3	ditto	·004	·012
No. 4	ditto	·040	·032

		SWIFTSURE.	
		v. = medium.	v. = medium.
Block No. 1	disturbance	·016	·008
No. 2	ditto	·036	·020
No. 3	ditto	·018	·010
No. 4	ditto	·023	·025

The mean of all these gives a disturbance = ·019, which differs very little from the former; but the greater slope, instead of exhibiting a greater, shows a less disturbance than the smaller slope: we may, however, consider them as equal, the difference being so extremely small, as not to be of any very definitive amount in experiments of this kind.

Experiments on the Disturbance of the Blocks, with Mr. Booth's new Rail, with 3 feet Bearings.

1st Block, firm	·018	·028	·018	·023
2nd Ditto, not firm	·036	·040	·036	·032
3rd Ditto, firm	·022	·013	·014	·011
4th Ditto, do.	·024	·020	·027	·023

Mean of the first and last two, which appear to have been firm, gives the disturbance ·020. We have, therefore, the following results:

Grand Junction, 3 feet 9 inches bearing, disturbance	·021
Ditto 5 feet bearing	·019
Mr. Booth's rail, 3 feet do.	·020

These show that the disturbance of the block is but little dependent on the length of bearing on a well consolidated base, and consequently, that the saving

supposed to be effected by reducing the amount of slope is not borne out by the experiments. I ought perhaps to observe, that, in the first instance, some experiments were made with the same view as the above, on which the disturbance was measured, by driving in a broad chiseled edge piece of iron between the chair and block, projecting beyond the latter to meet the deflectometer, which rather favoured the hypothesis in question; but it was admitted by all present, that they probably involved the motion of the chairs with that of the block, and were therefore by general consent rejected before any comparison was made; those above detailed, which were made the following morning, having been substituted for them.

Having, I hope, thus satisfactorily disposed of this side of the question, by admitting, for the sake of the argument, that the three blocks would not sink more than the five, let us now examine how far this admission is justifiable.

It is asserted in one Report, that whatever the number of blocks may be, each block is pressed with, or has to sustain, only the same weight during the passage of the train over it, and, therefore, whether on 3 feet bearings, or 5 feet, the sinking of the blocks will go on at the same rate; while others contend that, with the present distance of the wheels, the whole weight of an engine may fall on three blocks only in the 5 feet bearings, which would be distributed over five blocks in the 3 feet bearings; and,

therefore, although at the moment of the passage of a wheel over a block, that block is no more pressed in the one case than the other, it has, upon the whole, more pressure, or the effect of a greater number of blows to sustain in a given time when there are fewer, than when there are more, supports.

It seems to be impossible to deny this position, and I cannot doubt for a moment if rails upon a new ground or embankments were partly laid on 3 feet bearings, and partly on 5 feet bearings, the blocks being of the same size, that it would be found that the blocks in the latter case would sink faster than in the former, till the consolidations were fully effected; after which, I have little doubt the 5 feet bearing-blocks would be as efficient as the 3 feet, and that they would very soon become so in cuttings where the substrata are already well compressed. Another argument I have heard advanced, on the opposite side, is by assimilating the blocks of a rail to the piers of a bridge, which require to be broader as the number of piers are less, but this does not, I think, apply well to this question. In a bridge, the weight of structure is nearly every thing, the passing load being inconsiderable; whereas in a railway the load constitutes the principal weight to be resisted.

The conclusion to which I am brought, as to the relative expense of maintenance per block in 5 feet and 3 feet bearings, or, more generally, in long and short bearings, after well weighing all these points,

is, first, that in embankments, and where there is a soft sub-soil, the expense would be greater at first with the long bearings than with the short, but that it would ultimately become the same, although certainly never less; and, secondly, that on rocky, or very solid bottoms, the expense would be very nearly the same for the same number of blocks from first to last.

Such is my unbiassed opinion; it is, however, but an opinion; and I do think it would be very desirable to submit the question, if possible, to experiment on an embankment. Would it not be worth the trouble to lay down on the embankment at Kensal Green, a certain length of the Grand Junction rails, at 3 and 5 feet bearings, and to observe minutely the effect on each? The locomotion and train, I think, pass here seventy times a day, which would, in a very short time, decide the question, as far as embankments are concerned; and for the rest, I do not think there is any reasonable cause for doubt.

On the Best Form of Rail.

199. In the sections given in a preceding page for rails at different lengths of bearings, it will be seen that I have confined the breadth of the lower web to $1\frac{1}{2}$, or, at most, to $1\frac{2}{3}$ inches; and this has been done, although I am well aware that, to extend the breadth of the lower web, and to reduce its depth,

MISCELLANEOUS EXPERIMENTS.

heoretically give the strongest rail; in fact, the double T is, on paper, a stronger rail than a deep and less broad flanced rail, but I am quite sure it is not so in practice. The lower web comes no other way into use than as it is brought to a state of tension by the action of the centre rib; and although the fibres of the lower web lying entirely below the centre rib are brought into it, and that these fibres excite a similar effect laterally in those immediately contiguous to them, these again to the next, and so on, yet in metal like malleable iron this lateral effect is soon lost; so that the extreme fibres of the extended lower flanch become inefficient.

In fact is, this particular form of rail was proposed with a view to a certain advantage it was supposed to possess, viz., that it might be turned when the upper table had been worn down, but this has been shown in my former Report to be impracticable; and not fulfilling this condition, while in other respects it is disadvantageous, it should be at once rejected: I know it is said it may still be turned and used in side rails; but I reply, wherever it is used, it will be strongest if not turned. Again, it is stated, that both sides being alike, the rail-layers may select the side that fits best; but it would surely be better to have the rails made so uniform that no such choice was requisite. Again, it gives a broad bearing, in which, however, I see no advantage when carried to excess. And, lastly, it admits of

the rail being fixed by a wooden key or wedge; but is it not better, if possible, to avoid the wedge altogether? In fact, I can see no advantage this form of rail possesses, to compensate for its actual and obvious defects.

The proportions I have shown in the preceding diagrams, which resemble nearly the form of rail to which the prize was awarded, is, I am persuaded, the strongest and best; it being of course understood that these diagrams give only angular outlines, the salient and re-entering angles of which may be softened down or fortified according to the taste or other considerations of the engineer.

To convince Mr. Locke, and some other gentlemen, of the defect of the double T form, I had one of the rails taken up, and $\frac{1}{2}$ an inch cut away on each side from the lower flanch, reducing its breadth at the point of greatest strain, that is, in the middle of the bar, to $1\frac{1}{2}$ instead of $2\frac{1}{2}$ inches. It was then put into the press, and the strains brought on as usual, under the superintendence of Mr. Edward Woods and Mr. John Gray; Mr. Locke himself being obliged to leave just at the time the experiment was in progress.

Mr. Rathborn, Mr. Edward Cropper, and myself, were also present, and the result was, that the bar thus mutilated showed greater strength than the mean strength which Mr. Locke found to belong to it when whole. Now, although I am ready to grant that the bar was actually weakened, and that

this apparent anomaly is attributable to the imperfection of the press already pointed out, yet, on the other hand, it must be admitted that it could, with such a result, have lost but little of its strength, and that the iron thus abstracted, viz., nearly $\frac{1}{8}$ of the whole section, if judiciously introduced elsewhere, would undoubtedly give a much stronger rail^a.

On the Form of Chairs, and the means of securing the Rail to the Chair.

200. This is a delicate subject to approach, after so much has been done and written respecting it; and I know of only one qualification I possess, beyond that of common observation, to enable me to form a judgment upon it; which is, that I have no proposition of my own to recommend. I examined with every necessary attention all the different chairs, keys, filling-up pieces, &c., on the Liverpool and Manchester line, and must say that no one, in my estimation, is so simple nor so well adapted to preserve the bar steady, as that made to receive the plain T rail. Nor, in my opinion, would any other have ever been thought of, if it had not been for the

^a It is since this was written that the experiments have been made on the Southampton rails, which are still more objectionable from their extended lower web; but it must be admitted that these, where the iron was good, did not indicate the weakness anticipated from their extensions.

introduction of the lower web. The question therefore is, how are we to retain the advantages of the lower web, without losing the simplicity of lodgement due to the plain T rail? Now, a method of effecting this is proposed by Mr. Sinclair, in his recent Report, which consists in having a recess cut in the rolls, so that at the ends of the bar, and at every point of bearing, the rail shall form a plain rib equal in thickness to the breadth of the lower flanch, that is,

Fig. 1.

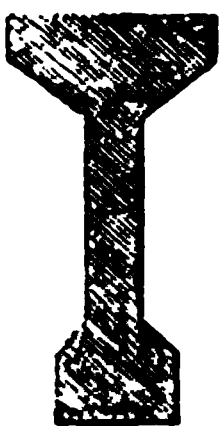


Fig. 2.

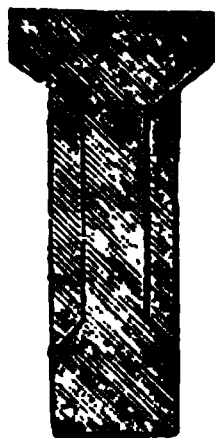


Fig. 3.

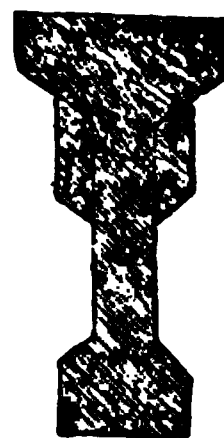


fig. 1 being the general section of the rail: about 3 inches at each bearing point would assume the plain T form shown in fig. 2, or, if more simple, to roll, the form fig. 3.

On inquiring of those best able to judge, I find it admitted that this proposition is practicable. I do therefore think, that we have in such a rail and chair all the good qualities that can be desired. 1st. We have all the simplicity and steadiness of lodgement due to the plain T rail. 2d. We retain all the advantages of the lower web. 3d. We have all the breadth of bearing that can be serviceable; and 4th, In case a central block sinks, we have a

much stiffer double bearing length than could be otherwise obtained.

This last, I conceive to be a decided advantage, although amongst other doctrines I have heard on this subject, one has been, that the small neck of the fish-bellied rail gives it less strength, and thereby enables the rail to bend and follow the block in its downward progress into the ground; and this was considered an advantage; but, I must say, if it be one, I am unable to see in what it consists.

I have only at present spoken of the lateral support of the rail in the chair; but it is necessary to prevent the rail rising in the chair, or rather, it is necessary to prevent the rail quitting the bottom of the chair when the latter sinks; and I do not think this can be more simply, nor more efficiently performed than with the present chair and pin proposed by Mr. R. Stephenson, omitting, however, his segmental piece.

In my former Report, having no idea it was possible to effect what Mr. Sinclair now proposes, I considered that the *whole* chair for the joint ends would have been useful; but Mr. Sinclair's present proposition offers, in my mind, such decided advantages, that I consider the whole chair to be rendered quite unnecessary, or rather, that each chair thus becomes a whole one.

It is proper here that I should notice also a provision which Mr. Locke has made in the bed of his chair, to prevent the rising up of the end of the rail,

while the rail itself is submitted to deflection by the passage of a load ; which I consider very useful. It is obvious that if the bed of the chair be left, as it commonly is, quite square, when the rail is deflected by a load in the middle, the whole bearing is brought upon the inside edge of the chair, and the short remaining part of the joint end is thrown up with such great force (although the motion is not much) as to render it impossible to restrain it. But what cannot be restrained by force is here judiciously prevented from occurring by giving a very slightly curved figure to the bottom of the chair, whereby the inside contact falls as the deflection proceeds, instead of allowing the end to rise ; and certainly such form of chair-bed I should strongly recommend, whatever chair may be adopted. The curvature required is, of course, very small, being scarcely perceptible to the eye, but it avoids a great strain upon the chair itself. And the other great and irresistible strain from contraction and extension, which is brought on when every thing is keyed tight, is also easily provided against, or will soon provide for itself, by the mode of fixing recommended ; that is, the small hole made by the pin in the rail may be a little enlarged, or it will soon enlarge itself sufficiently to admit the contraction and expansion to proceed without injury, and without reducing its holding power on the rail itself ; and that this action does take place in the rails thus laid down may be seen by examination, the ends of the pins and the holes on

the rails being both slightly polished, and the latter a little elongated. One of the pins in each rail also being rather harder driven than the rest, will answer the other object pointed out in my former Report; viz., to furnish a fixed point towards which the contraction may take place from each end of the bar. This was the suggestion of Mr. Woodhouse, which I omitted to notice in my former paper.

In some of Mr. Stephenson's chairs the pins are chisel-ended, instead of pointed. Of course, either the one or the other may be used that is found to answer best.

On the Formation of the Joints.

201. On carefully examining the joints of the rails on the Liverpool and Manchester line, I am disposed to estimate that about one in six of the plain butt joints are as perfect as can well be desired, and that another one in six are as bad as bad workmanship and negligence can make them; the remaining two-thirds varying in character between these two extremes. These circumstances naturally lead to the question—

On what does the goodness of the joints consist? and the answer is plain.

1st. On the uniformity of size and figure in the transverse section of the rail.

2d. On the straightness and evenness of the bar longitudinally.

3rd. On the squareness of the ends of the rail to the line of its length. And,

4th. On the uniformity of size and figure in the opening of the chair.

There can be no question, if these points could be perfectly attained, the joints would be perfect also ; and though perfection is not to be had, still a great approach towards it may be made with proper care, and would be made, if enforced in the condition of the contract, at a very inconsiderable charge. The practical genius and talent our iron-masters have at their command, stimulated by the large amount of the sum to be expended, would, if conditions were enforced, lead to the contrivance of simple means of effecting these objects within very close limits ; and, I will venture to say, that nothing like perfection in railways will be obtained till such conditions are made and complied with. It is, of course, understood that the means of producing the requisite degree of accuracy will rest with the iron-master, but the methods of gauging, or otherwise ascertaining how far the contract has been complied with, and the allowable amount of deviation, will be the business of the company.

I will not here undertake to prescribe either the limits of deviation, or the means of testing or gauging, which will both be best settled by the company's engineer, after duly weighing all the circumstances of the case ; but will merely add, that in Government contracts for shot, shells, &c., very little deviation is

allowed, and yet it is seldom found necessary to reject any of the articles sent in.

In the smaller shells, which are still considerably larger than the opening in a railway chair, and unquestionably much more difficult to cast, not more than a deviation of $\frac{1}{80}$ th of an inch is allowed, and I can see no reason why the railway chairs, and the end of the rails, should not be submitted to at least as close a gauge. To enforce this accuracy may, perhaps, incur some present charge, but do not the wear and tear of the rails and engines incur a much larger constant expense of maintenance? I am sure it is unnecessary for me to urge this point upon those proprietors who witnessed, during the experiments, the concussion on the rail exhibited by the deflectometer, which, of course, produced a like concussion on the engine and carriages. The whole of these were, doubtless, due to irregularities, of which the want of parallelism of the blocks and bad joints were the principal. Some persons present attributed them in part to flat places in the wheel; but if there are flat places in the circumference of the wheel, to what are these attributable but to bad joints? To be convinced of this we have only to consider what must be the effect of a blow on a wheel supporting a load of 3 tons, and moving with a velocity of 30 or 32 miles per hour, when such a body meets the end of a rail rising $\frac{1}{8}$ th, or, perhaps, nearly $\frac{1}{4}$ th of an inch above another; or when the joints are so open as to allow the wheel to fall from one

upon the other, with all the impetus due to such velocity.

In order to arrive at some estimate of this effect, a bad or open joint was selected, the deflectometer applied to the block, and the shock measured by the instrument. The rail was then taken up and relaid, so as to make the joint as close as usual, leaving the opening at the other end, and the effect was again taken, when it was found that the bad joint increased the force of concussion full 50 per cent.; that is, the engine had to sustain a shock from this circumstance one-half at least greater than was due to a very common joint, and probably double what it would have had to sustain at a good one.

The experiment above alluded to was our last, and was made by Mr. Edward Cropper and myself, with the *Swiftsure* engine. The following are the results :

	Bad joint.	Replaced joint.
Disturbance	·043	·032 great speed.
Ditto	·030	·016
Ditto	·031	·022
Ditto	·023	·015
	<hr/>	<hr/>
Mean disturbances .	·032	Mean ·021

Seeing, then, the obvious injury produced by bad joints, and the impossibility of having good ones, without a strict uniformity in the opening of the chair and in the size of the end, and bearing part of the rail, as well as in the perfect straightness of the bar, and the squareness of its end ; seeing, further,

that all these conditions may be insured at an inconsiderable charge for examination, when the articles are received, it will remain for the Directors to decide how far it will be advisable to adopt these suggestions.

I have not spoken of the half-lapped joint, because I think if the butt joints were well made, the half would be unnecessary; for, as I have already observed, there are perhaps about one-sixth of the present butt joints on the rail as good as any lapped joints could be made, and even much better than they would probably be made, unless cut by machinery, which, in the large bars the Directors will most probably adopt, would be attended with a great expense, without yielding an adequate advantage.

On the Fixing the Chair to the Block.

202. This question ought to be considered under two points of view; that is to say, simplicity of fixing and removing a chair, in case of necessity, and the firmness of the hold which it gives. In my former Report I recommended, for security of holding, a method which had been proposed for drilling a hole quite through the stone, and using a large-headed bolt, except that for reasons assigned in the Report, I proposed to countersink the stone below. I find, however, that it is a very prevailing opinion amongst those who ought to know, that this method

has been tried some years back, and found to be inefficient, and that it was ultimately rejected by its original inventor. I find also that, in general, the opinion of those who have considerable practice is, that the method now most commonly employed with a wooden plug and iron pin is, upon the whole, efficient and satisfactory; and under these circumstances, I feel myself bound to recommend the latter.

SUMMARY.

203. I have in the course of the preceding pages replied to every point which, by the resolution already referred to, I was requested to examine and to report upon. In so doing I have thought it right to explain the several principles and experiments from which I have drawn my conclusions; and these being thus disconnected, it may be well to state them here collectively and isolated from other matter. In this form they will stand thus:

1st. I am of opinion, that as far as is consistent with the amount of the first outlay, it is desirable to increase the weight or section of the rails, and to decrease proportionally the number of bearing blocks.

2nd. That in cuttings and other places furnishing a good firm bearing, the present size of blocks is sufficient; viz., allowing for the intermediate blocks 4 feet, and for joint blocks 5 feet, while the bearing

length does not exceed 5 feet ; but that on embankments they will probably require to be proportionally increased in size. But I recommend this to be put to the test of actual experiment.

3rd. I am of opinion that the cost of maintenance will, in the former case, after a short time, be in proportion to the reduced number of blocks, but certainly not less.

4th. I consider the double and equal flanged rail to be inferior, in strength and convenience of fixing, to that which is described and modified to suit different distances, in a preceding page.

5th. I consider Mr. Sinclair's proposition for rendering the rail plain at its points of bearing, to be in every respect recommendable.

6th. I am of opinion the form of chair, and method of fixing the rail in the chair, proposed by Mr. Stephenson, is as simple and efficient (adopting the plan of rolling of Mr. Sinclair) as can be desired.

7th. Yielding, as I am always ready to do, to practical opinions, when they are found pretty generally to agree, I am disposed to think the present mode of fixing the chairs to the blocks, with a wooden plug and iron pin, is, from its simplicity and convenience, the most recommendable.

Lastly. I am strongly convinced that no change or modification of form will produce any essential improvement, till greater uniformity be enforced in the figure and dimensions of the rails and chairs,

and greater attention paid to the parallelism of the blocks, and to a proper adjustment of the distances of the ends of the rails from each other, to allow for expansion and contraction.

In now concluding this Report, I think it my duty to state, that should it be found to contain any valuable facts or information, it is indebted for them to the facilities and conveniences with which the deputation were supplied by the Directors of the Liverpool and Manchester line, to whom also I feel great obligation for many personal accommodations and attention. And I am sure that I am only acting in accordance with the feelings of my coadjutors, Isaac Solly, and Thomas Tooke, Esqrs., who formed the London Deputation, in expressing our warm acknowledgments to Theodore W. Rathborn, Esq., for the hearty and hospitable manner in which he entertained us during the greater part of the time we were engaged in the experiments and investigations, which I have great reason to hope will be found to have led to some useful results, and to which we shall never recur without the most gratifying recollections of the bounteous and cheerful hospitality of Alerton Priory.

APPENDIX:

CONTAINING

ICAL INVESTIGATIONS ON THE EFFECT OF THE DE-
FLECTION OF RAILS, INCLINED PLANES, GRADIENTS, &c.

Determine the Influence of the Deflection of an Elastic Bar to the Motion of a Body passing over it, the Bar being supported at its two extremities.

Let ACB represent an elastic bar, supported at its middle point, and loaded at its extremities with two equal weights, w, w . The deflection of the two ends will be exactly the same as of the same bar supported at its ends and loaded with a weight $2w$ at its middle point.

Fig. 1.

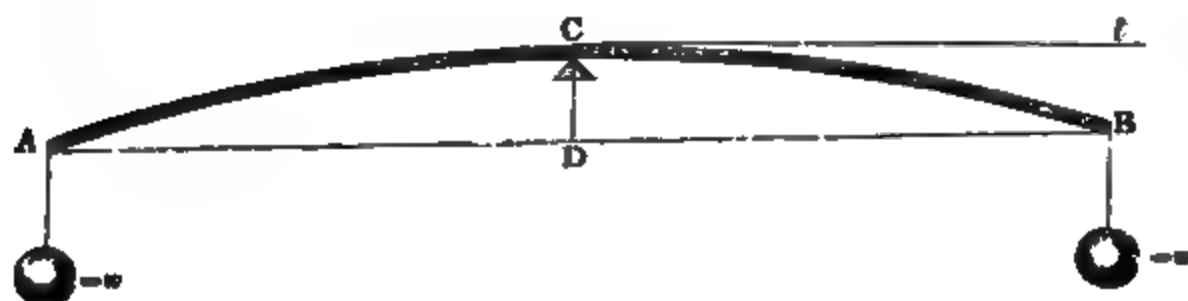
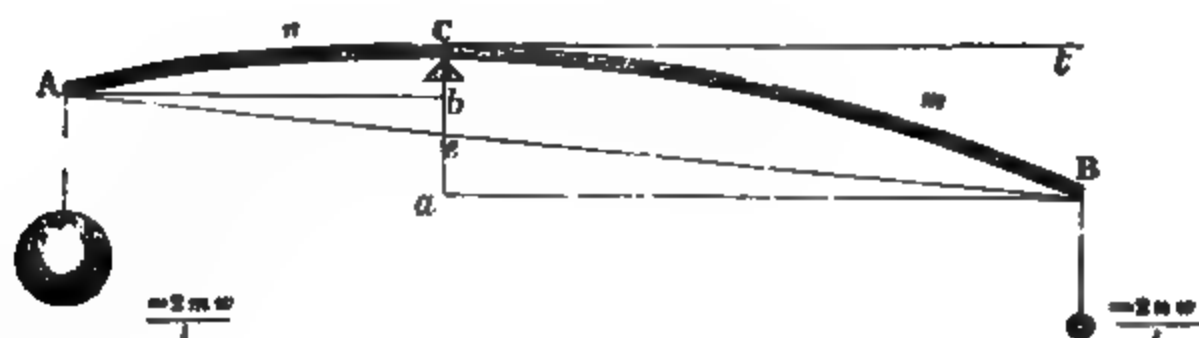


Fig. 2.



2. Let ACB , fig. 2, be the same bar supported at any point C , dividing the beam into two lengths m , n , and loaded at B by a weight $\frac{2nw}{l}$, and at A by a weight $\frac{2mw}{l}$ (l being the whole length), so that the beam may be still in equilibrio on the support C , and the sum of the two weights equal to $2w$, as before. Then Cb will be the deflection of the point A , and Ca of the point B , Ce being a mean deflection, as referred to the oblique line AB , and this deflection Ce will be the same as if the beam was supported at A and B in a horizontal line, and loaded at C with a weight $2w$, the deflections being considered as very small in comparison with the length.

In fig. 1, let the element of deflection at C be Δ , then the whole deflection, being as the element of deflection into the square of the length, we may represent $Ca = \delta$ by $\frac{1}{4}l^2\Delta$. But the element of deflection in the same beam is as the strain: and the strain at C in fig. 2, is to that in fig. 1, as $mn : \frac{1}{4}l^2$. Therefore, in fig. 2,

the element of deflection $\Delta' = \frac{4mn}{l^2}\Delta$,

and the deflection $Ca = \frac{4m^3n}{l^2}\Delta = \delta'$,

the deflection $Cb = \frac{4mn^3}{l^2}\Delta = \delta''$,

and $ba = \frac{4mn(m^2 - n^2)}{l^2}\Delta = \delta' - \delta''$.

Consequently, the sine of the inclination, or of the angle ABa

$$= \frac{4mn(m^2 - n^2)}{l}.$$

And this is precisely the inclination the tangent Ct would have, if the beam were turned about C till AB became horizontal, and therefore the same as the tangent Ct would have, if the beam were supported at its ends, and loaded at C with a weight $2w$; and it is this inclination which forms the impediment to the motion of the body along the plain face of the bar.

3. To find the point where this inclination is the greatest, we have

$$\begin{aligned} & \begin{cases} m+n=l \\ mn(m^2-n^2)=a \text{ max.} \end{cases} \\ \text{or, } & m(l-m)(2lm=l^2)=a \text{ max.} \\ \text{or, } & -2lm^2+3l^2m-l^3=a \text{ max.} \\ \text{whence, } & -6lm^2+6l^2m-l^3=0, \\ & m^2-lm=-\frac{1}{3}l^2 \\ & m=\frac{1}{3}l(1\pm\sqrt{\frac{1}{3}}) \\ & n=\frac{1}{3}l(1\mp\sqrt{\frac{1}{3}}). \end{aligned}$$

When m and n have these values, the inclination of the tangent is the greatest, and consequently at that point the resistance to the motion is the greatest. It is shown that the sine of the angle of inclination is expressed generally by

$$\frac{4mn(m^2-n^2)}{l^3}.$$

When $l=1$, this is $\frac{4}{3} \times \sqrt{\frac{1}{3}} = .384$.

For the inclination of a plane of half the length of the bar, viz. whose altitude is equal to the central deflection, viz. $\frac{1}{3}l^2\Delta$ —which this case is frequently but erroneously confounded) would when $l=1$ be proportional to $\frac{\frac{1}{3}l^2}{\frac{1}{3}l} = .5$. That is, the

greatest resistance a heavy load experiences in consequence of the deflection of the bar over which it passes, is to the constant resistance it would experience in ascending an inclined plane, whose height is equal to the central deflection, as .384 to .50, or nearly as 3 to 4. The former, moreover, acts only for an instant, and begins and terminates in zero, while the other remains constant throughout.

To compare the sum of all the resistances in the two cases, let us consider still $l=1$, then the general expression for the resistance at any point, viz.

$$\frac{4mn(m^2-n^2)}{l^3}$$

becomes $4(-2m^2+3m^2-m)$,

and this multiplied by the differential of m ,

gives $4(-2m^2+3m^2-m)dm$,

the integral of which between the values

$m=\frac{1}{3}$ and $m=1$, is $\frac{1}{3}$;

while the sum of all the constant resistances .5 for the half-length

$$=\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}.$$

That is, the sum of all the variable resistances to a load by the deflection of the bar over which it passes, is exactly half the resistance the load would experience in ascending a plane of the same half-length, and whose height is equal to the central deflection of the same bar.

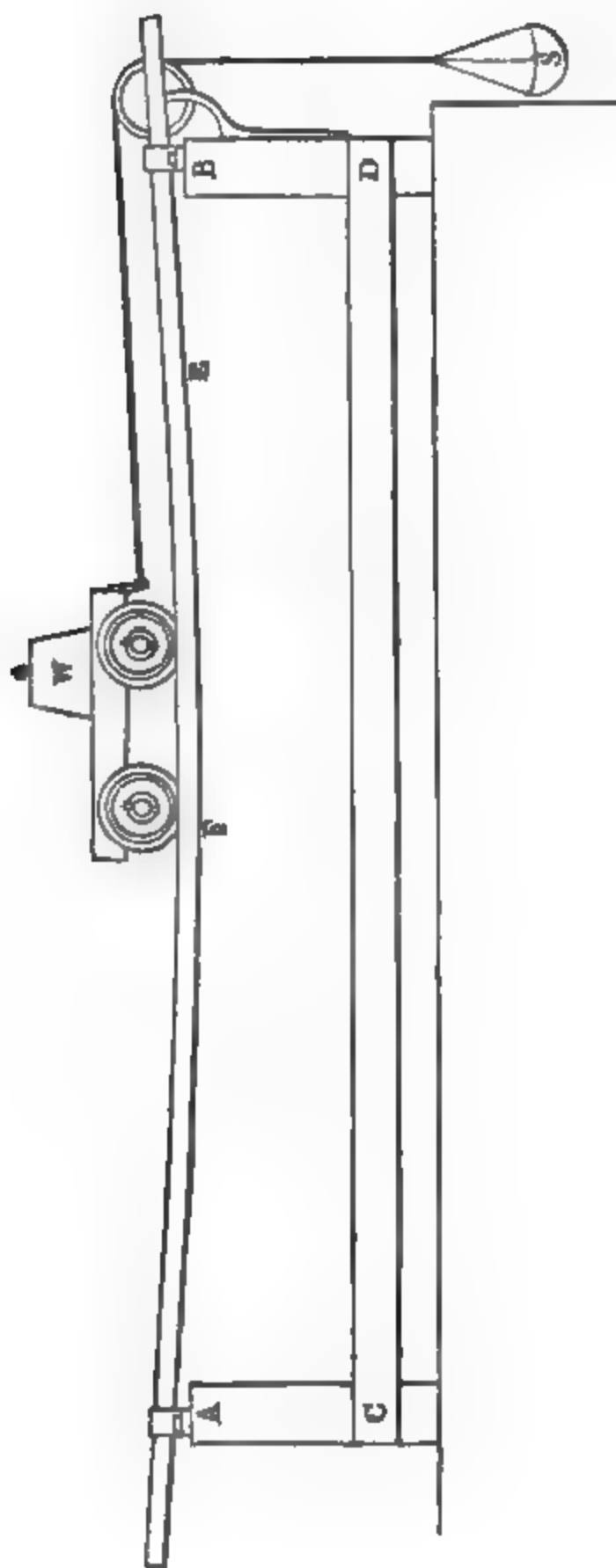
Now the resistance on such a plane, the central deflection being δ , which is to be considered the height of the plane, its length being $\frac{1}{2}l$, is $\frac{2\delta}{l}$, consequently the resistance of a bar only deflected to the same extent will be $\frac{\delta}{l}$.

4. It will be understood that this is the resistance to the ascent of the body from the middle of the bar up to the prop; and if, as has been assumed by some persons, as much power was gained in the descent as was lost in the ascent, the odds would be made all even, and the deflection of the bar would be no impediment; but that assumption is altogether erroneous, both in theory and practice. In fact, the gain from descent is so exceedingly small in such short planes as we are here considering, that it may be wholly rejected; so that in a plane supposed perfectly horizontal, the retardation, or additional resistance to the carriages, caused by the deflection of the bar, will be equivalent to the carriage being carried up a plane of half the whole length on a slope equal to $\frac{\delta}{l}$, the other half being horizontal, or, which is the same, on one entire ascending plane, whose slope is $\frac{\delta}{2l}$, where l is the distance between the props, and δ the central deflection. Having, thus, the resistance due to deflection estimated on a continually rising plane, the resistance per ton becomes known, and consequently the exact numerical increase of engine power which is necessary to overcome that resistance. Computing in this way, it appears that the effect of deflection on the several bars whose sections are given in p. 429, et seq., produce resistances equivalent to planes of the following slopes; viz.

Bearing distances.	Deflections.	Equivalent planes.	Increase power per ton.
ft. in.			lbs.
3 0	·024	1 in 3000	·75
3 9	·037	1 in 2432	·92
4 0	·041	1 in 2341	·95
5 0	·064	1 in 1875	1·2
6 0	·082	1 in 1756	1·3

5. These being important considerations in the economy of railways, and feeling that what is perfectly satisfactory to a mathematician, cannot be equally so to persons not in the habit of following such trains of reasoning, I had a little model made, representing one length of rail, the distance of the supports being 30 inches: the bars are drawn steel, $\frac{1}{4}$ inch by $\frac{1}{4}$; the load with the carriage weighs 134 ounces, and the deflection with that weight is nearly $\frac{1}{4}$ an inch. The model is represented in the following page, with the scale, in which weights are placed for illustrating the points in question. From A to B was laid a well-planed piece of wood, on which, in the first instance, the railway bars were secured at their proper parallel distance. The end of the model A being now raised, this plane was made to be truly horizontal; weights were then gradually put into the scale till that weight was found which just balanced the friction, and which was found to be exactly 5 ounces, including the scale.

The model was then placed in its natural position, the base C D accurately levelled, and the carriage placed on the unsupported bars, the weight being thrown as nearly as possible over the front wheels only; 5 ounces due to friction were introduced, and weights gradually added: as each ounce was introduced the carriage advanced, and with 16 ounces it rose over the point E, where the resistance was the greatest, and was then accelerated to the end. E, according to the preceding investigation, was a little beyond the half of the half-length, and the same was distinctly indicated by the experiment. At the lowest point of the curve the resistance was the same as on the horizontal plane, as it was also at the end B, which are both likewise consistent with the investigation.



The bars were now removed, and the plane already mentioned placed from A to B, inclining so that the bars passed exactly through the point F, when it was found that the weight necessary to balance the carriage and friction was $19\frac{1}{4}$ ounces. The greatest resistance, therefore, on the deflected bars was to the resistance on this plane as

$$(16-5) \text{ to } (19\frac{1}{4}-5), \text{ or as } 11 \text{ to } 14\frac{1}{4},$$

which is also very closely approximative to what is given by the theory. The only doubt, therefore, which can remain, is how far I ought to reject as inconsiderable any increase of power on the descending side. This point cannot be met experimentally, and I am therefore obliged here to depend only on demonstration. The case certainly involves no great difficulty of conception as a mere question of theoretical mechanics; having, however, been treated on different principles by persons of considerable scientific eminence, I should have been glad to have exhibited the effect experimentally; but as the whole turns upon velocity, this is impossible. The demonstration alluded to is involved in the principles explained in the following section.

On the Laws which govern the action of Locomotive Engines on Railways.

6. At this time, when a novel application of a powerful mechanical agent is being made over so many miles of this country, and different public companies are competing with each other to effect the same object by different lines, it is desirable that some certain rules should be established of estimating the effects of the same engine on different loads and of the several ascending and descending planes which necessarily occur in all, in order thereby to form a just comparison of their respective mechanical merits. These questions have been examined by different writers, but unfortunately without coming to any fixed conclusion; in fact, both the theory and practice in this branch of mechanics involve points of consideration which are liable to lead to some discrepancies, according to the views which may be taken of them.

One of the prevailing defects in many of these solutions is, that of assuming that the engine power required for different loads on a horizontal plane is proportional to the power of trac-

tion requisite to produce the motion; whereas the expense of engine power has no definite ratio to the force of traction, in consequence of the different forces which must be overcome before any motion can be impressed on the load.

Thus, for example, before any motion can be produced on the load, whether it be great or small, the following resistances must be overcome: viz.

1st. The friction of the engine gear.

2nd. The friction of the wheels and axles of the engine and tender.

3rd. The pressure of the atmosphere upon the surface of the pistons.

The power or quantity of steam thus expended every stroke of the engine, before any effect can be transmitted to the load, is very considerable, in many cases quite as much as is employed for actual traction.¹

7. Another source of error has been, according to my view of the subject, by confounding what may be called the statical and dynamical effects of friction. Thus, supposing the inclination of a descending plane to be such as just to balance the friction when the body is at rest, it has been taken for granted that it would continue to balance it at all velocities, without regard to the law of the force by which the body is affected, which is such that equal quantities of steam are producible in equal times, and consequently such that the pressure at any time is inversely proportional to the velocity,² whereas the force of gravity is constant at all velocities; or, which is the same, the whole retardation from friction down a given plane is the same at all velocities, whereas the effect of gravity depends only on the time of descent.

¹ Our engineers are in the habit of speaking of the power of high-pressure engines by the pressure of the steam as exhibited or limited by the safety valve, that is, by the pressure above the atmosphere, and this is quite correct while comparing the effective power of different engines; but in estimating the expenditure of steam to produce this disposable power, the whole elasticity of the steam must be considered.

² This is admitted as a general law in the production of steam in stationary engines; but it has been doubted whether it holds good in locomotives, because the draft being increased by the ejection of the steam from the

Amongst the writers who have contributed most to elucidate the laws of action in locomotive engines, we ought to distinguish M. Pambour, a French engineer, who, after many judiciously conducted experiments on the Liverpool and Manchester and on the Darlington lines of railway, has arrived at numerical results, which appear in every respect to be entitled to entire confidence: according to these—

1st. The friction of the engine gear alone, that is, without a load, amounts on an average of several engines, to 6 lbs. per ton of the weight of the engine, as applied to the circumference of the wheel.

2nd. That the friction of the wheels, axles, &c. of the engine and tender is 9 lbs. per ton.

3rd. That the friction of the waggons, without the engine and tender, is 8 lbs. per ton, including the weight of the waggons and load.

4th. That the friction on the engine gear is, at a medium, 1 lb. additional per ton, for every ton weight of the load and waggons.

M. Pambour, who, as far as I know, is the first writer who has distinctly introduced the pressure of the atmosphere on the pistons, estimates that pressure at 14·7 lbs. per square inch.

6th. Lastly, it is assumed, that equal quantities of steam are producible in equal times; and consequently, that the pressure on the piston, at any time, is inversely as the velocity.

8. Let now

W denote the tons' weight of the engine.

w the tons' weight of tender.

L the tons' weight of the waggons and load.

L' the gross load, including the engine, tender, &c.

Then the force necessary to be applied at the circumference of the wheel to balance these resistances alone, will be

$$6 W + 9 (W + w) + 9 L = 6 W + 9 L'.$$

cylinder to the chimney, it is supposed by some engineers that the more rapid the velocity the more will be the combustion: on the other hand it is contended, that although the ejection is more rapid, the steam is more rarefied, and that the effect is therefore the same, at least at all practicable velocities.

To this is to be added the pressure of the atmosphere, or its resistance to the motion of the pistons, viz.

$$2 d^2 \pi \times 14.7,$$

$d^2 \pi$ being the area of one piston in inches, and 14.7 the number of lbs. pressure per inch.

But this last resistance being only overcome with the velocity of the piston, must be transferred to the circumference of the wheel, where the other resistances are estimated. Taking therefore D to denote the diameter of the wheel, and l for the length of stroke, we have

$$4 D \pi : 2 l :: 2 d^2 \pi \times 14.7 : \frac{14.7 d^2 l}{D},$$

which is the force that must be applied at the circumference of the wheel to balance the pressure on the piston.

Let this be denoted by A , then the whole force requisite to balance the resistance on a horizontal plane, is

$$A + 6 W + 9 L'.$$

And as the sum of the first two terms is constant, call

$$A + 6 W = C,$$

then the whole resistance will be expressed simply by

$$C + 9 L'.$$

And suppose, that the observed horizontal velocity with this load is v , and it be required to determine the velocity the same engine would impress on a gross load L'' , we should have

$$(C + 9 L') v = v' (C + 9 L'').$$

$$\text{Whence } v' = \frac{C + 9 L'}{C + 9 L''} v.$$

9. In an observed experiment, let the weight of the engine $W=12$ tons, of the tender $w=6$ tons, and $L=82$ tons; and consequently $L'=100$ tons, and the velocity $v=25$ miles per hour. And in another case, let the load be one-half, or 41 tons, and therefore the gross load $L''=59$ tons; and let the dimensions of the engine be as follows, viz., diameter of piston 12 inches $=d$, the length of stroke $l=1\frac{1}{2}$ foot, and diameter of drawing wheels $D=5$ feet.

$$\text{Then } A = \frac{14.7 d^2 l}{D} = 635 \text{ lbs.}$$

$$\begin{aligned} 6 W &= 72 \\ \text{Then } C &= 707 \end{aligned}$$

And in the first case $9 L' = 900 \text{ lbs.}$

in the second $9 L' = 531 \text{ lbs.}$

And substituting these numbers in the above expression, we find

$$v = \frac{C + 9 L'}{C + 9 L'} v = 32\frac{1}{2} \text{ miles.}$$

So that, diminishing the load by one-half, only increases the velocity about $7\frac{1}{2}$ miles per hour.

If, on the other hand, the velocity $v = 25$ was that observed on the half load, we should have

$$v = \frac{707 + 531}{707 + 900} \times 25 = 19\frac{1}{2} \text{ miles.}$$

That is, the double load is carried by the same engine, and with the same expenditure of power, at nearly $\frac{4}{5}$ ths the speed of the single load,—results which are by no means inconsistent with practical experience.

On the Effect of Gradients.

10. As some difference of opinion exists on this subject, probably arising more from imperfect definition, than from any other cause, it may be well to examine the subject rather more in detail than would be otherwise requisite.

Let us therefore take a very simple theoretical case, by supposing a body free from friction and resistance to be moving along a horizontal plane, with a certain velocity, which we may assume to be 32 feet per second, and that it arrives at the foot of a plane, rising 16 feet; then, by the known laws of mechanics, the body in this particular case will arrive at the top of the plane, and at that point will have lost all its velocity; but if there it meets an equal descending plane, it will in its progress down, acquire at the bottom the same velocity it had at first. In this respect, therefore, it may be said to have lost no force, because its first and last velocities are equal; but, as the time of the body ascending one plane and descending the other, will be double that with which it would have passed over the same horizontal

distance with its first velocity, it will have *lost time*; and a loss of mechanical effect is thus sustained.

11. If now, instead of a body free from friction and resistance, we take the case of a locomotive engine, moving with the same velocity, and suppose it to possess, within itself, a power so exerted as just to balance the friction at all velocities, that is, as acting upon the piston throughout the journey with a uniform pressure, then this body will not mechanically differ from the former; that is, it will ascend and descend the plane according to the same laws, and there would still be no loss of power, but a loss of time only; for, according to this view of the question, the quantity of steam power expended would be the same as if the body had passed along the base of the two planes (rejecting the difference in the length of the base and plane itself as altogether inconsiderable).

It will be observed, however, that the nature of the steam power thus assumed, is not that which occurs in the actual machine: for, as the steam itself can only be generated at a certain rate, it follows, that its pressure will vary according to the rate of motion, and therefore, instead of being applied, as supposed above, only to overcome the friction, it will act on the ascending plane to aid in the ascent; and, on the other hand, on the descending plane the natural gravitating power will assist in overcoming the friction. The two forces thus act conjointly, and being subject to different laws, the question of gain or loss of power becomes rather complicated. If we examine our first two supposititious cases, it will be found, that the restoration of the original velocity depends upon the time of ascent and descent being equal, so that all the velocity lost by the ascent is regained in the descent; but in the actual case, the time of ascent exceeds that of the descent, and there is not therefore time for gravity to restore on the descending side all the velocity lost on the ascending side; and a loss both of time and power (which are equivalent in a locomotive engine) is sustained accordingly.

12. It is clear, that when a locomotive engine and train, proceeding with a given horizontal velocity, arrive at the foot of an ascending plane, its motion from that point will be retarded till

APPENDIX.

increased pressure of the steam is sufficient to balance the increased force of traction and friction, after which the motion will continue uniform. And when the engine and train, proceeding at the same velocity, arrive at the top of a descending plane, motion down will be accelerated till the reduced pressure of the steam due to the increased velocity is just such as to balance the difference between the two opposite forces of friction and gravity, when the descending velocity will become uniform also.

13. Let us now endeavour to get an expression for the accelerating forces above referred to.

We have seen, that with a gross load L' , the force of traction on a horizontal plane is expressed in lbs. by $C + 9 L'$; and let $\frac{L'}{D} = \frac{1}{f}$, be taken to denote the force as a fraction of the corresponding velocity being v , and let $\frac{1}{s}$ denote the sine of the plane, or the height divided by the length, and let y of ascent at any time, then the steam pressure p , as the velocity, and being equal to $\frac{1}{v}$, with a

velocity v will, at the velocity v' , be expressed by $\frac{v}{v' f}$.

The increased force of traction in lbs. will be $\frac{2240 L'}{s}$, and this will bring on an increased friction on the engine gear of $\frac{2240 L'}{8 s}$.

For we have seen, that the friction on the engine gear amounts to $\frac{1}{8}$ th of the whole force of traction: if, therefore, we again divide these terms by $2240 L'$, as before, we find that the actual forces in operation, are

Urging force . . . $\frac{v}{v' f}$, or steam pressure.

Retarding force . . . $\frac{1}{f}$, the original retarding force.

Do. do. . . $\frac{1}{s}$, the weight of body on the plane.

Do. do. . . $\frac{1}{8 s}$, increased friction of engine gear.

And therefore the whole variable force is

$$\frac{v}{vf} - \frac{1}{f} - \frac{1}{s} - \frac{1}{8s} = \frac{v-v'}{v'f} - \frac{9}{8s}.$$

14. Precisely the same forces are in action on the descending plane, but $\frac{1}{s}$ is now an urging force, and $\frac{1}{8s}$ acts as a reduction of the force $\frac{1}{f}$. The expression, therefore, for the descending force is

$$\frac{v-v'}{v'f} + \frac{9}{8s}.$$

And therefore,

$$\frac{v-v'}{v'f} \pm \frac{9}{8s} = \phi$$

will be a general expression for the variable force with which the engine is urged along any plane ascending or descending; and by the known laws of variable motions, we shall have

$$\frac{v-v'}{v'f} \pm \frac{9}{8s} = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

It is not necessary to stop here to reduce and integrate this expression, because, if we did, it would lead to merely a theoretical result not applicable to the real case; we may therefore pass it over without injury to the inquiry, the real object of which is, not to ascertain what would be the result, according to the theory of these forces, but what is the practical result. Theoretically, we may find, on a descending plane, a velocity of 100 or 200 miles per hour, or even an infinite velocity; but practically, if these results were attainable, they would be inadmissible; leaving therefore this part of the subject, let us endeavour to ascertain as nearly as we can the real practical effect.

15. In the first place, it may be observed, that although for want of having obtained the integration in question, we cannot estimate the immediate effect of the acceleration or retardation; we may in all cases determine the velocity of ascent or descent after the acceleration ceases, that is, after the motion becomes uniform; for in this case, the preceding value of the force ϕ becomes zero, so that

APPENDIX.

$$\frac{v-v'}{fs} \pm \frac{9}{8s} = 0, \text{ or that}$$

$$\frac{v-v'}{fs} = \mp \frac{9}{8s}.$$

From this we may ascertain the uniform velocity due to any slope, or the slope which will give any proposed velocity.

Suppose, for example, it were required to find the inclination which would produce a final uniform velocity $= 2v$. Substituting $2v$ for v' , we find,

$$\frac{1}{2f} = \frac{9}{8s}. \quad \text{Or } \frac{1}{s} = \frac{4}{9f}.$$

to find the slope that will give an ultimate uniform velocity greater than the uniform velocity v , we have only to set $v' = \frac{1}{2}v$, and we obtain,

$$\frac{1}{6f} = \frac{9}{8s}. \quad \text{Or } \frac{1}{s} = \frac{4}{27f}.$$

And this is perhaps the greatest increased speed that can, with a due regard to safety, be admitted on a descending plane; and it is therefore the greatest slope that can be safely descended with the steam admission valve fully open.

16. It will be observed, that after the motion thus becomes uniform, the loss and gain of time and power on the ascending and descending planes compensate each other, so that, upon the whole, the space passed over is the same with the same power, as if the path had been horizontal; the actual loss sustained being only during the period of acceleration; but there is this important consideration, that the time of acceleration always exceeds that of retardation, and we must not therefore conclude, that because there is no loss when the motions are uniform, that there is therefore no loss actually sustained; or that that loss is inconsiderable.

Let us suppose, for example, that

$$\frac{1}{f} = \frac{9}{8s}. \quad \text{Or } \frac{1}{s} = \frac{8}{9f}.$$

In this case we find, that the velocity of ascent, after the retardation ceases, is half the horizontal velocity, that is, $v' = \frac{1}{2} v$; and, on the descending plane, the velocity after the acceleration ceases, is infinite, or $v' = \text{infinity}$.

The former velocity $v' = \frac{1}{2} v$ is very soon attained, but the latter can never be practically acquired; therefore, theoretically, the gain on the descending side of any given plane whatever, can never compensate the loss on the ascending side. This, though apparently an extreme case, is one which frequently occurs in practice, and is selected here to put the question in a strong light; but a similar effect takes place in all ascending and descending planes, that is, the time of acceleration will always exceed that of retardation, and a loss both of time and power must be sustained accordingly.

17. The actual loss thus occasioned cannot, as has been stated, be ascertained theoretically without the integration of the preceding formula, nor is it important that it should while looking only for practical results; because the velocities thus computed could not be admitted into practice with a due regard to safety: let us, therefore, now confine ourselves wholly to the question as limited by considerations of prudence, that is, by claiming no more advantage for the descending planes than is consistent with safety.

These limitations must be somewhat arbitrary, but the following are perhaps agreeably to the usual practice.

1. That no plane on which the train would be accelerated with the steam wholly shut off, ought to be descended with more than the uniform horizontal velocities. Such are all planes having a slope greater $\frac{1}{s} = \frac{8}{9f}$, and on which of course the brake must be applied to prevent acceleration.

2. That all those planes on which the ultimate velocity would exceed $\frac{1}{2}$ th of the original horizontal velocity, and in descending which, therefore, the admission of steam must be partly shut off, ought not to be descended with more than $\frac{1}{2}$ ths of the original velocity. Such are all planes between

$$\frac{1}{s} = \frac{8}{9f} \text{ and } \frac{1}{s} = \frac{4}{27f}.$$

All planes of less slope than this last will, soon after the descent of the body commences, take up their uniform velocity without shutting off any steam, and the speed down them may be computed from the formula

$$\frac{v-v'}{vf} = \frac{-9}{8s}$$

any sensible error.

in all cases the ascending velocity, which soon becomes may be computed by the formula

$$\frac{v-v'}{vf} = \frac{9}{8s},$$

the former of which gives

$$v' = \frac{8vs}{8s-9f},$$

and the latter

$$v' = \frac{8vs}{8s+9f}.$$

18. Hence, in estimating the mechanical advantage of a descending plane, we must claim nothing for those whose slopes are equal to or exceed $\frac{1}{s} = \frac{8}{9f}$.

For all planes whose slopes fall between

$$\frac{8}{9f} \text{ and } \frac{4}{27f}$$

we may claim an increased velocity of $\frac{1}{3}$ th.

For planes of less slope than $\frac{4}{27f}$ the advantage may be computed by the first of the above formulæ.

And in all cases the reduced velocity on the ascending plane by the latter formula.

19. The best way of exhibiting these effects will be by computing the lengths of equivalent horizontal planes, that is, the lengths of horizontal planes which would be passed over in the same time, and with the same power as the ascending or descending planes in question, and taking these lengths as the measure of their mechanical effects.

Thus, planes sloping more than $\frac{8}{9f}$ (descending), will have for their equivalent horizontal plane one of equal length to the planes themselves; descending planes having slopes between

$$\frac{8}{9f} \text{ and } \frac{4}{27f},$$

will have their equivalent horizontal planes $\frac{4}{3}$ ths of their own lengths. And planes of less slope than $\frac{4}{27f}$ will have their equivalent planes $\frac{8s-9f}{8s}$ times their own length; and

Lastly, all ascending planes will have their equivalent planes $\frac{8s+9f}{8s}$ times their own length.

20. By way of illustration, the following Table has been computed, taking the dimensions already given of the locomotive, page 469, with a gross load of 100 tons.

According to those data,

$$\frac{C+9L'}{2240L'} = \frac{707+900}{224000} = \frac{1}{139} = \frac{1}{f};$$

and taking the several planes, each 1 mile, the length of the equivalent planes for the ascending side are given in column 2, and the equivalent descending planes in column 3; and column 4 shows the mean of two, ascending and descending.

Thus the time and power required to ascend a plane of 1 in 90, one mile in length, would carry the train 2.74 miles on a horizontal plane. The time to descend it would be the same as to go over the same mile horizontally, and the mean of the two 1.87, that is, a mile of such plane would require the same time to pass and repass it as would admit the train to pass and repass 1.87 mile on a level.

21. Table showing the equivalent horizontal lines to the several ascending and descending planes as given below. The power and dimensions of the Engine being as stated in p. 469. The gross load, including Engine, &c., 100 tons.

Gradients or inclined planes.	Equivalent horizontal lines.		Mean effect.
	Ascending.	Descending.	
1 in 90	2.74	1.00	1.87
1 100	2.57	1.00	1.78
1 120	2.31	1.00	1.65
1 140	2.12	1.00	1.56
1 160	2.00	.83	1.41
1 180	1.87	.83	1.35
1 200	1.78	.83	1.30
1 250	1.63	.83	1.23
1 300	1.52	.83	1.17
1 350	1.46	.83	1.14
1 400	1.39	.83	1.11
1 500	1.31	.83	1.07
1 750	1.21	.83	1.03
1 1000	1.16	.85	1.01
1 1500	1.10	.90	1.00

It will have been observed that as the expression $C + 9 L'$ involves a constant quantity C , the value of the fraction $\frac{1}{f}$ will vary with the load. Thus, supposing the gross load to be 50 tons instead of 100 tons, we should have

$$\frac{C + 9 L'}{2240 L'} = \frac{1}{97} = \frac{1}{f}.$$

The length of the equivalent planes, therefore, change with the load, and the following Table is computed for the same engine, with a load of 50 tons.

22. Table showing the equivalent horizontal lines to the several ascending and descending planes, as given below. The power and dimension being as stated in p. 469. The gross load, including the Engine, &c., 50 tons.

Gradients or inclined planes.	Equivalent horizontal lines.		Mean effect.
	Ascending.	Descending.	
1 in 90	2·21	1·00	1·61
1 100	2·09	1·00	1·54
1 120	1·91	1·00	1·45
1 140	1·78	·83	1·39
1 160	1·68	·83	1·25
1 180	1·60	·83	1·21
1 200	1·54	·83	1·18
1 250	1·44	·83	1·13
1 300	1·36	·83	1·09
1 350	1·31	·83	1·07
1 400	1·27	·83	1·05
1 500	1·22	·83	1·03
1 750	1·15	·85	1·00
1 1000	1·11	·89	1·00
1 1500	1·07	·93	1·00

23. The two cases above computed, of gross weights of 100 tons and 50 tons, are about the mean of the luggage and passenger trains on the Liverpool and Manchester line. And in estimating the loss occasioned by gradients on any proposed line, we may take the one or the other accordingly as the traffic may be expected to consist mostly of luggage or passengers.

The following Table shows the computed equivalent length of a line of railway from Croydon to Dover ; the data being assumed as stated in the Table.

24. Table showing the lengths of the equivalent horizontal planes for the several gradients on the South Eastern line, between Croydon and Dover. Engine as before; assumed gross weight, 100 tons.

Distance.		Gradients.	Rise or Fall.	Equivalent horizontal lines from Croydon.		Equivalent horizontal lines from Dover.		Data employed.
M.	Ch.			M.	Ch.	M.	Ch.	
0	22		Level.	0	22	0	22	Weight of engine .. 12 tons.
1	12	1 in 150	Rise.	2	28	1	12	Do. tender .. 6 "
1	58	1 100	Ditto.	4	34	1	56	Waggons and loads 82 "
1	14	1 150	Ditto.	2	22	1	14	Gross weight 100 "
2	56	1 330	Ditto.	3	79	2	20	Friction of load 8 lbs. per ton.
1	68	1 360	Fall.	1	43	2	53	Engine and tender 9 lbs. do.
1	50	1 100	Ditto.	1	50	4	14	
7	0	1 330	Ditto.	5	67	10	28	Engine gear } 72
5	0	1 528	Ditto.	4	14	6	40	without load }
1	0	Level.	1	0	1	0	Additional at 1 lb. per ton.
1	40	1 880	Rise.	1	61	1	24	
2	40	1 330	Fall.	2	7	3	56	Diameter of wheel . . . 5 ft.
3	0	1 880	Fall.	2	49	3	43	Length of stroke 1 "
4	0	1 1320	Rise.	4	38	3	53	Diameter of piston 12 inches.
1	0	1 2640	Fall.	0	77	1	4	Pressure of atmosphere,
3	0	Level.	3	0	3	0	14.7 lbs. per inch.
9	0	1 609	Rise.	11	27	7	40	
2	68	1 2950	Fall.	2	60	2	79	
3	49	1 330	Rise.	5	27	3	1	
5	40	1 380	Ditto.	7	60	4	47	
1	42	1 100	Fall.	1	42	3	73	
1	71	1 330	Ditto.	1	46	2	63	
5	53	Level.	5	53	5	53	
0	76	1 338	Fall.	0	63	1	32	
69	37			79	9	79	37	Mean 79 23

Whence it appears that the effect of the several gradients will cost an expenditure of time and power which would have carried the train 10 miles further on a horizontal plane; being a loss of power of about 10 per cent.

It will be observed, that in the preceding Tables the whole time of ascent is considered as if it were made with the uniform velocity, whereas the commencement of the ascent is more rapid in consequence of the original velocity; it is, however, assumed that the little time thus gained is lost after the train reaches the top of the

plane, by its having to regain its original horizontal velocity. A similar remark applies to the time of descent.

To obtain a practical case, in order to compare the preceding rules with practice, I wrote to Mr. R. Stephenson, and was furnished by him with the following :

WHARNCLIFFE ENGINE.

	ft.	in.
Diameter of driving wheels	4	6
Length of stroke	1	6
Diameter of piston	0	12
Mean speed, horizontal plane, with a load of 100 tons	20 miles.	
Mean speed up the Rainhill plane of $\frac{1}{96}$, with a load of 50 tons	12 „	

Weight of engine, 12 tons ; tender, 6 tons.

Let us now assume the horizontal velocity of 20 miles, as given, and compute what the ascending velocity ought to be :

First, $100 + 18 = 118$ gross load,

$$6w = 72$$
$$A = \frac{14.7 d^2 f}{D} = 705$$

}

with 118 tons.

$$\frac{118 \times 9 = 1062}{C + 9 L' = 1839}, \text{ and } \frac{1839}{2240 L} = \frac{1}{140} = \frac{1}{f}.$$

Again,

$$6w = 72$$
$$A = 705$$

}

with 68 tons.

$$\frac{68 \times 9 = 612}{1389}, \text{ and } \frac{1389}{2240 L'} = \frac{1}{110} = \frac{1}{f}.$$

And as

Miles. Miles.

$$1389 : 1839 :: 20 : 27$$

$$21$$

the rate a load of 50 tons would be carried on a horizontal plane by the same engine, we have, therefore, by the formula

$$\frac{v-v'}{vf} = \frac{9}{8s}$$

$$v' = \frac{8s}{8s+9f} = 12\frac{1}{2} \text{ miles,}$$

the velocity of ascent, which, according to Mr. Stephenson's practical experience, is 12 miles per hour; as close an approximation as can be expected in such a case.

The following Table contains a number of other practical examples, which will enable the reader to form a comparison of the results with the formula. They are taken from the experiments by M. Pambour, on levels and planes of $\frac{1}{89}$ and $\frac{1}{96}$.

FROM PAMBOUR.

		Load and tender.	Descent or level.	Speed, miles.	Press of steam.	Ascent.	Speed, miles.	Press of steam.
July 17, 1834.	} Atlas	27·45	$\frac{1}{2} \frac{1}{2} \frac{1}{2}$	26·47	54	$\frac{1}{2} \frac{1}{2}$	14	56
July 23.		39·40	not given			$\frac{1}{2} \frac{1}{2}$	6	55
July 31.		40·15	level	16	27 $\frac{1}{2}$	$\frac{1}{2} \frac{1}{2}$	7·5	51
Aug. 4.		44·26	not given			$\frac{1}{2} \frac{1}{2}$	3·75	61
July 24, 1834.	} Fury	56·16	level	17·14	55	$\frac{1}{2} \frac{1}{2}$	6·31	66·5
July 24, 1834.		48·8	level	17·50	55	$\frac{1}{2} \frac{1}{2}$	15	67
Aug. 4.		37·97	level	25·00	52·5	$\frac{1}{2} \frac{1}{2}$	13·33	55
Aug. 1, 1834.	} Vesta	33·15	level	29	50	$\frac{1}{2} \frac{1}{2}$	14·11	55
Aug. 16, 1834.		37·45	not given			$\frac{1}{2} \frac{1}{2}$	3·25	58
Do.		39·05	not given			$\frac{1}{2} \frac{1}{2}$	3·0	56·5
Aug. 15, 1834.	} Leeds	38·15	level	22·5	46·5	$\frac{1}{2} \frac{1}{2}$	10	48·5
July 22, 1834.		39·07	not given			$\frac{1}{2} \frac{1}{2}$	11·42	57·5
July 22.	Do.	41·32	not given			$\frac{1}{2} \frac{1}{2}$	18·75	57·5

		Inches.	Stroke.	Diam. W.	Weight.
Atlas . .	Diam. piston	12	16 in.	5 feet.	11·40 tons.
Fury		11	16	5	8·20
Vesta		11 $\frac{1}{2}$	16	5	8·71
Leeds		11	16	5	7·07
Vulcan		11	16	5	8·34

Atlas	26·47	:	14	or	1	:	·53	} Mean 1 : ·52
Do.	16	:	7·5		1	:	·47	
Fury	17·14	:	6·31		1	:	·37	
Do.	17·50	:	15		1	:	·85	
Do.	25·00	:	13·33		1	:	·53	
Vesta	29·00	:	14·11		1	:	·48	
Leeds	22·5	:	10		1	:	·44	

7)367

·52

TABLE

Showing the Specific Gravity, and the Weight of a Cubic Foot of various Building Materials.

The specific gravity of rain water being 1000.

MATERIALS.	Specific gravities.		Weight of a cubic foot in lbs.	
	From	To	From lbs.	To lbs.
WOODS.				
Acacia (false)	748	820	46.75	51.25
three-thorned	Mean	676	Mean	42.25
Ash (dry)	690	845	43.12	52.81
Beech (mean sort)	696	854	43.50	53.37
(dry)	690		43.12	
Birch	720		45.00	
Box (Dutch)	1030	1328	64.37	83.00
(Turkey)	950	1024	59.37	64.00
Cedar (Indian)	1315		82.18	
Cedar (various countries)	453	753	28.31	47.06
(of Libanus)	486	603	30.37	37.68
Cherry Tree	672	741	42.00	46.31
Chestnut (Sweet)	535	685	33.45	42.81
Chestnut (Horse)	483	657	30.18	41.06
Cowrie	579		36.20	
Cypress	644	655	40.25	40.93
Elm (green)	693	940	44.41	58.75
(seasoned)	553	588	34.56	36.75
Fir (Norway Spruce)	512		32.00	
(American)	465		29.06	
Larch (seasoned) (red)	496	640	31.00	40.00
(white)	364		22.75	
Mahogany (Spanish)	816	852	51.00	53.30
(Honduras)	560		35.00	
Oak (green)	1063	1216	66.43	76.03
(Irish Bog)	1046		65.37	
(Adriatic)	993		62.06	
(American)	752		47.00	
(English) (dry)	625		39.06	
(Dantzic)	755		47.24	
Pear Tree (dry)	646	708	40.37	44.25
Pine (American Pitch) (dry)	741	936	46.31	58.50
(Scotch) (dry)	529	696	26.81	43.50
(Memel and Riga)	466	553	29.12	34.56
(American)	368		23.00	
Plane	538	648	33.62	40.50
Poona	635		39.95	

TABLE—(CONTINUED).

MATERIALS.	Specific gravities.		Weight of a cubic foot in lbs.	
	From	To	From lbs.	To lbs.
WOODS.				
Poplar	374	529	24·37	33·06
Sycamore	590	645	36·87	40·31
Teak (dry)	657	832	41·06	52·00
Walnut Tree (green)	920		57·50	
(dry)	616	735	38·50	45·93
Willow (green)	619		38·68	
(dry)	404	568	25·25	35·50
STONES AND CEMENTS.				
Basalt	2478	3000	154·87	187·50
Brick (common)	1557	2000	97·31	125·00
(stock)	1841	2168	115·06	135·50
(Dutch clinker)	1482		92·62	
(Welsh fire)	2408		150·50	
Brick-work	Mean	1520	Mean	95·00
Chalk	2315	2657	144·68	166·06
(Clunch)	1869	2657	116·81	166·06
Flint	2580	2630	161·25	164·37
Granite	2624	3000	164·00	187·48
Marble	2580	2840	161·25	177·50
Mortar (hair) dry	1384		86·50	
(various) dry	1414	1393	88·37	118·31
Plaster, cast	1286		80·37	
Puzzolano	2570	2850	160·62	178·12
Serpentine	2561	2683	160·06	167·68
Slate	2512	2888	157·00	180·50
Stone (Bath)	1975	2494	123·43	155·87
(Blue lias limestone)	2467		454·18	
(Bramley fall)	2506		156·62	
Stone, (mean of various kinds)	2000	2686	125·00	167·87
Stone-work	Do.	Do.	Do.	Do.
(Yorkshire paving)	2356	2507	147·25	163·37
Tile (common)	1815	1858	113·43	116·15
EARTHS, ETC.				
Clay (common)	1919		119·93	
(with gravel)	2560		160·00	
Coke	744		46·50	
Coal	1269	1526	79·31	95·37
Earth (common)	1520	2016	95·00	126·00
Gravel	1749		109·80	
Lime (quick)	843		52·68	
Marl	1600	2870	100·	179·37
Sand (quartz)	2750		171·87	
(common)	1454	1886	90·87	117·87

TABLE—(CONTINUED).

MATERIALS.	Specific gravities.		Weight of a cubic foot in lbs.	
EARTHS, ETC.	From	To	From lbs.	To lbs.
Shingle	1424	..	89.00	..
Water (Rain)	1000	..	62.50	..
(Sea)	1027	..	64.18	..
METALS.				
Brass (cast)	8100	..	506.25	..
(wire, plate)	8441	.. 8544	527.66	.. 534.00
Copper (cast)	8607	..	537.93	..
(sheet)	8785	..	549.06	..
Iron (bar)	7600	.. 7800	475.00	.. 487.50
(cast)	7200	.. 7600	450.00	.. 475.00
Lead	11352	.. 11407	709.80	.. 712.93
Pewter	7248	..	453.00	..
Platina	21531	..	1345.63	..
Steel	7780	.. 7840	486.25	.. 490.00
Tin	7291	.. 7299	455.68	.. 456.18

THE END.

PRINTED BY W. HUGHES,
KING'S HEAD COURT, GOUGH SQUARE.

Fig 2

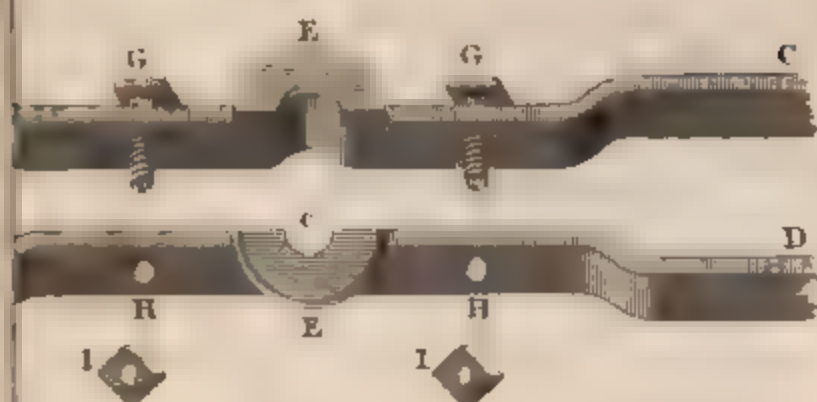


Fig 3



Fig 1



TABLE—(CONTINUED).

MATERIALS.	Specific gravities.		Weight of a cubic foot in lbs.	
EARTHES, &C.	From	To	From lbs.	To lbs.
Shingle	1424	..	89.00	..
Water (Rain)	1000	..	62.50	..
(Sea)	1027	..	64.18	..
METALS.				
Brass (cast)	506.25	..
(wire, plate)	8544	527.56	534.00
Copper (cast)	537.93	..
(sheet)	549.06	..
Iron (bar)	7800	7800	475.00	487.50
(cast)	7600	7600	450.00	475.00
Lead	2	11407	709.30	712.93
Pewter	453.00	..
Platina	1345.63	..
Steel	7840	486.25	490.00
Tin	7299	455.68	456.18

THE END.

PRINTED BY W. HUGHES,
KING'S HEAD COURT, COUGH SQUARE.

Fig 2

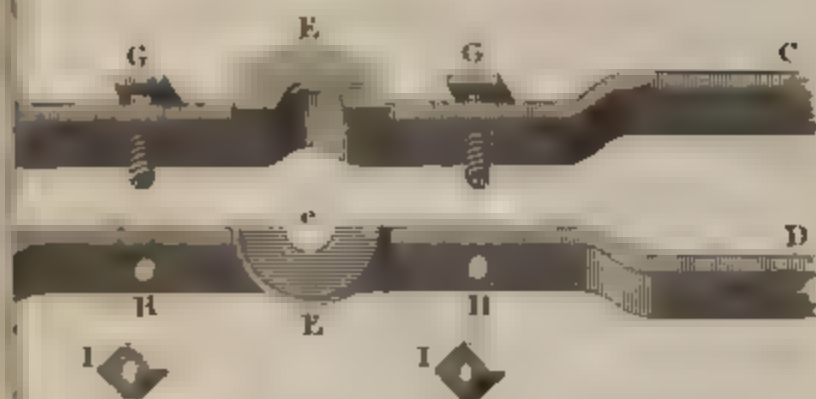


Fig 3



Fig 1





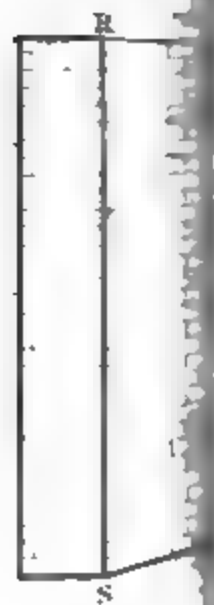


Fig 3

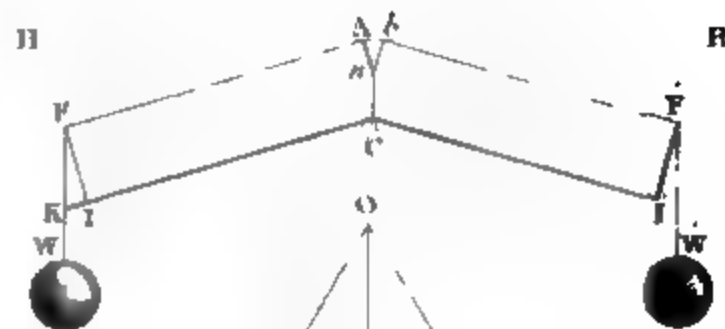


Fig 4

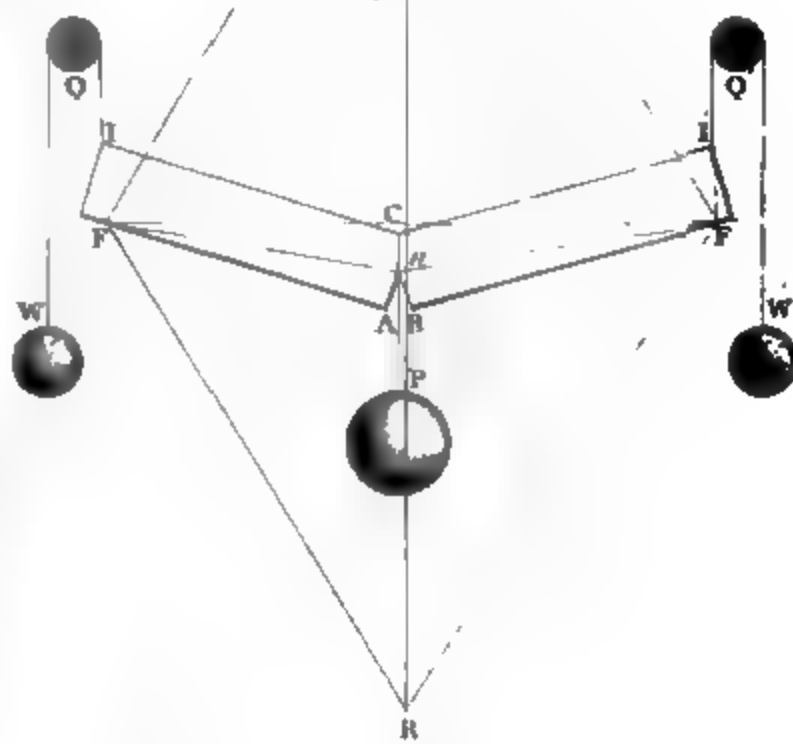


Fig 5

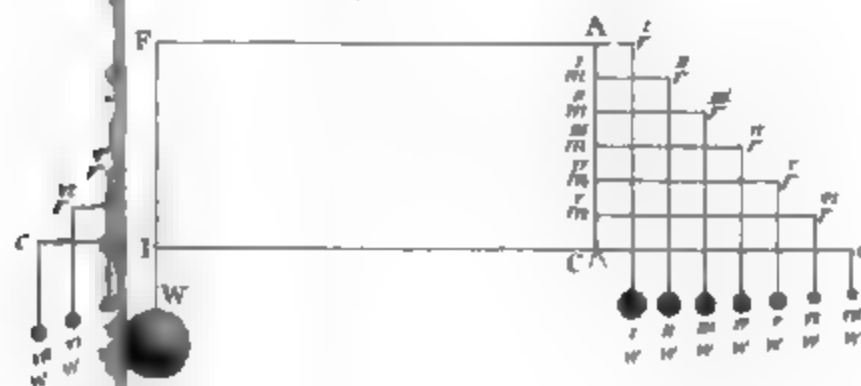


Fig. 1.

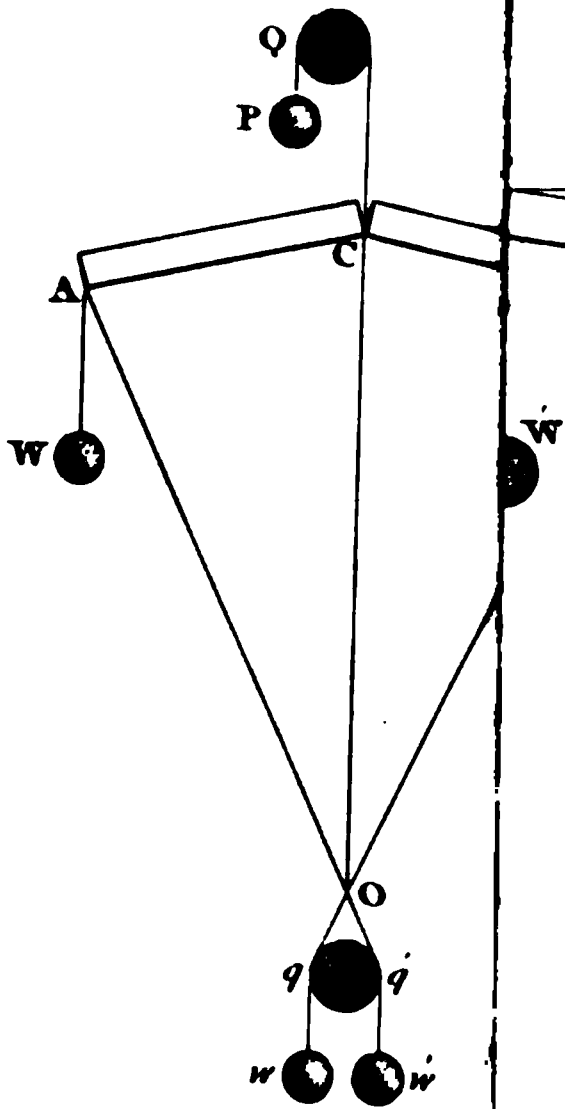


Fig. 4.

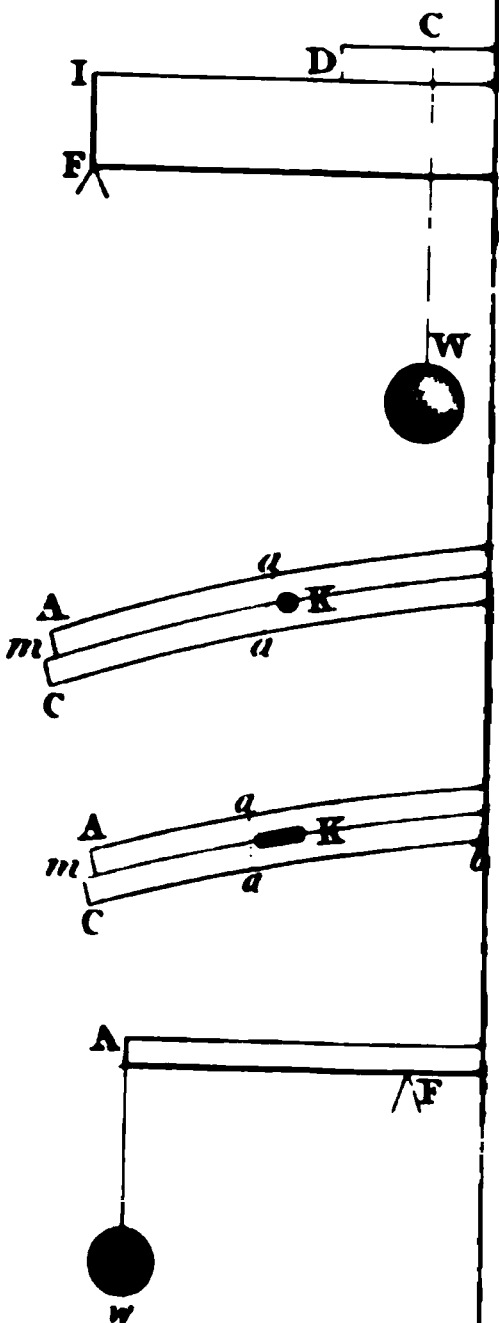


Fig. 6.

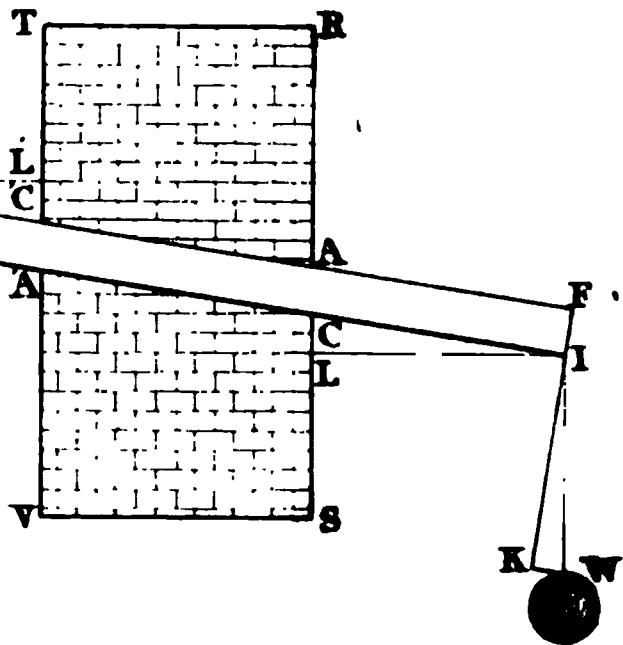


Fig. 7.

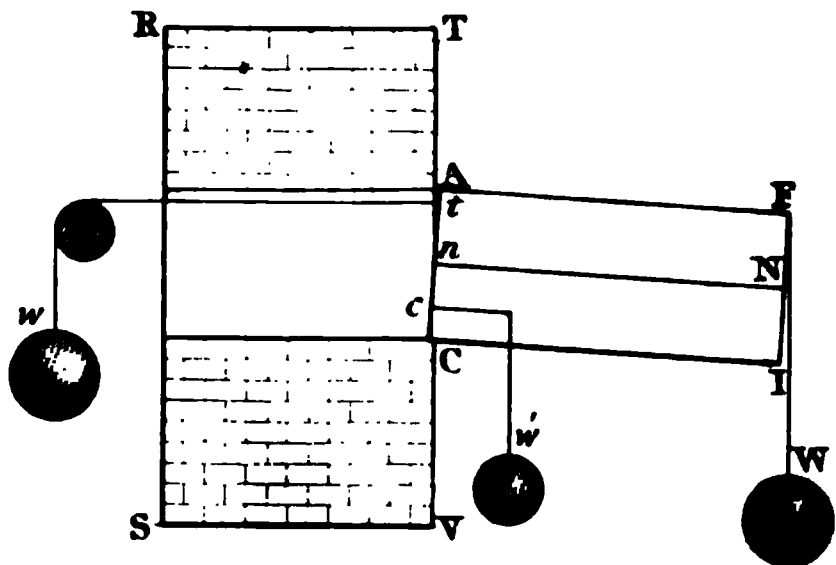
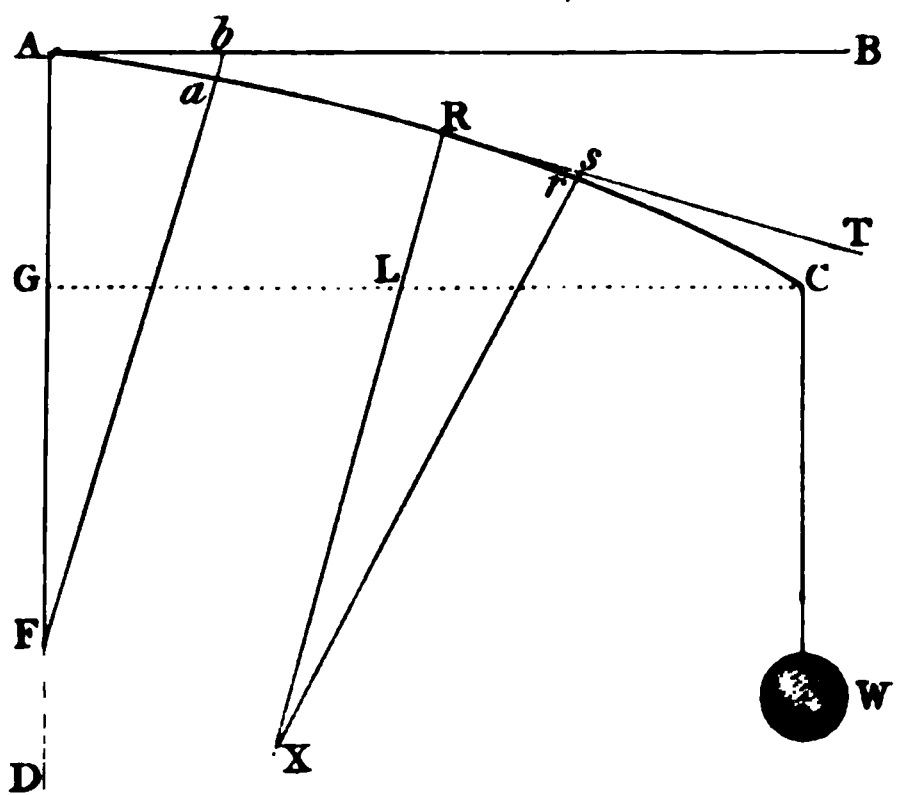
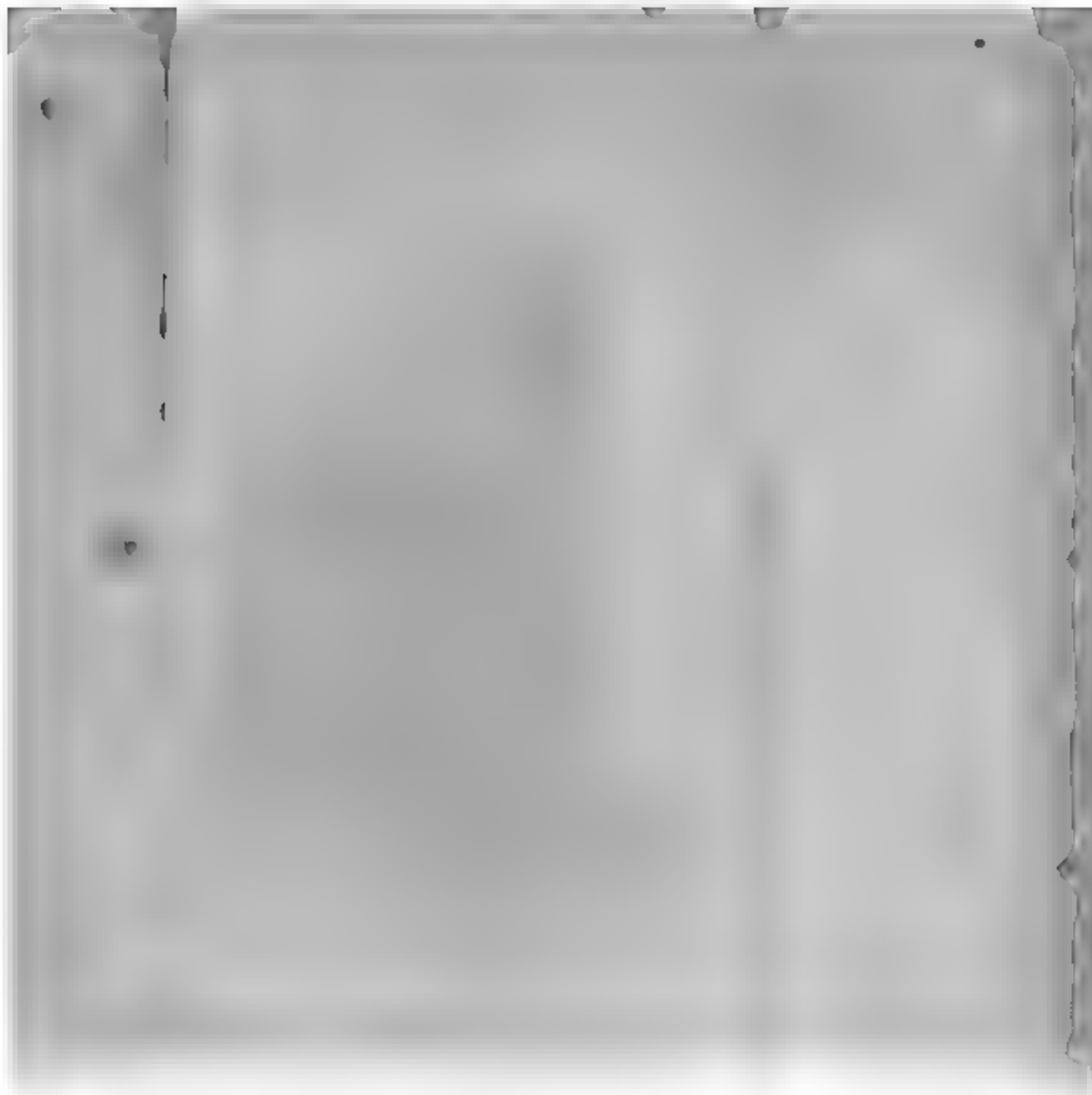


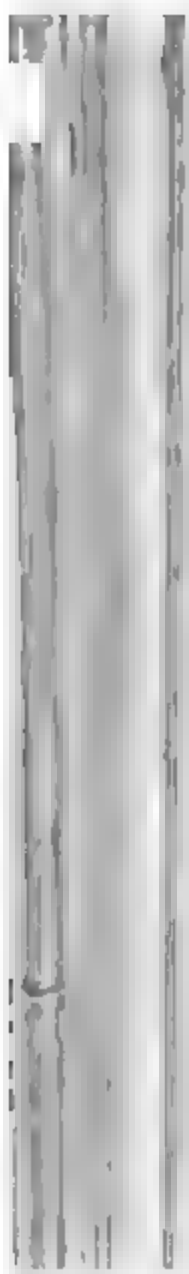
Fig. 9.

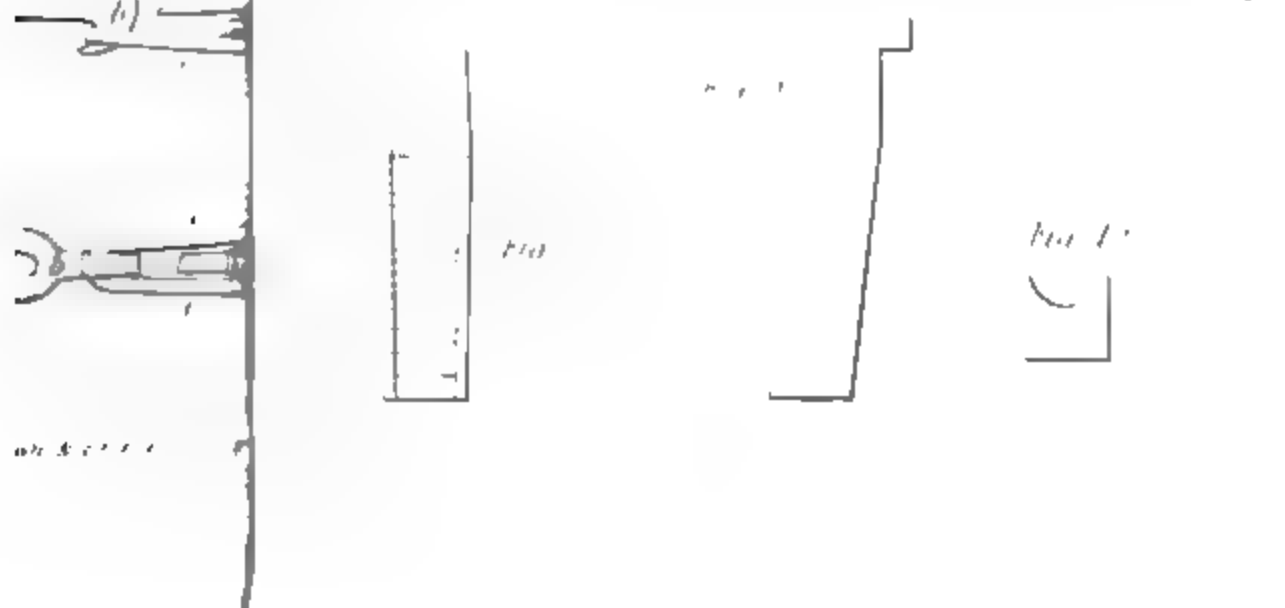
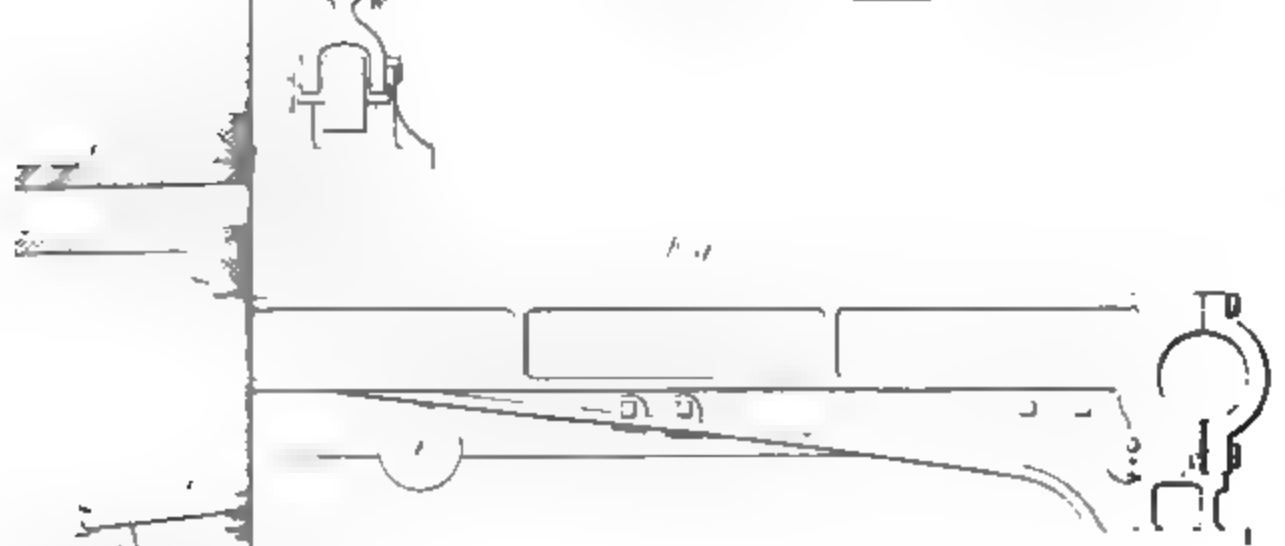
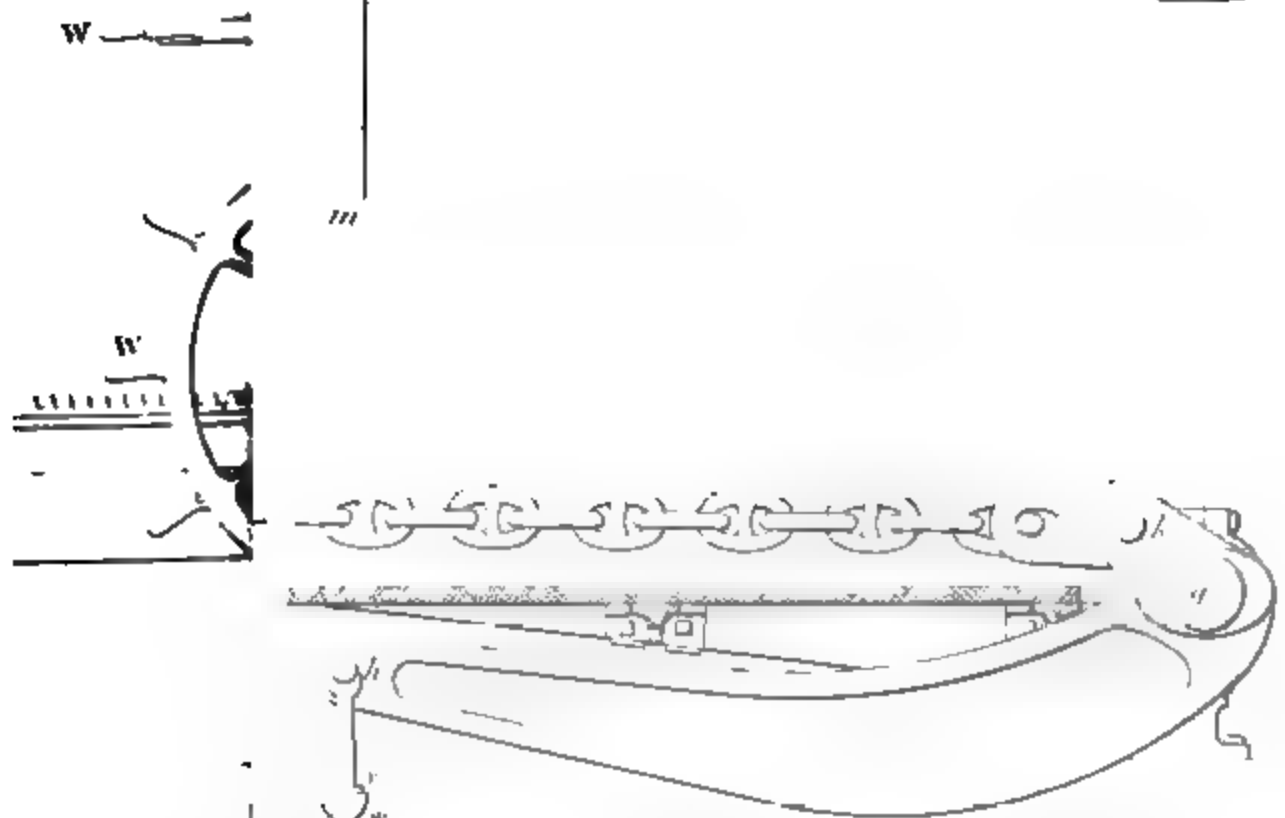
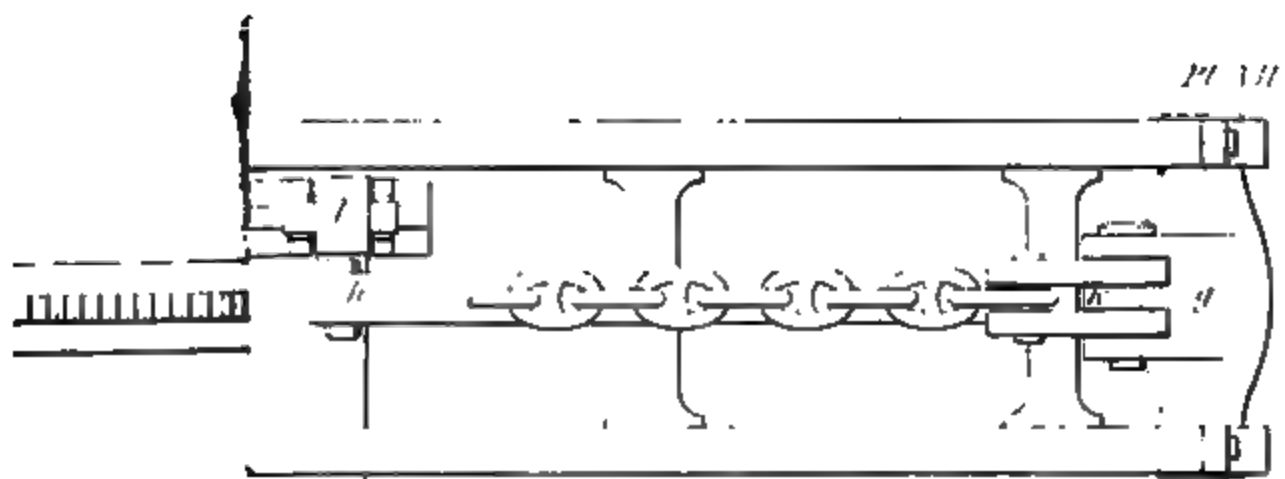












PUBLISHED BY MR. WEALE.

In 4 vols. royal 8vo, bound in 3 vols. half morocco, Price £4. 10s.

THE THEORY, PRACTICE, AND ARCHITECTURE OF BRIDGES OF STONE, IRON, TIMBER AND WIRE; WITH EXAMPLES ON THE PRINCIPLE OF SUSPENSION:

Illustrated by 138 Engravings and 92 Wood-cuts.

Divisions of the Work.

THEORY OF BRIDGES. By James Hann, King's College, London.

GENERAL PRINCIPLES OF CONSTRUCTION, &c. Translated from Gauthey.

THEORY OF THE ARCH, &c. By Professor Moseley, King's College, London.

PAPERS ON FOUNDATIONS. By T. Hughes, C.E.

ACCOUNT OF HUTCHESON BRIDGE, GLASGOW, with Specification. By Robert Stevenson, C.E.

MATHEMATICAL PRINCIPLES OF DREDGE'S SUSPENSION BRIDGE.

ESSAY AND TREATISES ON THE PRACTICE AND ARCHITECTURE OF BRIDGES.
By William Hosking, F.S.A., Arc^t. and C.E.

SPECIFICATION OF CHESTER DEE BRIDGE.

**PRACTICAL DESCRIPTION OF THE TIMBER BRIDGES, &c. ON THE UTICA AND
SYRACUSE RAILROAD, U. S.** By B. F. Isherwood, C.E., New York.

Description of the Plates—General Index, &c., &c., &c.

In 1 large and thick royal 8vo vol., with several Plates, extra cloth boards,
Price £1. 10s.

PAPERS ON IRON AND STEEL,

PRACTICAL AND EXPERIMENTAL, WITH COPIOUS ILLUSTRATIVE NOTES.

By DAVID MUSHET, Esq.,

Honorary Member of the Geological and the Quebec Literary and Historical
Societies; of the Institution of Civil Engineers of London; Corresponding
Member of the Wernerian Natural History Society, Edinburgh.

In 3 vols. 4to, with more than 200 Engravings, Price £4. 8s.

WEALE'S QUARTERLY PAPERS ON ARCHITECTURE.

The object of this entirely new work is to treat of Architecture as a fine and
useful Art; together with occasional Biography and Reviews of Books.

Professional Architects and promoters of this Art are invited to send their
contributions, addressed to Mr. WEALE.

In 3 vols. 4to, with numerous Illustrations, Price £2. 14s.

WEALE'S QUARTERLY PAPERS ON ENGINEERY AND MECHANICS.

This work embraces continually varied Papers, useful to the Civil Engineer
and Machinist. Much space is devoted to subjects foreign to the usages
of this country, illustrated by Engravings, with lucid descriptions, developing
principles and practice useful to the Practical Engineer and Student; Biography
of deceased Engineers, together with Reviews of Books, &c.

The aid of the Professional Engineer and Amateur is solicited. Communi-
cations to be addressed to Mr. WEALE.

7

WORKS PUBLISHED BY MR. WEALE.

In 8vo, upwards of 500 pages, Price 8s.

✓ **LEMENTARY INVESTIGATION OF THE THEORY
OF NUMBERS,**

Application to the Indeterminate and Diophantine Analysis, the
al and Geometrical Division of the Circle, and several other curious
al and Arithmetical Problems.

✓ **ETER BARLOW, Esq., F.R.S., M. Inst. C.E., and of several other
Learned Societies and Academies.**

In 1 royal 8vo volume, 28 Plates, extra cloth boards, Price 12s.

ENSAMPLES OF RAILWAY MAKING ;

Which, although not of English Practice, are submitted, with Practical Illus-
trations, to the Civil Engineer, and the British and Irish Public.

This Volume is divided into parts :

1. Preliminary Observations recommendatory of the adoption of a more
economical mode of Railway making in connexion with the great lines already
in operation, and a much further extension of a principle of Railways for less
advantageous traffic than the great lines afford, yet essential for the develop-
ment of the resources of the country, by employment of material of a less
cost. Landed proprietors would find their advantage in the improvement of
their land by a quick transit, and, consequently, more ready sale for their
produce. In some instances the material is on their estates.

2. Mechanical Works on the Utica and Syracuse Railroad, explanatory,
with specification and cost of this one of the best constructed Railroads in the
United States, made over swamps, creeks, and valleys, at a cost of £3600
per mile.

3. Historical, statistical, and scientific Account of the Railways of Belgium,
from 1834 to 1842, by E. Dossion, Assoc. Inst. C.E., explanatory of the Rail-
road from Ostend on the coast to Cologne in Prussia.

In 8vo, extra cloth boards, with Plates, a Second Edition, with considerable
Additions, Price 18s.

**A PRACTICAL TREATISE ON LOCOMOTIVE ENGINES
UPON RAILWAYS ;**

The construction, the mode of acting, and the effect of Engines in conveying
heavy loads ; the means of ascertaining, on a general inspection of the Machine,
the velocity with which it will draw a given load, and the results it will pro-
duce under various circumstances and in different localities ; the proportions
which ought to be adopted in the construction of an Engine, to make it answer
any intended purpose ; the quantity of fuel and water required, &c. ; with
Practical Tables, showing at once the results of the Formulæ : FOUNDED UPON
A GREAT MANY NEW EXPERIMENTS made on a large scale, in a daily practice
on the Liverpool and Manchester, and other Railways, with different Engines
and Trains of Carriages. To which is added an APPENDIX, showing the
expense of conveying Goods by means of Locomotives on Railroads.

By COMTE F. M. G. DE PAMBOUR.





TA 410 .B25 1845 C.1
A treatise on the strength of
Stanford University Libraries



3 6105 035 267 173

TA 410
B25
1845

DATE DUE

STANFORD UNIVERSITY LIBRARIES
STANFORD, CALIFORNIA
94305

